An Analysis of Recycled Content Claims under Supply Uncertainty and Demand Benefit

Aditya Vedantam
Krannert School of Management, Purdue University, West Lafayette, IN 47907, USA avedanta@purdue.edu,

Ananth Iyer
Krannert School of Management, Purdue University, West Lafayette, IN 47907, USA aiyer@purdue.edu,

Paul Lacourbe
CEU Business School, Budapest, Hungary lacourbep@ceu.edu

In this paper, we study how voluntary recycled content claims made by a manufacturer are impacted by recyclate supply uncertainty and demand benefit for recycled products from environmentally conscious businesses and governments. We identify two types of recycled content claims - product specific claims for which manufacturer makes the recycled content commitment for each product (e.g., paper napkins made from 90% post consumer recycled paper) and company wide claims where the recycled content averaged across products has to meet or exceed the claim (e.g. “[...] incorporate average of 10% recycled PET across all soft drink containers [...]”). We illustrate each type of claim with industry examples and formulate a two stage stochastic model with recourse to understand the dynamics. In the first stage, the manufacturer chooses the fraction of recycled content and type of claim to make and in stage two, it procures recyclate and virgin raw material to meet the claim. We model two sources of recyclate for manufacturer - a low cost but variable supply from municipal (local) curbside recycling and an expensive but unlimited supply from other (non local) sources. We characterize the optimal actions for each type of claim and find that a unique recycled content decision exists for general conditions of demand curve irrespective of the supply distribution. Finally, we look at conditions under which the optimal product specific claim and profits are larger than the company wide claim.

Our contribution is three fold: First, we show recyclate supply constraints impact the recycled content decision made by manufacturers. For example, we find that lowering variability of municipal supply only increases both the optimum claim and profits when non local sources of recyclate are expensive. Second, we study the demand benefit for recycled products from ‘buy recycled’ procurement and find that when the cost of non local purchases is above a threshold, a higher demand benefit from recycled content can actually decrease the recycled content claims in products. Thirdly, we describe conditions when easing supply constraints and creating demand side incentives for recycled content can create win-win conditions i.e., increasing both manufacturer profits and environmental features of products. Overall, we suggest that adopting a two pronged approach of easing recyclate supply constraints and creating demand incentives for recycled content can encourage manufacturers to design products with higher recycled content and greater profits.

Key words: Environmental Operations, Recycling, Sustainability, Voluntary Claims, Flexibility
1. Introduction

Manufacturers make recycled content claims either at the product level (‘product specific claims’), or on an aggregate level (‘company wide claims’). The latter is advertised by companies in sustainability reports, while the former appears as a label on product packaging. Companies making such voluntary claims are guided by rules set by the Federal Trade Commission (FTC) to regulate the content of environmental marketing claims. The FTC can take action, under the FTC Act, if a marketer makes a deceptive or unfair claim that is inconsistent with the guidelines\(^1\). To signal the veracity of their claims, manufacturers are also adopting third party certification to validate and increase reliability of their recycled content claims. Table 1 provides examples of companies and associated recycled content claims made at the aggregate or the product level.

<table>
<thead>
<tr>
<th>Company</th>
<th>Product</th>
<th>Claim</th>
<th>Claim Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chipotle</td>
<td>Paper napkins</td>
<td>Made from 90% post consumer recycled unbleached paper</td>
<td>Product Specific</td>
</tr>
<tr>
<td>Tide</td>
<td>Cleaning product packaging</td>
<td>Bottles made from 25% or more post-consumer recycled plastic</td>
<td>Product Specific</td>
</tr>
<tr>
<td>Starbucks</td>
<td>Coffee cup sleeves</td>
<td>85% post consumer fiber cup sleeve</td>
<td>Product Specific</td>
</tr>
<tr>
<td>Coca Cola</td>
<td>Plastic bottle</td>
<td>Source 25% of PET Plastic from recycled or renewable material by 2015</td>
<td>Company Wide</td>
</tr>
<tr>
<td>Pepsi</td>
<td>Plastic bottle</td>
<td>Incorporate average of 10% recycled PET (rPET) in primary soft drink containers in the US</td>
<td>Company Wide</td>
</tr>
<tr>
<td>Nestle</td>
<td>Plastic bottle</td>
<td>Increase recycling rate for PET bottle to 60% by 2018</td>
<td>Company Wide</td>
</tr>
</tbody>
</table>


Recycled Content Supply

Manufacturers making recycled content claims have to procure recycled input from municipal and private sources to produce the desired recycled content. However, the limited and uncertain nature of the recycled input supply from municipal sources impacts claims. For example, despite being

\(^1\) Source: Guides for the use of environmental marketing claims FTC (2014):  
1. All claims about pre- and post- consumer content in product must be substantiated.  
2. For products with partial recycled content, the percent of recycled material in product must be clearly stated to avoid consumer deception about amount of recycled content in product.
committed to make their packaging with 100% rPET, Nestle Waters North America (NWNA) was only able to incorporate 50% recycled content in all their bottled water packaging due to inadequate supply of quality rPET at reasonable prices (Washburn (2013)). In the glass and aluminum industries, beverage container manufacturers like Saint Gobain and Alcoa are facing shortages of recycled input in states like Indiana (Gazette (2012)). Part of the reason for such shortages is that the majority of recyclable feedstock ends up in landfills. In 2011, about 250 million tons of solid waste was generated in the US out of which about 35% was recycled while the rest of was either incinerated or ended up in landfills (EPA (2011)). The waste that ended up in landfills consists of potentially valuable recyclable feedstock like recycled glass (cullet), plastic (rPET), recyclable paper and aluminum cans.

The supply of recycled input is also subject to inherent uncertainties such as consumer recycling behavior and variable weather conditions for e.g., cold days where less recyclables might be put out on the curb for pickup by residents. As an illustration of this variability, we collected data from the Streets and Sanitation Department for the City of West Lafayette in 2012. Figure 1 shows the variability in the amount of curbside and drop off recyclables collected separately (not commingled) monthly by the Streets and Sanitation department in 2009. The data shows that a total of 392 tons of newspaper were collected in 2009, with an average monthly collection of 32.6 tons (coefficient of variation: 0.37). Similarly, a total of 230 tons of glass were collected, with an average monthly collection rate of 19.2 tons (coefficient of variation: 0.32). Finally, a total of 535 tons of plastic were collected, with an average monthly collection rate of 44.6 tons (coefficient of variation: 0.2). We note that the numbers reported were the amounts collected from the municipal stream...
(and not the amounts actually recovered after recycling). One could surmise that as a result of contamination and separation costs, the variability in the amount of good quality recycled input that is actually recovered is higher.

