Sourcing Decisions with Stochastic Supplier Reliability and Stochastic Demand

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Supplier sourcing strategies are a crucial factor driving supply chain success. In this paper, we investigate the implications of uncertain supplier reliability on a firm’s sourcing decisions in an environment with stochastic demand. In particular, we characterize specific conditions under which a firm should choose a single versus multiple supplier sourcing strategy. In an environment with both uncertain demand and supply, we characterize the total order quantity, the number of suppliers selected for order placement, and the allocation of the total order quantity among these selected suppliers. For deeper managerial insight, we also examine the sensitivity of the optimal sourcing decisions to interactions between uncertainties in product demand and supply reliability. We show that sourcing from a single supplier is an optimal strategy for environments characterized by high levels of demand uncertainty or high salvage values. A numerical analysis based on data obtained from an office products retailer further reinforces our analytical results. In addition, we also find that when minimal order quantities are imposed, there are situations where it is not optimal to place an order with the lowest cost supplier.

Key words: sourcing; supplier selection; supply chain management

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1. Introduction

Supply mismanagement can have grave financial consequences for firms relying on suppliers for crucial items. For example, Hendricks and Singhal (2003) show that buying firms reporting supply chain disruptions due to supplier glitches typically experience a 12% decrease in shareholder returns and that the performance of these same firms is on the average much lower when compared with their pre-disruption metrics. Moreover, these authors also show that a firm suffering from supply chain disruptions also incurs long-term negative effects on its stock price (Hendricks and Singhal 2005). A firm’s sourcing strategy is typically operationalized as three key interrelated decisions: (a) the criteria for qualifying approved suppliers; (b) the supplier(s) selection from the approved base for order placement; and (c) the order quantities to place with each selected supplier (Burke et al. 2007). Although purchasing managers tend to consider multiple supplier attributes such as cost, quality, reliability, and delivery in making these decisions, industry surveys indicate that cost primarily drives these actual decisions (Verma and Pullman 1998). In line with this, Cohen and Agrawal (1999) found that while supply managers would like to develop long-term relationships, they often engage in short-term contracting based on costs to fulfill their immediate demand for products.

Even sourcing decisions made during a single selling period impact a firm’s long-term financial results. For example, a shortage of toys during a peak season due to supplier problems significantly impacted Mattel’s profitability in late 2007 (Bapuji and Beamish 2008). Thus, if firms do not assure adequate supply to satisfy the demand for a season, they may unnecessarily stock-out of items and suffer opportunity costs, or be left with an oversupply of products that must be deeply discounted. From an analytical perspective,
this points to the usefulness of the traditional news-
vendor model, which balances costs of overage and
underage to identify an ordering policy for seasonal
products with uncertain demand. However, this
approach does not explicitly consider the unreliable
or uncertain characteristic of supply. In reality, there is
substantial industry evidence (see Hendricks and
Singhal 2003 for multiple examples) which doc-
uments that after an order is placed with a supplier,
there is a chance that a proportion of that order will
not be satisfied. Less than complete order fulfillment
from a supplier may arise for various reasons in-
cluding defect rates, order rationing from supply
shortages, or a delay in transit. Regardless of the rea-
son, there is a risk that a given supplier will deliver
less than the pre-specified order quantity.

Based on this, the primary focus of this paper is on
analyzing a buying firm’s optimal supplier selection
and order allocation decisions when there is both
upstream (i.e., supply) and downstream (i.e., demand)
uncertainty. Our analysis is restricted to a single-
product and single-period setting so that we can
obtain structural insights into these decisions. More
specifically, our purpose is to: (a) determine the
optimal number of suppliers which the firm should
select for order placement; (b) structurally character-
ize the quantity to be ordered from each selected sup-
plier; (c) perform a sensitivity analysis of key param-
eters driving the supplier selection and quantity
 allocation decisions; (d) identify conditions under
which a single supplier sourcing or a multiple supplier
sourcing strategy is optimal; and (e) experimentally
analyze the impact of minimum order quantities on the
supplier selection and quantity allocation decisions.

The remainder of the paper is organized as follows.
A review of the relevant literature is included in
Section 2 and in Section 3, the profit maximizing
newsvendor model adapted to incorporate supply un-
certainty is introduced. Structural results are derived
and discussed in Section 4, and a numerical analysis
demonstrates and extends these results in Section 5.
Finally, Section 6 summarizes our conclusions.

2. Relevant Literature

Rather than review all of the current literature on
supplier sourcing, the reader is referred to Elm-
aghlayy (2000) and Minner (2003) who provide
excellent and comprehensive summaries of prior lit-
erature in this area. Because the stochastic supplier
reliability issue integrated in this paper is analogous
to the random yield problem in manufacturing, Yano
and Lee (1995) also provide an extensive review in
this domain. Given that our paper addresses sourcing
decisions under conditions of stochastic supplier yield
and stochastic demand, we review in detail prior
work which has addressed both these issues.

Gallego and Moon (1993) address the supplier yield
problem in a newsvendor framework. Specifically,
they consider minimizing the upper bound on cost in
a setting with known mean and variance of demand,
but the demand distribution is unknown. The base
case analysis is extended to consider the situation
where the buying firm pays for all units ordered from
a single source with binomial yield. They show that
this extension results in higher purchase quantities
and costs as compared with the base case. In com-
parison, our paper differs substantially because we
focus explicitly on total cost minimization without
any distributional assumptions on supplier yields.

Anupindi and Akella (1993) concentrate on supply
uncertainty and characterize several scenarios where
single sourcing is preferable to dual sourcing in such a
setting. All these scenarios examine the quantity
allocation decision between two suppliers in the
presence of supply and demand uncertainty. The sce-
nario labeled Model II is closest to that examined in
our paper where they assume that each of the two
suppliers delivers a random proportion (i.e., yield) of
the order quantity to the buyer. In the general setting,
their key result is that with equal supplier costs, it is
optimal to source from both suppliers while if sup-
plier costs are not equal, this is not necessarily the
case. For the special case of exponential demand and
normally distributed yields, they are able to show
that, regardless of the supplier cost structures, both
suppliers will receive an order quantity that is mod-
erated by the parameters of the yield distribution. In
a similar setting, this paper makes some additional
contributions. First, we consider a setting with a sig-
ificantly larger number of suppliers. Second, without
making any distributional assumptions on the sup-
plier yields, we are able to structurally characterize
the order quantity for each supplier. Finally, we are
also able to develop a simple decision rule to deter-
mine the optimal number of suppliers to select for
order placement.

Bassok and Akella (1991) introduce the Combined
Component and Production Problem (CCOPPP). The
problem is to identify ordering and production levels
of a critical component and its parent finished good
for a single period with uncertainty in both demand
for the finished good and component supply. Simi-
larly, Gurnani, Akella, and Lehoczky (2000) decide
ordering and production levels for a two component
assembly system facing random final product
demand and random yield from two suppliers, each
providing a distinct component. Using a substantively
different modeling framework, the basic result from
this stream of research (i.e., supplier diversification is
a preferred sourcing strategy) is extended in our work
for the general case of an arbitrary number of suppli-
ers. With respect to supplier characteristics, we are
also able to show that cost drives the selection of suppliers from the approved base.

Kouvelis and Li (2008) consider replenishment decisions for a constant rate demand environment from a supplier with uncertain lead times. They analyze the potential benefits of the second supplier in supplier diversification for an initial order and as an emergency response backup for a second order. Parlar and Wang (1993) analyze a two supplier setting with uncertain yields in an EOQ and newsvendor framework. They hypothesize that under varying supplier prices and different supplier yields, the buyer will place an order with both suppliers. For the EOQ setting (i.e., known demand), they obtain analytic results. For the newsvendor setting, they propose an approximate solution method to determine order quantities. In contrast, our focus is on analyzing a larger supplier base with yield and demand uncertainty in a newsvendor setting. We are also able to exactly characterize the quantity that should be ordered from each supplier selected for order placement and further show that under certain parametric conditions, it might be optimal to place an order with a single supplier.

Agrawal and Nahmias (1997) analyze the supplier sourcing decision in the presence of yield uncertainty and deterministic demand when there are fixed costs associated with sourcing from each individual supplier. They establish structural results which can be exploited to determine the number of suppliers which should be selected for order placement and lot sizes. The major obvious difference in our paper is that, when demand is uncertain but supplier fixed costs are zero, we are also able to characterize (a) which suppliers should be selected for order placement and (b) how much to order from each supplier. In addition, an experimental investigation of a multiple supplier setting with uncertain yield and uncertain demand in the presence of high supplier minimum order quantities also illustrates additional insights in our paper.

A recently published paper analyzes a scenario similar to the one adopted in this paper and also uses the newsvendor setting. Given uncertain exogenous demand, Dada et al. (2007) examine the newsvendor’s procurement problem when suppliers are unreliable. However, the focus of this paper is substantively different from ours as they examine the differences in sourcing strategies when suppliers are completely reliable as compared with when they are unreliable. Using this motivation, the authors derive results which show that the total order quantity would be greater in an unreliable supplier setting as compared with a reliable supplier setting while the end consumer service level would be lower in the former setting as compared with the latter setting. In line with the results in our paper, they also find that, in an unreliable supplier setting, supplier selection is based strictly on costs while quantity allocation for these selected suppliers is driven by each individual suppliers’ reliability parameters. We also derive some additional results for the unreliable supplier setting as compared with those presented by these authors. First, we present structural results for determining: (a) the exact number of suppliers who should receive an order; and (b) the exact quantity of units ordered from each of these suppliers. Second, we also present a sensitivity analysis of key parameters on both the supplier selection and order quantity decisions. Finally, we also identify parametric conditions under which a single supplier sourcing strategy would be an optimal choice for the firm.

In sum, we offer substantive new insights into the supplier selection and quantity allocation decisions in the presence of both downstream and upstream uncertainty. In the next section, we present our modeling framework and corresponding structural results.