Major drivers for this variability are the operational and financial constraints involved in the municipal collection process. In many communities, collection of recyclables by municipal agencies is done through single stream recycling, under which different waste streams are commingled to reduce collection costs. This causes mixing of recycled material and leading to contamination which reduces yield. In plastics recycling, typically 5 to 15% of collected material is lost during processing because of contamination due to presence of paper labels, product residue, unwanted variety of plastic, or other materials (Selke (2006)). Further, lack of recycling laws in some states, e.g., the absence of container deposit laws for beverage containers, causes recycle rates of glass to be around 20%, as compared to states with container deposit regulation, which see return rates of over 70% (Iyer (2010). Increasing export of recycled material to countries willing to pay higher prices of recyclable feedstock is also cited as a major reason for supply constraints.

In many communities, responsibility of collection and subsequent processing of recycled input is being shared by both government, where significant curbside collection infrastructure is already in place, and industry, where manufacturers are forming public-private partnerships and industry coalitions for increasing recycling. Examples include the Recycling Reinvented Coalition (http://www.recycling-reinvented.org) in the plastics packaging industry, the Paper recycling coalition (www.paperrecyclingcoalition.com) and public private partnerships in the Glass industry (www.indianarecycling.org). Manufacturers and retailers are making it convenient for consumers to recycle by setting up separate collection bins, promoting within store recycling (e.g., Starbucks (2012)) and using reverse vending technologies, which allow for easy sortation of recyclables.

**Demand Incentives for Recycled Content Claims**

Manufacturers making recycled content claims also benefit from green procurement initiatives followed by environmentally conscious businesses and governments. The US federal government is a major buyer of recycled products through “buy recycled” procurement mandates. The Comprehensive Procurement Guideline (CPG) program in the EPA provides a list of designated products and recommends recycled content levels to look for when procuring these products. For example, in the
paper products category, the EPA recommends purchasing uncoated printing and writing paper with at least 30% recycled content (EPA 2007). Several state governments have incorporated buy recycled requirements into their procurement. California’s State Agency Buy Recycled Campaign (SABRC) is an effort to implement law requiring state agencies to buy recycled-content products.

Environmentally conscious businesses also have ‘buy recycled’ procurement initiatives to encourage products containing recycled content. For example, Best Buy’s paper procurement policy seeks to buy paper with the highest recycled content possible (BestBuy (2013)). In the construction industry, the US Green Building Council has developed a system in Leadership in Energy and Sustainable Design (LEED), whereby buildings get LEED credits for using recycled materials and inputs in construction, with the intent of increasing demand for buildings built using sustainable materials. For example, in new constructions and major renovations, the LEED system gives credits for using more recycled input: 1 credit for 10% recycled content, 2 credits for 20% recycled content and 3 credits for 30% recycled content. The credits are used to give environmental certification of LEED Certified Silver, Gold and Platinum to the buildings which command a higher rental and sale price premium over noncertified buildings and have broader demand from property investors and tenants (Council (2005), Fuerst and McAllister (2011)). Thus products with recycled content see an increasing but possibly non linear demand benefit, from governments and businesses that advocate green procurement.

**Research Goals**

Our model of the cost benefit of recycled content, and the opportunities to influence the supply of such content and associated stakeholders, is motivated by a case written by Iyer (2010). This paper looks at whether manufacturers should make recycled content claims on a product or an aggregate basis, and the impact of operational parameters such as supply variability of recycled material and demand impact of recycled content claims on manufacturer profitability and recycled content. Our research goal is the following: Given an uncertain availability of recycled material and associated demand side impact of recycled content claims, to develop a model that answers the following questions: (a) Are profit maximizing recycled content claims different under an product specific vs. aggregate commitment?, (b) How do approaches to stabilize recycled content availability and schemes to increase the demand impact of recycling claims impact recycled content claims? and, (c) Are there parameter values under which making a product specific or company wide claim,
leads to both increased profits and greater environmental impact i.e. win-win conditions?.

Production Economics associated with Recycled Content

The economics of recycling has been the subject of much interest and debate since the 90’s. For some materials (like glass) the savings in processing cost due to using recycled glass reduced virgin material usage, and lower emissions, as well as savings on disposal costs by avoiding landfill fees make recycling viable. Every ton of recycled glass (cullet) used, over a ton of raw material are saved, including 1300 pounds of sand, 410 pounds of soda ash and 380 pounds of limestone. Manufacturing glass from recycled input reduces energy costs by 2 to 3% for every 10% of cullet used in the manufacturing process and saves raw material and carbon requirements. A recent cradle to cradle life cycle assessment commissioned by the Glass Packaging Institute states that incorporating a higher fraction of cullet in production decreased the total life cycle costs of the end product. The study evaluated the impact on both transportation and production and found that the increase in emissions from transportation is more that offset by decrease in emissions from production since cullet requires much less energy to manufacture (GPI (2013)). However, for other materials like High Density Polyethylene (HDPE), the costs of processing recycled material can obliterate any other savings. Recycling of products that cost more the benefits they provide: examples being e-waste and plastics like HDPE, are not within the scope of this paper. It is natural that these industries seldom voluntarily recycle & rarely makes recycled content claims, which make them good candidates for mandated take back laws. In this paper, we focus our attention on materials used in making products with low durability like glass bottles and plastic packaging which, once collected, are viable to recycle due to production cost benefits but are limited and variable in supply owing to contamination in municipal streams.

The rest of the paper is organized as follows. In Section 2, we survey the relevant literature. In Section 3 we discuss the model assumptions and analyze product specific and company wide claims in Sections 4,5 and 6. In Section 7, we discuss the managerial implications of our work and conclude.

2. Literature Review

Previous papers have explored the impact of various types of regulation, from mandated minimum recycled content standards, to taxes on raw material, to product take back, to tax credits for purchase of recycling equipment to deposit/refund schemes and advance disposal fees, and their
impact on costs faced by businesses. Our paper explores the impact of voluntary environmental claims. We however focus on manufacturers that choose recycled claims voluntarily to maximize their profits. In this section, we summarize the literature on environmental regulation including work in the marketing and remanufacturing literature.