3. Modeling Framework

3.1. Preliminaries

The sourcing decision we focus on consists of a two-stage supply chain with \( N \) (\( i = 1, \ldots, N \)) suppliers and a single buying firm. We assume that the \( N \) suppliers have been pre-qualified such that they all meet minimum sourcing standards set by the firm.\(^1\) The key decision variables for the firm are the number of units \( q_i \) to order from each supplier \( i \) (\( i = 1, \ldots, N \)) given that the supplier quotes a constant per unit cost of \( c_i \) to the buying firm. The optimal number of suppliers which receive an order is denoted by \( n^* \) (\( n^* \leq N \)).

In addition, the firm has some knowledge of each supplier’s historical quantity reliability. Let \( g_i(\cdot) \) denote the continuous probability density function associated with the proportional yield \( r_i \) for each supplier \( i \). To maintain tractability, we assume that this density function is twice differentiable and \( r_i \) and \( \sigma_i \) represent the mean and standard deviation, respectively. Obviously the assumed reliability distribution is such that \( r_i \) (i.e., the realized yield) is <1. In line with prior research, we also assume that \( \sigma_i \leq r_i \) or that the coefficient of variation is \( \leq 1 \) (e.g., Agrawal and Nahmias 1997 assumed that \( 3\sigma_i \leq r_i \) for a normally distributed random variable for supplier yields).

While we do not make any assumptions concerning the specific reliability distribution, the firm’s minimal reliability requirements impose some mild restrictions on the shape of the distribution. To illustrate, consider a firm that requires each of its suppliers to have an average historical yield of at least 90%. Because all realized values of the reliability distribution are >0 and <100%, it is likely that the reliability distribution will be left-skewed.
The firm makes a sourcing decision to satisfy total demand $x$, which is uncertain with density function $f(x)$ and distribution function $F(x)$. In line with the newsvendor framework, we assume that selling price per unit ($p$) is known and fixed as are the unit salvage value ($s$) for unsold stock and unit underage cost ($u$) for unsatisfied demand. The standard assumption that $p > c_i > s$ ($\forall i$) is assumed to hold for our analysis. A list of the notation used in this paper is included in Table 1.

The objective of the firm is to determine the appropriate order quantities for each supplier such that the expected profit associated with satisfying demand is maximized. Utilizing the framework from the traditional newsvendor problem (Silver et al. 1998), the objective function given below in Equation (1) maximizes the single period expected profits $E(\pi)$ for the firm. In addition, non-negativity constraints (see Equation (2)) are also included in our formulation. subject to:

$$q_i \geq 0, \quad \forall i,$$  (2)

where $Q = \sum_{i=1}^{N} r_i q_i$.

In order to obtain structural insights into optimal sourcing policies, we make the problem more tractable by assuming that demand is uniformly distributed with parameters $\{a,b\}^2$. Based on this assumption, the following key result stated in Lemma 1 below holds.

**Lemma 1:** The expected profit function shown in Equation (1) is concave in the order quantities $q_i$ when demand is uniformly distributed with parameters $\{a,b\}$.

**Proof:** See Appendix.

All of the analysis that follows is based on the result of this lemma.

### 3.2. Analysis

Before presenting our analysis, we first assume that suppliers are indexed in non-decreasing order of per unit costs (i.e., $c_1 \leq c_2 \leq \ldots \leq c_N$). Given that we have shown that our objective function is concave in the decision variables when demand is uniformly distributed with parameters $\{a,b\}$, the following conditions are necessary and sufficient to identify a global optimum to our sourcing problem:

$$\frac{\partial E(\pi)}{\partial q_i} \leq 0, \quad \forall i,$$  (3)

$$q_i \geq 0, \quad \forall i,$$  (4)

$$q_i \left[ \frac{\partial E(\pi)}{\partial q_i} \right] = 0, \quad \forall i.$$  (5)

We now proceed to derive a specific functional form for the optimal sourcing policy for the buying firm. The optimal order quantity assuming the firm knows the number of suppliers it will place an order with is defined by the result of Theorem 1 below.

**Theorem 1:** Assuming that the firm wants to source its total requirements from the first $n^*$ ($k = 1, \ldots, n^*$; $n^* \leq N$) suppliers, the optimal quantity sourced from each supplier $k$ is as follows:

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**Table 1 Notation Used in the Paper**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Number of suppliers</td>
</tr>
<tr>
<td>$q_i$</td>
<td>Quantity of units ordered from supplier $i$</td>
</tr>
<tr>
<td>$c_i$</td>
<td>Cost per unit quoted by supplier $i$</td>
</tr>
<tr>
<td>$n^*$</td>
<td>Number of suppliers for which $q_i &gt; 0$</td>
</tr>
<tr>
<td>$r_i$</td>
<td>Proportion yield for supplier $i$</td>
</tr>
<tr>
<td>$g_i(r)$</td>
<td>Probability density function associated with yield of supplier $i$</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>Standard deviation of the yield associated with supplier $i$</td>
</tr>
<tr>
<td>$x$</td>
<td>Demand (a random variable)</td>
</tr>
<tr>
<td>$f(x)$</td>
<td>Cumulative distribution function associated with demand</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Mean demand</td>
</tr>
<tr>
<td>$p$</td>
<td>Unit selling price</td>
</tr>
<tr>
<td>$s$</td>
<td>Unit salvage value for unsold stock</td>
</tr>
<tr>
<td>$u$</td>
<td>Unit underage cost (cost of not satisfying demand)</td>
</tr>
<tr>
<td>$Q$</td>
<td>Total expected quantity received by the buying firm</td>
</tr>
<tr>
<td>$a$</td>
<td>Minimum demand parameter when demand is uniform</td>
</tr>
<tr>
<td>$b$</td>
<td>Maximum demand parameter when demand is uniform</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Profit function for the firm</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>Portion of demand adjusted by the critical ratio associated with supplier $i$</td>
</tr>
<tr>
<td>$K_i$</td>
<td>Critical fractile associated with supplier $i$</td>
</tr>
<tr>
<td>$V_i$</td>
<td>Inverse of the coefficient of variation of yield for supplier $i$</td>
</tr>
<tr>
<td>$m_{0i}$</td>
<td>Minimum order quantity for supplier $i$</td>
</tr>
</tbody>
</table>
In Equation (6), 
\[
q_k^* = \frac{\frac{V_k}{\sigma_i^2} \left\{ \sum_{j=1, j\neq k}^{n^*} (\beta_k - \beta_j)V_j^2 + \beta_k \right\}}{\left( \sum_{j=1}^{n^*} V_j^2 \right) + 1},
\]
where \(\beta_i = K_i(b - a) + a; K_i = \frac{p + n - c_i}{p + n - 3},\) and \(V_i = \frac{\bar{r}_i}{\sigma_i}.
\]

**Proof:** See Appendix.

Although supplier selection and order quantity allocation is cost-based, the exact quantity ordered from each supplier is moderated by the mean and variance of the entire selected supplier set. Other specific insights which emerge based on this result are as follows:

- The firm will always order a positive quantity from the lowest cost supplier (because \(\beta_1 \geq \beta_2 \geq \ldots \geq \beta_n\)). Given that all suppliers meet minimal sourcing standards, this result points to the fact that cost is the primary driver as compared with reliability in supplier selection.

- Assuming a multiple sourcing strategy is optimal (i.e., \(n^* \geq 2\)), the lowest cost supplier will not necessarily be allocated a higher order quantity as compared with a higher cost supplier. For example, consider a case where \(n^*\) suppliers have been selected for order placement. Then it is easy to show based on Equation (6) that any supplier \(j (j = 2, \ldots, n^*)\) will be allocated a higher order quantity as compared with supplier 1 if:

\[
(b_1 - b_j) \leq \frac{\left( \sum_{k=2, k\neq j}^{n^*} \beta_k V_k^2 (\bar{r}_1 \sigma_j^2 - \bar{r}_j \sigma_1^2) \right) - \left( \sum_{k=2, k\neq j}^{n^*} V_k^2 + 1 \right)(\beta_1 \bar{r}_1 \sigma_j^2 - \beta_j \bar{r}_1 \sigma_1^2)}{\bar{r}_j (\bar{r}_1 + \bar{r}_j)}.
\]

This result is indicative of the fact that although supplier selection for order placement is cost-based, the quantity allocation is determined by cost and reliability parameters of each selected supplier.

An additional structural issue relates to whether it is possible for the firm to determine the optimal number of suppliers to select for order placement (i.e., determine \(n^*\)). This result is characterized in Corollary 1 below.

**Corollary 1:** Supplier \(n^*\) (\(n^* \leq N\)) such that for all suppliers \(k = 1, \ldots, n^*,\) \(q_k^* > 0\) can be identified such that:

\[
\beta_{n^*} > \frac{\sum_{j=1}^{n^*-1} \beta_j V_j^2}{1 + \sum_{j=1}^{n^*-1} V_j^2}
\]

and

\[
\beta_{n^*+1} \leq \frac{\sum_{j=1}^{n^*} \beta_j V_j^2}{1 + \sum_{j=1}^{n^*} V_j^2}.
\]

**Proof:** This is a direct result of observing that in Equation (6), \(q_k^* > 0\) as long as \(\left\{ \sum_{j=1, j\neq k}^{n^*} (\beta_k - \beta_j)V_j^2 \right\} + \beta_k \geq 0.\) Thus, by iteratively solving the equation starting at \(k = 1,\) we can determine the value of \(n^*\).

One consequence of this property is that if suppliers are indexed in non-decreasing order of per unit costs, then if for some supplier \(m, q_m^* = 0,\) then \(q_k^* = 0\) for all suppliers \(k = m + 1, \ldots, N.\) This result is indicative of the fact that costs rather than supplier reliabilities are the key drivers for selecting suppliers who will receive a positive order quantity. Thus, it does not really matter whether a higher cost supplier might be more reliable as compared with a lower cost supplier because the latter will always be chosen first for a quantity allocation before the former. Dada et al. (2007) derive a similar result by analyzing the Type I service levels for alternate suppliers. However, we provide precise ratios for determining the optimal number of suppliers to source from in Equations (7) and (8) above.