Palmer and Walls (1997) compare deposit refund systems with mandated minimum recycling content standards, focusing on states that have mandated manufacturers of newsprint to use a minimum amount of secondary/recycled materials in manufacturing. Atasu and Van Wassenhove (2012) explore the benefits when regulators take an operations perspective in designing environmental policy. Atasu et al. (2009) focus on the design of efficient take back legislation, when producers are responsible for end-of-life recycling of their products, satisfying a minimum government mandated recycling/collection rate. Jacobs and Subramanian (2012) model the impact of sharing responsibility for product recovery between various supply chain partners under extended producer responsibility. Both Atasu et al. (2009) and Jacobs and Subramanian (2012) consider product recovery under government mandated recycling targets. Their approach differs from ours in that, while in their case the regulator sets mandated collection/recycling targets, in our model the manufacturer voluntarily chooses a recycling claim. Ata et al. (2012) examine how regulatory mechanisms like price premiums for electricity (Renewable Energy Certificates) and capital subsidies, can affect operational decisions of waste-to-energy (WTE) firms. Assuming independent waste generated across periods, they find that depending on rural/urban settings, regulators can choose different policy instruments to align consumer and WTE firm objectives. Plambeck and Wang (2009) show how e-waste regulation affects new product introduction. Market based policy instruments have also been explored by Subramanian et al. (2007) and Kroes et al. (2012).

The impact of consumers’ increasing appreciation for environmental friendly products, and the marketing impact of sustainability claims made by corporations have been studied by Straughan and Roberts (1999), Arora and Henderson (2007) and Sen and Bhattacharya (2001). Peattie (2001) reviewed the evolution of “green marketing” and divided the process into three stages: ecological marketing, environmental marketing, sustainable marketing. Other studies find that as the ‘green consumer’ segment becomes more sophisticated they examine environmental attributes and related claims more carefully (Ginsberg and Bloom (2004), Mohr and Webb (2005)).
Our work also has thematic parallels to the literature on remanufacturing, which models consumer’s environmental friendliness through their differential appreciation for a remanufactured product versus a new product (See Subramanian and Subramanyam (2012), Atasu et al. (2008) and Debo et al. (2005)). Atasu and Souza (2011) investigate the impact of product recovery on quality choice. Guide et al. (2006) suggest that management can proactively influence product acquisition and thus impact the profitability of remanufacturing. Toktay et al. (2000) focus on the return process of a remanufacturing system using the single use camera of Kodak.

To our knowledge however, the link between voluntary recycling content claims and manufacturer operations has not been modeled in the literature. We seek to fill this gap.

3. Model & Assumptions

<table>
<thead>
<tr>
<th>Table 2 Notation.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cost and Revenue Parameters</strong></td>
</tr>
<tr>
<td>$s$</td>
</tr>
<tr>
<td>$c_{m}$</td>
</tr>
<tr>
<td>$c_{e}$</td>
</tr>
<tr>
<td>$c_{v}$</td>
</tr>
<tr>
<td>$h$</td>
</tr>
<tr>
<td><strong>Decision Variables</strong></td>
</tr>
<tr>
<td>$r$</td>
</tr>
<tr>
<td>$i \in {p, c}$</td>
</tr>
<tr>
<td>$y_{t}$</td>
</tr>
<tr>
<td>$z_{t}$</td>
</tr>
<tr>
<td><strong>Other notation</strong></td>
</tr>
<tr>
<td>$x_{t}$</td>
</tr>
<tr>
<td>$\xi_{t}$</td>
</tr>
<tr>
<td>$[\underline{A}, \bar{A}]$</td>
</tr>
<tr>
<td>$D_{i}(r)$</td>
</tr>
<tr>
<td>$\Psi_{i}(r)$</td>
</tr>
<tr>
<td>$\Pi_{t}(x_{t})$</td>
</tr>
</tbody>
</table>

We formulate the model in two stages:

**Stage One (Design Stage):** In stage one, the manufacturer voluntarily commits to using a fraction ‘$r$’ of recycled content in the product and announces whether the commitment will be met for each unit of product or on average across several batches of the product. We call the latter a
company wide claim (denoted by subscript ‘c’) and the former a product specific claim (subscript ‘p’). The manufacturer’s stage one problem is to maximize the stage one expected profit $\Psi_p(r)$ for each type of commitment:

$$\max_{0 \leq r \leq 1} \Psi_i(r), \quad i \in \{p, c\}$$

We refer to tuple $(r, i)$, where ‘r’ is the fraction of recycled content declared and ‘i’ denotes the associated commitment, as a recycled content claim. At the end of stage one, the recycled content claim made by the manufacturer determines the demand to be met in stage two, which is deterministic and depends on the fraction of recycled content and the type of commitment, is represented as $D_i(r)$ for $i \in \{p, c\}$. When there is no ambiguity about the context, we drop the subscript and the dependence on $r$ to write demand as $D$.

**Stage Two:** In stage two, the manufacturer purchases recycled input to satisfy the demand while meeting the commitment. We model this stage over $T$ periods and consider two sources of recycled input - an inexpensive but random local supply stream (municipal curbside recycling) and an expensive but unlimited non local supply of recycled input (manufacturer self collection, imports, commodity markets etc).

Figure 2 Timeline of events.
The timeline of events is as shown in Figure 2 and described below:

1. At the beginning of period $t$, the manufacturer receives a quantity of recycled input $\xi_t$ from the municipal source, where $\xi_t$ is independent and identically distributed on $[\underline{A}, \bar{A}]$ with a twice continuously differentiable distribution function $\Phi$ and density function $\phi$ with $\phi(x) > 0$ for all $x \in [\underline{A}, \bar{A}]$. The manufacturer pays $\$c_m$ per unit of recycled input received from the local source.

2. If the initial inventory is $x_t$, then $x_t + \xi_t$ is the total recycled input available to be used in production.

3. The manufacturer decides the amount of recycled input ($y_t$) to use in the current period and satisfies rest of the demand ($D - y_t$) with virgin raw material. The manufacturer can purchase recycled input from non local sources at $\$c_e$ per unit and virgin raw material at $\$c_v$ per unit.