Corollary 1 also helps to determine when a single sourcing strategy might be optimal for the firm because such a strategy corresponds to showing that \(n^* = 1.\) If this is the case, we know that \(\beta_1 \geq 0\) (based on Equation (7)) which is always the case. Based on Equation (8) above and replacing \(n^* = 1,\) it is obvious that single sourcing is optimal when:

\[
\left( \frac{c_1}{\bar{r}_1} \right)^2 \leq \frac{(c_2 - c_1)(b - a)}{b(p + u - c_2) + a(c_2 - s)}.
\]

The expression on the left hand side of Equation (9) is the squared coefficient of variation for the lowest cost supplier, while the expression on the right hand side of the equation is an adjusted cost differential between the first and second supplier. Equation (9) also helps to identify how key parameters drive the choice of the firm towards a single sourcing strategy. When the squared coefficient of variation of the first supplier is relatively small in comparison with costs, then the first supplier will receive the complete order. Recall that the implicitly assumed reliability distribution is such that \(\sigma_i \leq \bar{r}_i\) and that \(\bar{r}_i < 1\) for all suppliers \(i.\) Therefore, in many cases, it is likely that the squared coefficient of variation is very small and hence, it is likely that a single supplier strategy is appropriate.

A key feature of the relationship shown in Equation (9) is that the expression is independent of the reliability distribution of every other supplier (i.e., for suppliers \(j = 2, \ldots, N\)). Specifically, the only parameter associated with supplier 2 is the unit cost. Consequently, if supplier 2 has relatively higher unit cost as compared with supplier 1, it is likely that a
single sourcing strategy will be optimal. An alternate explanation of this result focuses on the cost and reliability characteristics of the minimum cost supplier. If supplier 1 (i.e., the lowest cost supplier) is sufficiently reliable, then a single sourcing strategy will be optimal and no other suppliers need to be considered.

This result is consistent with single sourcing conditions with only two suppliers in Anupindi and Akella (1993). However, in our N supplier model, the demand distribution also drives the decision of whether to choose a single sourcing strategy. More specifically, higher levels of mean demand lead a firm to diversify its total order and source from multiple suppliers. In contrast, a higher degree of demand variability likely leads the firm to opt for a single sourcing strategy. Thus the uncertainty in demand and uncertainty in yield for supplier 1 both play a key role in determining whether a single sourcing strategy is optimal. Essentially, a higher variability in demand coupled with a lower uncertainty in supplier 1 yield would result in single sourcing.

In addition, we can also see the impact of other news-vendor related parameters on the firm’s sourcing strategy. From the traditional newsvendor model (i.e., without supplier choice or reliability variability), we know that in response to increases in price, underage costs, and salvage values, the buying firm should optimally increase the total order quantity. We can now develop further insights into the impact of these factors on the single vs. multiple supplier sourcing decision. In particular, if the price per unit or the underage cost increases, then the buyer may optimally source from more than one supplier. Conversely, if the salvage value per unit increases, then a single supplier sourcing strategy is more likely to be optimal. In this situation, the risk associated with buying from a single supplier is partially offset by the higher salvage value for the leftover units.

We now consider the situation where all suppliers have equivalent costs. Given equal supplier costs, let $c = c_i$ which implies $b = b_i$ for all $i$. In this case, the firm will always choose to source from all N suppliers (i.e., $n^* = N$). The quantity allocation for each supplier under this case is:

$$q_i^* = \frac{V_i b_i}{1 + \sum_{i=1}^{N} V_i^2}, \quad \forall i. \quad (10)$$

From this expression, the entire portfolio of supplier reliabilities directly impacts the order quantities such that the buying firm realizes diversification benefits. The order quantity for a particular supplier depends not only upon its unique reliability function, but also on the reliability of the other available suppliers. This optimal order quantity for an individual supplier increases in response to the following: (a) an increase in the mean reliability of that supplier, (b) a decrease in the standard deviation of reliability of that supplier, (c) a decrease in the mean reliability of other suppliers, and (d) an increase in the standard deviation of reliability of other suppliers. Consequently, the order quantity allocated to any supplier is moderated by its relative quantity reliability as compared with all other suppliers in the approved base.

To summarize the results of our analysis, when purchase cost differentials exist within the supplier pool, either a multiple or single supplier strategy could be optimal. In each of these cases we examined, a buying firm’s optimal sourcing strategy depends on trade-offs between supply cost reductions and supplier reliability profiles. Further, it is possible for the firm to optimally determine the number of suppliers $n^*$ from which it should source and appropriate allocation of its total requirements among the $n^*$ suppliers selected. When the suppliers all have similar costs, a fully diversified sourcing strategy is best. In the next section, we conduct a numerical analysis to further explore some of the interactions between supplier reliability parameters, firm level demand, and minimum order quantity restrictions on optimal sourcing strategies.

4. Numerical Analysis

4.1. Experimental Design

A numerical analysis based on data obtained from an office products retailer is shown in this section. In our Base Case the approved supplier base consists of four suppliers ($N = 4$) from which the retailer can source its requirements for a single item. Parameter settings for this benchmark case are provided in Table 2. Additional examples are also shown to illustrate the sensitivity of our model to changes in specific parameters. For Case A we consider the impact of the relative cost dominance among suppliers by changing the quoted unit cost vector from the Base Case. For Case B and Case C, we illustrate the impact of changes to salvage value ($s$) and underage cost ($u$) respectively, on optimal selection and allocation decisions. By varying demand or supplier reliability parameters one at a time, we create four more sample problems. Specifically, we vary the mean demand (Case D), the range of demand (Case E), the mean reliability of supplier 1 (Case F), and the standard deviation of the reliability of supplier 1 (Case G). Finally, we impose a minimum order quantity for supplier 1 in Case H.