4. Any recycled input not used in production is carried over to the next period. The manufacturer can salvage leftover recycled input in the last period at local cost.

The manufacturer thus makes operational decisions in stage two regarding quantity of recycled input to buy and amount to carry over to meet the recycled content claim made in the design stage one. Next we formulate the stage two problem for both claim types. The periods are indexed in decreasing order.

**Problem formulation for product specific claim:**
Let $\Pi_t(x_t) =$ maximum expected profit for the $t$ period problem. Then,

$$\Pi_t(x_t) = \mathbb{E}_{\xi_t} \Pi_t(x_t, \xi_t) \quad \text{where,}$$

$$\Pi_t(x_t, \xi_t) = \max_{y_t} \left[ s D_p(r) - c_m \xi_t - c_e(y_t - x_t - \xi_t)^+ - c_v(D_p(r) - y_t) - h(x_t + \xi_t - y_t)^+ + \Pi_{t-1}(x_{t-1}) \right]$$

and $$x_{t-1} = (x_t + \xi_t - y_t)^+ \quad \forall t$$

subject to $r D_p(r) \leq y_t \leq D_p(r)$ and the notation $x^+ = \max(x, 0)$. The first term in (3) is the revenue from selling $D_p(r)$ units of the finished product at price $s$ in period $t$. The manufacturer meets this demand with a mix of recycled input and virgin raw material where the recycled input is obtained from local or non-local sources. The second term is the cost of local recycled input; the third term is the cost of non local purchases when the amount to be used in production $y_t$ is greater than the available supply $x_t + \xi_t$; the remaining demand $D_p(r) - y_t$ is met with virgin raw
material; the fifth term is a cost of holding the remaining recycled input for the next period and the last term is the optimal profit for the $t - 1$ period problem with initial inventory of recycled input $x_{t-1} = (x_t + \xi_t - y_t)^+$. The constraints require that amount of recycled input used each period exceed the minimum commitment ($y_t \geq rD_p(r)$) but not exceed demand ($y_t \leq D_p(r)$). The terminal value function $\Pi_0(x) = c_m x \forall x \geq 0$. The stage one expected profit can be written as:

$$\Psi_p(r) = \Pi_T(0)$$

where $x_T = 0$ i.e., the manufacturer starts stage two with no initial inventory.

**Problem formulation for company wide claim:** For the company wide claim, we use an additional state variable, $\chi_t = $ minimum commitment left to be fulfilled in period $t$. If $\Pi_t(x_t, \chi_t) =$ maximum expected profit for the $t$ period problem with initial inventory $x_t$ and remaining commitment $\chi_t$ then,

$$\Pi_t(x_t, \chi_t) = E_t \Pi_t(x_t, \chi_t, \xi_t) \text{ where,}$$

$$\Pi_t(x_t, \chi_t, \xi_t) = \max_{y_t} \left[ sD_c(r) - c_m \xi_t - c_v(y_t - x_t - \xi_t)^+ - h(x_t + \xi_t - y_t)^+ - c_v(D_c(r) - y_t) + \Pi_{t-1}(x_{t-1}, \chi_t - y_t) \right]$$

subject to $0 \leq y_t \leq D_c(r)$ and $\chi_1 = 0$. The terminal value function is $\Pi_0(x) = c_m x \forall x \geq 0$. The difference from the product specific claim is in additional state variable and the last term of (6) where the minimum commitment for the next period is decreased by the amount of recycled input used to $\chi_t - y_t$. The constraint $0 \leq y_t \leq D_c(r)$ requires that the amount used is non negative and not exceed demand while $\chi_1 = 0$ is requires that the commitment be satisfied by the end of $T$ periods. Similar to the above formulation, we can write the stage one expected profit as:

$$\Psi_c(r) = \Pi_T(0, TrD_c(r))$$

where the manufacturer commits to using $\chi_T = TrD_c(r)$ across $T$ periods i.e., $rD_c(r)$ on average and starts stage two with no initial inventory. We next state the major assumptions in formulating the model.
3.1. Assumptions

1) Costs are stationary over time with $c_m < c_v < c_e$.

The first inequality is driven by the significantly lower processing cost of recycled input than virgin raw material. This is true of recycled glass (cullet), recycled aluminum or recycled paper pulp, where processing recycled input uses much less energy than raw material and also has indirect benefits like longer furnace/equipment life and lower greenhouse gas emissions. On average, the prices of recycled glass have also been much lower (“prices of soda ash have more than doubled in last few years” - Iyer (2010)) than virgin raw material. The second inequality is driven by significant collection costs of recycled input since the manufacturer has to set up its own in store collection of recyclables or in case of glass, import from other states with bottle bills. Lastly, we assume that the cost of procuring local recycled input $c_m$ is lower than $c_e$ since in the latter case, the collection cost is borne by the municipality which typically has a wider network of collection infrastructure (e.g. curbside recycling) for recyclables with greater penetration into residential and commercial areas and can leverage economies of scale by pooling different waste streams.

2) $D'(r) > 0$ for all $r$.

We argue that there are demand side benefits of incorporating a higher recycled content in products. Firstly, the federal government is providing several incentives through “buy recycled” procurement mandates. For example, the Comprehensive Procurement Guideline (CPG) program in the Environmental Protection Agency (EPA) provides a list of designated products and recommends recycled content levels to look for when procuring these products. In the paper products category for example, the EPA recommends purchasing uncoated printing and writing paper with at least 30% recycled content. In the construction industry, the US Green Building Council has developed a system in Leadership in Energy and Sustainable Design (LEED), whereby buildings get LEED credits for using recycled materials and inputs in construction, with the intent of increasing demand for building products that incorporate recycled materials. For instance, in new constructions and major renovations, the LEED system gives credits for using more recycled input: 1 credit for 10% recycled content, 2 credits for 20% recycled content and 3 credits for 30% recycled content. The credits are used to give LEED ratings of Certified, Silver, Gold and Platinum to the buildings which have an indirect demand impact. Another example is suppliers that see a demand lift from companies that are encouraging sustainable procurement. For example, suppliers to Walmart have a greater chance of being made the “preferred supplier” if they incorporate more recycled content in their products.
3) $\bar{A} < D(0)$

We assume that the maximum possible supply from municipal stream is less than the demand. This assumption is reasonable since there are significant yield issues in the municipal recyclables stream whereby a significant quantity of recyclables end up in landfills due to inadequate sorting at the consumer end, contamination due to mixing with other waste streams (e.g. different grades of plastic, colored glass with non colored glass etc.) or lack of adequate sorting equipment at recyclers. Further, in the long run equilibrium the supply of recycled input is constrained by the amount sold in previous periods, which also supports our assumption.