4.2. Results

The summary results from our numerical experiments (see Table 3) are as follows. First, consider the impact...
of the purchase cost structures by comparing optimal sourcing strategies for the Base Case vs. Case A. The application of Theorem 1 is illustrated in this situation as we specifically identify the impact of supplier costs on the quantity allocation choice. In the Base Case, the cost differentials are significant enough such that the firm optimally places an order with the two lowest cost suppliers. In contrast, Case A’s relatively small cost differentials are such that the firm optimally places an order with the three lowest cost suppliers. Also, in terms of expected profits, having a priority supplier that is cost dominant as in the Base Case (c1 = 621) is preferable and leads to less diverse order quantity allocation profiles. Furthermore, conditions that would lead to single sourcing can be determined via Equation (8). For example, in the Base Case, any one of the following improvements in supplier 1’s profile would eliminate selection of any and all other competing suppliers: c1 ≤ 619.60 or V1 ≥ 36.5 (via σ1 or r1).

Secondly, we illustrate the impact on optimal sourcing decisions attributable to changes in salvage values (s) or underage costs (u). By comparing Case B to the Base Case, the impact of salvage value on this newsvendor’s optimal sourcing strategy can be observed. Higher salvage values tend to lead a newsvendor with upstream and downstream stochasticity to single source. As a counterbalance, a higher underage cost tends to favor multiple sourcing and oftentimes leads a newsvendor to substantially allocate quantities to higher cost suppliers (see Case C). From these two examples, we note the sensitivity of Equations (6) and (9) to a, b, r1, and D, the impact of changes in demand illustrate the analytic results shown in Equation (9). The optimal number of suppliers selected increases in response to higher levels of mean demand. Therefore, if a firm anticipates significant demand growth, it should consider enlarging its supplier base. However, due to the presence of supply risk, a news vendor firm has greater risk of exposure to a glut of product if supplier delivered quantities exceed expectations. Case E illustrates how supply risk coupled with greater demand variability leads a news vendor firm to decrease the number of suppliers it selects. In this example, to hedge the increased risk inherent in greater demand variability, the firm allocates its entire order quantity to the low cost supplier.

The impact of the first supplier’s reliability on the optimal sourcing strategy is illustrated by comparing the Base Case to Case F. In general, the low cost supplier’s coefficient of variation impacts both the number of suppliers selected and the corresponding total order quantity allocation. When the mean reliability of the first supplier is reduced, the firm sources from the same number of suppliers but allocates a smaller proportion of its total order quantity to the first supplier. Note also that the firm’s total order quantity substantially exceeds maximum demand due to the lower expected yield of the lowest cost supplier. In Case F, the increased uncertainty in reliability for the low cost supplier does not affect the

<table>
<thead>
<tr>
<th>Case ID</th>
<th>Parameter changed</th>
<th>q1</th>
<th>q2</th>
<th>q3</th>
<th>q4</th>
<th>⨿</th>
<th>Firm profit ($)</th>
<th>n^x</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Base)</td>
<td>NA</td>
<td>6950</td>
<td>1032</td>
<td>0</td>
<td>0</td>
<td>7982</td>
<td>517,487</td>
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</tr>
<tr>
<td>A</td>
<td>c = [626.5,627.5,628.5,629.5]</td>
<td>4141</td>
<td>2659</td>
<td>1179</td>
<td>0</td>
<td>7979</td>
<td>484,400</td>
<td>3</td>
</tr>
<tr>
<td>B</td>
<td>s = 600</td>
<td>8734</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8734</td>
<td>578,287</td>
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</tr>
<tr>
<td>C</td>
<td>u = 480</td>
<td>5401</td>
<td>2747</td>
<td>0</td>
<td>0</td>
<td>8148</td>
<td>410,405</td>
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</tr>
<tr>
<td>D</td>
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<td>12,654</td>
<td>2303</td>
<td>0</td>
<td>33,526</td>
<td>2,129,577</td>
<td>3</td>
</tr>
<tr>
<td>E</td>
<td>a = 6500, b = 8500</td>
<td>7638</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7638</td>
<td>471,087</td>
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<tr>
<td>F</td>
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<td>6901</td>
<td>1848</td>
<td>0</td>
<td>0</td>
<td>8749</td>
<td>514,042</td>
<td>2</td>
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<tr>
<td>G</td>
<td>σ1 = .0577</td>
<td>2779</td>
<td>5199</td>
<td>0</td>
<td>0</td>
<td>7978</td>
<td>499,861</td>
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<tr>
<td>H</td>
<td>mq1 = 10,000</td>
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<td>7975</td>
<td>0</td>
<td>0</td>
<td>7975</td>
<td>488,118</td>
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</table>
optimal number of suppliers selected. However, the low cost supplier’s share of the total order quantity suffers as the relative certainty of supplier 2’s reliability outweighs its relative unit cost deficiency. Consequently, this results in a substantial increase in the second supplier’s share of the newsvendor’s business.

Lastly, we consider the impact of minimum order quantities on the firm’s optimal sourcing strategy. Case H illustrates a situation where the lowest cost supplier also has a fairly high minimal order quantity. Here, the firm optimally sources the entire order from supplier 2 instead of suppliers 1 and 2 as in the Base Case. The total profit also significantly decreases because the firm no longer sources from the lowest cost supplier. If we consider a minimum order quantity as a surrogate for fixed order costs, then we see that cost is not always the driver for supplier selection for quantity allocation.

As a final note, we also conducted similar experiments as outlined in this section with a homogeneous supplier base. For these example problems with homogeneous suppliers, total order quantity and thus expected profit are robust with respect to changes in an individual supplier’s reliability or minimum order quantities. We believe this to be relevant for risk averse decision makers operating in an environment where consistency in performance and output is desirable.

5. Conclusions
This paper provides structural insights illustrated with numerical analysis for characterizing a news- vendor’s optimal sourcing strategy in the presence of general upstream yield and uniformly distributed downstream uncertainties. Our analytic results directly address supplier selection and order quantity allocation decisions, while our numerical results offer managerial guidelines regarding supplier qualification criteria.

In the context of the supplier selection decision, our results provide quantitative theoretical support for observed practice. For example, Verma and Pullman (1998) find that while supply managers recognize the importance of quality, cost primarily drives their supplier selection decisions. Consistent with findings in Dada et al. (2007), our basic results also show that “cost is an order qualifier.” Specifically, we find that a supplier’s cost (and not its reliability) is the key supplier selection criterion. Consequently, the lowest cost supplier will always receive a share of the total order quantity. It follows that if all pre-qualified suppliers have equivalent costs, then it is optimal to place an order with all suppliers in the pool. An exception to this rule occurs when the lowest cost supplier has a restrictively high minimum order quantity.

We also address the choice between single sourcing and multiple sourcing by integrating multiple upstream sources of supply uncertainty as well as randomness in demand. We propose a simple ratio (see Equation (9)) to analytically determine whether or not a single supplier strategy is appropriate. This ratio reflects a supply base driven trade-off between the first supplier’s reliability and its cost advantage relative to other suppliers. Essentially, if a supplier has a large cost advantage and a reliability distribution with a high mean and a low standard deviation, then a single supplier strategy is likely best. However, as illustrated in our benchmark numerical example, multiple sourcing may be optimal even when the priority (low cost) supplier also possesses the lowest covariance of reliability.

We also characterize how the newsvendor’s downstream market may influence the optimality of sourcing from a single supplier versus multiple suppliers. Our analysis favors a single supplier strategy when mean demand is low and multiple sourcing when mean demand is high. Somewhat surprisingly, an increase in the variability in demand favors a single sourcing strategy. In this situation, the buying firm tends to hedge the financial risk induced by increased demand uncertainty by leveraging the lowest cost source of supply. An additional contribution of our analysis highlights the sensitivity of optimal supplier selection and requirements allocation decisions to multiple parameters such as the cost of unsatisfied demand ($u$) and salvage value ($s$). Specifically, relatively high values of $u$ provide incentives to formulate a multiple sourcing strategy, while relatively high values of $s$ may predispose the newsvendor to single source.

We are also able to structurally characterize the optimal order quantities for selected suppliers. For given downstream market parameters, each selected supplier will receive an order amount based on its unit cost, mean reliability, and variance in reliability. Of supply side interest is that while the lowest cost supplier is guaranteed to receive a positive order, it will not necessarily receive the largest order.

Finally, some intuitive insights on the number of suppliers that the newsvendor should pre-qualify are as follows. The fixed costs of qualifying a supplier to ensure that it meets a minimal set of criteria based on quality, costs, and delivery can be exorbitant. Our analysis suggests that sourcing from a single supplier with very low costs results in a higher expected profit than other multiple-supplier-sourcing strategies. Thus, if a buying firm can source from a cost dominant single supplier, then it may be wise to develop that single supplier and make efforts to ensure that this priority supplier’s reliability, quality, and delivery are sufficient to meet the buying firm’s
needs. Again, this may be especially true if the downstream market provides relatively high salvage value for unsold product.

Our analysis also shows that if several suppliers are very close in cost or the downstream market is characterized by relatively high selling prices or unsatisfied demand costs, then the buying firm should consider qualifying and developing multiple sources. These findings demonstrate the impact of the newsvendor critical fractile parameters beyond the well-established determination of Type I service levels and attendant base stock levels. Specifically, these parameters not only influence the total quantity to source, they also significantly influence the optimal number of suppliers from which to source and the allocated shares among the selected suppliers.