Other assumptions are that we permit carry over of recycled input and not finished product, and a unit conversion ratio between recycled input and virgin raw material to finished product. The former assumption gives more flexibility to the manufacturer while the latter can be generalized by assuming appropriate material conversion ratios.

4. Stage Two Problem

In this section we provide the optimal actions in stage two of the problem for both claim types. The recycled content fraction $r$ is known at this stage.

4.1. Optimal Policy of Product Specific Claim

**Lemma 1.** The manufacturer purchases from non local sources in period $t$ only if the available total supply $x_t + \xi_t$ is less than the minimum commitment $rD$.

Under a product specific claim, the manufacturer’s commitment requires that a fraction $r$ of the demand $D$ each period be manufactured using recycled input. Lemma 1 states that if the available total supply from on hand inventory and local sources in period $t$, is less than the minimum commitment $rD$, the manufacturer purchases from non local sources to meet the remaining commitment, but does not exceed the commitment. We expect that when $x_t + \xi_t > rD$ the manufacturer carries part of the supply after meeting the periods commitment to future periods, incurring a holding cost but potentially lowering expensive non local purchases in the future. The next proposition states that the optimal amount carried over follows a threshold-type policy.

**Proposition 1.** For a recycled content $r$, there exist a threshold $z_t(r)$ in period $t$, such that the optimal amount of recycled input used ($y^*_t$) and carried over ($z^*_t$) respectively are given by,
(y^*_t, z^*_t) = \begin{cases} 
(rD, 0) & \text{if } x_t + \xi_t < rD \quad (\text{Region } R_1) \\
(rD, x_t + \xi_t - rD) & \text{if } rD \leq x_t + \xi_t < rD + z_t(r) \quad (\text{Region } R_2) \\
(x_t + \xi_t - z_t(r), z_t(r)) & \text{if } rD + z_t(r) \leq x_t + \xi_t < D + z_t(r) \quad (\text{Region } R_3) \\
(D, x_t + \xi_t - D) & \text{if } x_t + \xi_t \geq D + z_t(r) \quad (\text{Region } R_4)
\end{cases}

where z_t(r) depends only on r i.e., independent of \xi_t, and z_T(r) \geq ... \geq z_t(r) \geq z_{t-1}(r) \geq ... \geq z_1(r) = 0.

Proposition 1 identifies the optimum amount of recycled input used and amount carried over every period. The four regions described depend on the magnitude of total supply. In Region \( R_1 \), the total supply (on hand inventory and local supply) of recycled input is less than the minimum commitment and the manufacturer purchases from non local sources to meet the commitment. In Region \( R_2 \), the total supply exceeds the commitment and the manufacturer carries over whatever is left after meeting the minimum commitment. In Regions \( R_3 \) and \( R_4 \), the manufacturer exceeds the minimum commitment. Unlike in \( R_2 \), the amount carried over in \( R_3 \) is fixed at \( z_t(r) \), which depends only on \( r \) and the index of the period, not on the realization of supply. Thus, the manufacturer carries over \( z_t(r) \) in period \( t \), and uses the remaining supply in production for meeting demand. Lastly, in Region \( R_4 \), the total supply exceeds demand and the manufacturer satisfies all the demand with recycled input and carries over the leftover recycled input to the next period. An interesting observation from the proposition is that the manufacturer might choose to exceed the commitment in some periods, especially in Regions \( R_3 \) and \( R_4 \).

We also expect that the amount carried over should depend on the relative magnitudes of the holding cost and cost of non local purchases. For example, if the holding cost of recycled input is higher than the cost of non local purchases, the manufacturer would not carry over recycled content but would purchase from non local sources whenever it is short of the commitment. Intuitively also, if the municipal supply of recycled input is likely to be ‘high’ enough to meet the commitment each period, the manufacturer would again not carry, since it can meet the commitment with the municipal supply alone. This suggests that the amount carried over should depend on the relative magnitude of the holding and purchase costs, as well as the distribution and magnitude of supply.

The dependence of the threshold on problem parameters is explored further in Section 5.

4.2. Optimal Policy of Company Wide Claim

Proposition 2. For a company wide claim \( r \), if the realization of municipal supply and remaining commitment in period \( t \) are \( \xi_t \) and \( \chi_t \), respectively, then the optimal actions are: if \( \xi_t \leq \bar{y}_t = \max(0, \chi_t - (t-1)D) \) then \( y^*_t = \bar{y}_t \), else \( y^*_t = \xi_t \) for \( 2 \leq t \leq T \) and \( y^*_1 = \max(\xi_1, \chi_1) \).
Under a company wide claim, the manufacturer has more flexibility to comply with the commitment, and can thus postpone costly decisions like non local purchases of recycled input to later periods. The flexibility afforded by aggregation means that the manufacturer does not need to carry inventory. This suggests linkages between the company wide and product specific claims, because the former provides sourcing flexibility by allowing aggregation between periods, whereas, in the latter, holding between periods provides the sourcing flexibility to meet the stated claim each period.

5. Stage One problem

In stage one, the manufacturer chooses the optimum recycled content fraction and type of claim to maximize the stage one expected profit. Since we do not assume a specific functional form of demand, \( \Psi_c(r) \) need not be concave. To gain insights, we thus first solve the stage one problem for a two period problem i.e., \( T = 2 \). In Proposition 3, we first provide conditions for existence of a unique recycled content decision \( r \) for the company wide claim, and in Proposition 4 we show analogous conditions for the product specific claim. We then use numerical results to explore the impact of many \( (T > 2) \) periods.

**Proposition 3.** If \( D_c(r) \) is log concave in \( r \) and \( 1 + \frac{D_c(1)}{D_c'(1)} > \frac{s - c_v}{c_e - c_v} \), then \( \Psi_c(r) \) has a unique optimum that is given by the unique root of,

\[
\Psi'_c(r) = \frac{2(s - c_v)}{D_c'(r)} - \frac{(c_e - c_v)}{D_c(r)} \Phi_1 \Phi_2[2rD_c(r)]
\]

where \( \Phi_1 \) is the cumulative distribution function of the random variable representing total municipal supply \( \xi_1 + \xi_2 \).

The first term in (8) is the marginal profit if all demand were to be met only with virgin raw material. \( D_c'(r) \) is the marginal demand and \( s - c_v \) is the base margin i.e., the profit margin if virgin raw material was the only input used. The second term in the expression, is the expected marginal cost, which is composed of three terms: the first term is the unit cost of non local purchases less the cost of virgin raw material; the second is the marginal commitment \( \frac{d(2rD_c(r))}{dr} = 2(rD_c'(r) + D_c(r)) \) where \( 2rD_c(r) \) is the total minimum commitment for two periods; the third term is the probability that the manufacturer will make an expensive non local purchase, which occurs with probability \( p_c(r) = \Phi_1 \Phi_2[2rD_c(r)] \) i.e., if the total municipal supply is less than the commitment. Thus, incorporating a greater recycled content in the design stage has three effects: (1) an increase
in demand leading to an increase in revenue, (2) an increase in the total minimum commitment, and (3) a greater likelihood that the manufacturer has to make non local purchases in stage two.

The manufacturer chooses the optimum recycled content fraction to balance the marginal revenue with the future expected marginal cost. The first order condition in (8) confirms the intuition that changes at the product design stage should account for the impact on expected future revenues and costs. Proposition 3 requires that the demand function \( D_c(r) \) be log-concave for the optimum recycled content to be unique. The class of log-concave demand functions are quite broad - it includes all concave functions and convex demand functions like the exponential \( (e^r) \) and power function \( (r^a, a \geq 0) \). Possibly the simplest case is the linear demand function \( D(r) = a + br, b > 0 \), which is both log-concave and log-convex. We use a linear demand function to get insights into the model, though we can extend the results to other demand functions as well.

Thus, three takeaways from the analysis so far are that: (1) the recycled content decision is unique if the demand function is log-concave in \( r \), (2) the optimum \( r \) decision involved balancing the marginal revenues with expected future marginal costs, and (3) our result does not require any specific form of the supply distribution. The conditions above provide a starting point to deriving similar conditions for the product specific claim. Note that the two period company wide claim is equivalent to a single period product specific claim - consider a manufacturer making a product specific claim over a single period in stage two with commitment \( 2rD \) and municipal supply \( \xi_1 + \xi_2 \).

Before we provide conditions for the product specific claim, we first explicitly write the amount carried over in the two period case. This in turn, provides insights into why the expected profit function has a unique maximizer. Lemmas 2 and 3 give the optimal amount carried over and the expression for the threshold \( z_2(r) \).

**Lemma 2.** For a given product specific recycled content claim \( r \) and supply realization \( \xi_2 \), the optimum carry over amount is given as follows:

\[
 z^*_2 = \begin{cases} 
 0 & \text{if } \xi_2 \leq rD \\
 \xi_2 - rD & \text{if } rD < \xi_2 \leq rD + z_2(r) \\
 z_2(r) & \text{if } rD + z_2(r) < \xi_2
\end{cases} 
\]

where \( z_2(r) \) is given by Lemma 3.

The regions are the same as that defined in Proposition 1 except that \( R_4 = \{ \emptyset \} \) since we let the starting inventory \( x_2 = 0 \) and by assumption have \( \xi_2 \leq \bar{A} < D \). In Region \( R_1 \), the manufacturer carries over no recycled input; in Region \( R_2 \), the manufacturer carries over all remaining municipal supply after meeting the minimum commitment \( \xi_2 - rD \); in Region \( R_3 \) the manufacturer carries
Figure 3  Plot of the amount carry over $z^*_2$ versus $r$ for a fixed $\xi_2$.

The curve in bold shows how the amount carried over $z^*_2$ varies with the recycled content fraction for a given supply realization $\xi_2$. When the recycled content is low (including $r = 0$), the manufacturer does not carry over recycled input; as $r$ increases the amount carried increases along $z_2(r)$ until the supply constraint is reached, after that the manufacturer carries over $\xi_2 - rD$ i.e., the amount of recycled input leftover after meeting the minimum commitment $rD$. When $r$ is sufficiently high, the manufacturer again does not carry over, instead purchases from non local sources each period to meet the claim. The figure identifies the Regions $R_1$, $R_2$ and $R_3$ and the amount carried over is always less than the demand $D(r)$. In the next lemma we provide the equation for $z_2(r)$ and see how it varies with the problem parameters.

**Lemma 3.** Let $h < c_e - c_v$. The threshold $z_2(r)$ is,

$$
z_2(r) = \begin{cases} 
0 & \text{if } 0 \leq r \leq \tilde{r}_1 \\
 r D - \Phi^{-1}\left[\frac{h}{c_e - c_v}\right] & \text{if } \tilde{r}_1 \leq r \leq \tilde{r}_2 \\
 z & \text{s.t. } -h + (c_e - c_v) \Phi[rD - z] - (h + c_v - c_m) \bar{\Phi}[D - z] = 0 & \text{if } r > \tilde{r}_2 
\end{cases}
$$
where \( \tilde{r}_1 = \left\{ r \bigg| rD = \Phi^{-1}\left[ \frac{h}{c_e - c_v} \right] \right\} \) and \( \tilde{r}_2 = \left\{ r \bigg| \bar{A} - \Phi^{-1}\left[ \frac{h}{c_e - c_v} \right] = D(1 - r) \right\} \). Moreover, \( z_2(r) \) is continuous and differentiable everywhere except at \( r = \tilde{r}_1 \) and \( r = \tilde{r}_2 \). Further, \( z_2(r) \) is non increasing in \( h \) and non decreasing in \( r \) and \( c_e \).