There are several future areas of research related to this model which warrant further investigation. First, a more detailed model could be developed which endogenously incorporates appropriate criteria for approving suppliers. In this context an alternative model may be proposed such that suppliers selected for sourcing meet some minimum qualifying reliability parameter (i.e., supplier \( i \) is only selected for order placement such that \( \bar{r}_i \leq \text{COV} \) where \( \text{COV} \) is the maximum allowable coefficient of variation set by the firm). Second, integrating our analysis on supplier selection/quantity allocation with the pre-qualification decision might be another potential research avenue. In this case, it would be possible to integrate costs related to supplier pre-qualification and investigate the impact of these costs on conditions when single versus multiple sourcing strategies would be the preferred choice for the firm. Third, we assume that demand is uniformly distributed to facilitate the development of simplified expressions. Other types of demand distributions could be explored to enhance the generalizability of the results derived here. Finally, the focus of this model is on decision making at the buying firm; future research could incorporate the supplier’s optimal selection and selling quantity decisions in the presence of multiple downstream distribution outlets.

Acknowledgments
We would like to thank the editorial team for their guidance with this paper.

Notes
1These minimum standards could be related to quality, flexibility, and delivery since we explicitly consider supplier costs and reliabilities as parameters in our problem.

2Although the primary reason to assume it uniformly distributed is related to tractability issues, it can also be argued that given a fixed market price \( p \), demand around this fixed price is uniformly distributed.

Appendix

**Lemma 1:** The expected profit function shown in Equation (A1) is concave in the order quantities \( q_i \), when demand is uniformly distributed with parameters \( [a,b] \).

**Proof:** Given that demand is uniformly distributed with parameters \( [a,b] \), the specific functional form of Equation (A1) is:

\[
\text{Max } E(\pi) = -\frac{u(a+b)}{2} - \frac{a^2L'}{2} + \sum_{i=1}^{N} [r_i q_i (p + u - c_i)] \\
+ aL' \left[ \sum_{i=1}^{N} \bar{r}_i q_i \right] - \frac{L'}{2} \left\{ \sum_{i=1}^{N} M_i q_i^2 \right\} \\
+ \left[ 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \bar{r}_i \bar{r}_j q_i q_j \right].
\]

(A1)

where \( L' = \frac{p+u-c}{a+b} \) and \( M_i = \alpha_i^2 + \bar{r}_i^2 \). Note that in order to establish concavity of this expected profit function, we need to show that all of the \( k \) principal minors of the Hessian must alternate in sign starting with a negative value when \( k = 1 \). We now proceed to show this is true.

Given \( N \) suppliers, the Hessian is:

\[
H_N = \begin{bmatrix}
-M_1 & -\bar{r}_1 & \cdots & -\bar{r}_1 & -\bar{r}_1 \\
-\bar{r}_2 & -M_2 & \cdots & -\bar{r}_2 & -\bar{r}_2 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
-\bar{r}_{N-1} & \cdots & \cdots & -M_{N-1} & -\bar{r}_{N-1} \\
-\bar{r}_N & \cdots & \cdots & -\bar{r}_N & -M_N \\
\end{bmatrix}
\]

(L'H\_N)

The principal minor of \( H_N \) is:

\[
|H_N'| = (-L')^N \left\{ \sum_{i=1}^{N} \bar{r}_i^2 \prod_{j \neq i} \sigma_j^2 \right\} + \prod_{i=1}^{N} \sigma_i^2
\]

(A3)

Because \( \bar{r}_i > 0 \) and \( \sigma_i^2 > 0 \), the sign of the principal minor is determined by \( (-L')^N \). Now based on the standard assumptions of the newsvendor model and the fact that \( b \sim a \sim 0 \), we know that \( L' > 0 \). This implies that the sign of the determinant of \( H_N \) is simply \((-1)^N\). Now starting with:

- \( k = 1 \), the sign of \(|H_1|\) is determined by \((-1)^1 < 0\);
- \( k = 2 \), the sign of \(|H_2|\) is determined by \((-1)^2 > 0\);
- \( k = 3 \), the sign of \(|H_3|\) is determined by \((-1)^3 < 0\);

and so on. Thus, all the \( k \) principal minors of \( H_N \) alternate in sign and the expected profit function in Equation (A1) is concave. \( \Box \).
THEOREM 1: Given that the firm wants to source its total requirements from the first \( n^* \) suppliers, the optimal quantity sourced from each supplier \( k \) is as follows:

\[
q_k^* = \frac{V_k}{\alpha_k \left( \frac{\sum_{j=1, j \neq k}^{n^*} (\beta_k - \beta_j)V_j^2}{\sum_{j=1}^{n^*} V_j^2} \right) + 1}, \tag{A4}
\]

where \( \beta_i = K_i(b - a) + a; K_i = \frac{p + u - s_i}{p + a - s_i} \) and \( V_i = \frac{r_i}{a_i} \).

PROOF: We know that Equation (A1) is concave in the decision variables. Now assuming that \( q_k^* > 0 \) for \( k = 1, \ldots, n^* \), we know that through the complementary slackness condition in Equation (A5):

\[
\frac{\partial E(\pi)}{\partial q_k^*} = 0 \quad \text{for } k = 1, \ldots, n^*. \tag{A5}
\]

Solving these \( n^* \) simultaneous equations for \( q_k^* \), we obtain the result in Equation (A4). This concludes the proof of this Theorem 1.

References