The last equation provides some insight into the tradeoffs in the choice of \( z_2(r) \). At \( z_2^* = z_2(r) \) the manufacturer balances the unit cost of holding a unit of recycled input with the potential of avoiding a non local purchase (if the total supply is less than the commitment in period one) and cost of holding and salvaging if the recycled input is unused (if total supply exceeds demand in period one). If \( 0 \leq r \leq \tilde{r}_1 \) i.e., if \( h > (c_e - c_v)\Phi[rD] \) then the threshold is equal to 0. In words, if the unit holding cost exceeds the (expected) savings from avoiding a future non local purchase then since carrying costs are relatively high the manufacturer has no incentive to hold. When \( \tilde{r}_1 \leq r \leq \tilde{r}_2 \), then threshold can be written in closed form as \( z_2(r) = rD - \beta \) where \( \beta = \Phi^{-1}\left[ \frac{h}{c_e - c_v} \right] \). Thus, a higher holding cost causes the manufacturer to carry over less (lower \( z_2(r) \)) while a higher recycled content level or higher costs for non local purchases causes the manufacturer to carry over more (a higher \( z_2(r) \)) to protect against uncertainty of supply in period one. It can also be seen that while \( z_2(r) \) is continuous and increasing in \( r \), the slope has breakpoints at \( r = \tilde{r}_1, \tilde{r}_2 \). Lemmas 4 and 5 provide intuitive results that will be useful in showing that existence of a unique maxima of \( \Psi_p(r) \).

**Lemma 4.** The stage one lookahead probability that the manufacturer purchases from non local sources in stage two,

\[
p_c(r) = \Phi[rD] + \int_{\xi_2 \in R_1} \Phi[rD] + \int_{\xi_2 \in R_2} \Phi[2rD - \xi_2]\phi(\xi_2)d\xi_2 + \int_{\xi_2 \in R_3} \Phi[rD - z_2(r)]\phi(\xi_2)d\xi_2,
\]

is increasing in \( r \) if \( (rD)' > D' \forall r \).

A higher per period commitment (a higher \( r \)) under the same local supply distribution should intuitively require a greater probability of future non local purchases. The first term is the probability of non local purchase in the first period and the rest of the terms represent the probability of non local purchases in the second period, conditional on the amount carried into period one, given by Lemma 2. The condition in the above lemma says that the marginal increase in the quantity of recycled input \( (rD)' \) is greater than the marginal increase in demand \( D' \). This is reasonable assumption that translates to simple conditions on the parameters of the demand function. For example, if \( D = a + br \) then, we need \( a > (2r - 1)b \) which is true if the base demand \( a \) is greater than \( b \).
**Lemma 5.** The stage one lookahead probability that there is leftover recycled input that is salvaged at the end of stage two,

\[ p_e(r) = \int_{\xi_2 \in \mathbb{R}_2} \Phi[D(1+r) - \xi_2] \phi(\xi_2) d\xi_2 + \int_{\xi_2 \in \mathbb{R}_3} \Phi[D - z_2(r)] \phi(\xi_2) d\xi_2, \tag{10} \]

is decreasing in \( r \) if \((rD)' > D' \forall r\).

Lemma 5 states that the amount of leftover recycled input, that is salvaged at the end of period one, is stochastic decreasing in \( r \). Using Lemma 4 and 5 we are now able to give conditions for the existence of a unique recycled content decision for the product specific claim.

**Proposition 4.** If \( D_p(r) \) is log concave in \( r \), \((rD_p)' > D'_p\) for all \( r \) and \( 1 + \frac{D_p(1)}{D'_p(1)} > \frac{p - c_v}{c_c - c_v} \), then \( \Psi_p(r) \) has a unique optimum which is the unique root to the first order condition,

\[ \frac{d\Psi_p(r)}{dr} = 2(s - c_v)D' - (c_c - c_v)(rD)' p_e(r) + (h + c_v - c_m)D' p_e(r) - (rD)' k(r) \tag{11} \]

where, \( p_e(r) \) and \( p_c(r) \) are as defined and

\[ k(r) = \int_{rD}^{D(1+r) - A} k_1(r, \xi_2) \phi(\xi_2) d\xi_2 + \int_{D(1+r) - A}^{rD + z_2(r)} k_2(r, \xi_2) \phi(\xi_2) d\xi_2 \]

where \( k_1(r, \xi_2) = -h + (c_c - c_v)\Phi[2rD - \xi_2] \) and \( k_2(r, \xi_2) = -h + (c_c - c_v)\Phi[2rD - \xi_2] - (h + c_v - c)\Phi[D(1+r) - \xi_2] \).

Proposition 4 provides conditions for the product specific stage one profit to have a unique maxima. We find that conditions on the demand function, specifically that the demand is log-concave in \( r \), the rate of increase of the amount of recycled input required to meet the claim is greater than the increase in demand, and the cost of non local purchases be sufficiently high, are sufficient to ensure a unique maximizer. Note that we do not require conditions on the supply distribution except that it be continuous and differentiable.

### 6. Pareto Improving Conditions and Sensitivity Analysis

We next provide sufficient conditions under which making a product specific claim can generate greater manufacturer profits and that the consumers see a higher recycled content claim i.e., a win–win solution. We then perform sensitivity analysis on the recycled content decision. For simplicity, we will work with linear demand functions, \( D_c(r) = a + b_cr \) and \( D_p(r) = a + b_pr \), although the results can be generalized to other log-concave functions as well. The base demand \( a \) is the same for both claim types, but the rate of increase of demand can be different: \( b_p \) for the product specific claim and \( b_c \) for the company wide claim where \( b_p, b_c > 0 \).
Proposition 5. If $b_p > b_c$ and $c_e < c_v + \frac{b_v(s - c_v)}{\sqrt{a^2 + 4b_vA}}$ then $r^*_p > r^*_c$ and $\Psi^*_p > \Psi^*_c$.

If the cost of non local purchases is low, and purchasing programs provide preference to product specific recycled content claims over aggregate claims, then a manufacturer that makes a product specific claim is more profitable and chooses a higher recycled content fraction that if it made a company wide claim. For example, in the context of beverage manufacturers that often make aggregate claims due to supply issues like lack of availability of recycled PET, this means that if the federal government adopts ‘green purchasing’ rules that gives preference to product specific recycled content claims then manufacturers can be motivated to make product claims which are win-win for manufacturer and consumers. In Proposition 5 we examine the sensitivity of the demand side benefit on the optimum company wide profits and recycled content claim.

Proposition 6. The optimal stage one expected profit $\Psi^*_c$ is increasing in the demand benefit $b_c$. If $c_e < c_v + \frac{b_v(s - c_v)}{\sqrt{a^2 + 4b_vA}}$ then $r^*_c = \min\left(\frac{s - c_v}{2(c_e - c_v)} - \frac{a}{2b_c}, 1\right)$ is non decreasing and concave in $b_c$. However, there exists a threshold $\bar{c}_e$ such that if $c_e > \bar{c}_e$ then the optimum recycled content $r^*_c$ is decreasing in $b_c$.

The proposition states that while ‘buy recycled’ procurement always increases manufacturer profits, it might not always lead to greater recycled content. Indeed, when non local purchases are expensive and municipal supply is constrained, demand side incentives do not cause the manufacturer to choose a greater recycled content in product design - it is instead optimal for the manufacture to lower recycled content claims. This confirms the intuition that supply constraints could impact designs by fostering manufacturer choices with lower recycled content in products, even with demand incentives.

However, if the costs of non local purchases are sufficiently low, and demand side incentives are provided, win-win situations are obtained, whereby the manufacturer obtains a higher profit from incorporating a greater fraction of recycled content in the product design. It is also interesting to note in this region, that the optimal recycled content is a concave function of the demand benefit, suggesting that there are decreasing returns to scale, so ‘green procurement’ based on recycled content would have a smaller impact on design after a point. For example, in states with bottle bills, the manufacturers have lower cost of non local purchases. In such states, providing incentives through buy recycled purchasing can act a driver to increase recycled content claims and manufacturer profits.
The next proposition examines the value of reducing the variability of municipal supply. Recently, municipal agencies have moved towards consolidating the recyclables stream, called “single stream recycling”, by which consumers dispose recyclable plastics, paper, glass etc. in a single bin. It is argued that by reducing the sorting effort on part of the residents, the municipality can encourage residents to divert greater amount of recyclables from the regular waste stream (trash) that ends up in landfill. However, commingling of recyclables into a single bin also leads to significant contamination, which creates random yields from the recycling process. In the next proposition, we examine how increased supply variability impacts the the optimum recycled content design decision.

**Proposition 7.** Under a mean preserving transformation \( X_\alpha = \alpha X + (1 - \alpha)\mu \) where \( X = \frac{\xi_1 + \xi_2}{2} \) and \( \mu \) is the mean of the municipal supply per period, the manufacturer’s expected profit increases with decreasing supply variability. The optimal recycled content however, decreases in supply variability if \( c_e - c_v > \frac{b_e(s - c_v)}{\sqrt{a^2 + 4b_c\mu}} \) and increases otherwise.

The random variable \( X \) represents the average municipal supply over two periods and \( X_\alpha \) is the mean preserving transformation of \( X \). When \( \alpha = 0 \), \( X_0 = \mu \) and as \( \alpha \) increases less weight is placed on the mean and more on \( X \). In other words, as \( \alpha \) increases the random variable “spreads out” around the mean. Under such a transformation, Proposition 7 states that lowering the variability of the supply stream, keeping mean the same, increases both manufacturer profits and recycled content claims only if the cost of non local purchases is higher than a threshold. If instead, the cost is lower than the threshold, then reducing variability decreases the recycled content claim. Since non local purchase cost is low, the manufacturer would prefer the upside of a higher realization of a more variable supply, hence would increase the claim if variability increases.

7. **Numerical Analysis**

In this section we provide numerical results for larger numbers of time periods (i.e., \( T > 2 \)). We consider discrete values for the recycled content fraction between 0 and 1, with a step size of 0.01, i.e., \( r \in \{0, .01, .02, ..., .99, 1\} \). The local supply was chosen to be normally distributed with mean \( \mu = \{30, 80\} \) and standard deviation \( \sigma = 5 \). The base demand \( a = 100 \), so that \( \Phi[D(r)] \geq \Phi[a] \approx 1 \). We use a linear function to link demand to recycle content claim, \( D_p(r) = a + b_pr \). The other parameter values were set at \( s = 1, c_v = 0.1, c_m = 0.01, h = 0.1, a = 100, b_e = 10 \) unless otherwise
Figure 4  Plot of thresholds for the product specific claim with $T = 4$ and $s = 1, h = 0.1, c_v = 0.1, c_m = 0.01, c_e = 2, a = 100, b_p = 10, r = 0.5$. $\xi$ is normally distributed across all periods with mean $\mu = 30$ and $\sigma = 5$. 

(a) Plot of thresholds vs $t$ for different $c_e$ ($r = 0.5$) 
(b) Plot of thresholds vs $c_e$ ($r = 0.5$) 
(c) Plot of thresholds vs $t$ for different $h$ ($r = 0.5$) 
(d) Plot of thresholds vs $h$ ($r = 0.5$) 
(e) Plot of thresholds vs $t$ for different $r$ 
(f) Plot of thresholds vs $r$
Figure 5  Plot of optimum product specific profit with respect to \( r \) for \( T = \{1, 2, 3, 4\} \) and \( D_p(r) = a + b_p r \) where 
\[
a = 100, \ b_p = 10, \ s = 1, \ c_v = 0.1, \ c_m = 0.01, \ c_e = 0.2, \ h = 0.1 \quad \text{and} \quad \xi \text{ is normally distributed across all periods}
\]
with \( \mu = 30, \sigma = 5 \).

mentioned. We explore how the optimal recycled content claim \( r^*_p \) and profits \( \Psi^*_p \) vary with parameters.

In Figure 4, we plot the thresholds \( z_t(r) \) for \( t = 2, 3, 4 \) when the length of time horizon is \( T = 4 \). We see in Figures 4(a), (c), (e) the thresholds \( z_t(r) \) are increasing in \( t \). This is intuitive since we expect that closer to the end of the horizon less recycled input will be held for the future and more would used up in the current period. When \( r = 0 \), as in Figure 4(e), the thresholds are all 0 since nothing is carried over when no claim is made and demand is known in advance. Figures 4(b), 4(d) and 4(f), we plot each period’s threshold for different values of holding, non local purchase cost and the recycled content fraction. We seek to illustrate the trade off between holding cost and expensive purchases.

In Figure 4(b), the thresholds in each period are plotted against range of costs \( c_e \). We find that as \( c_e \) increases, the thresholds increase, which is intuitive, since the manufacturer carries over more each period to avoid expensive purchases to meet future commitments. Similarly in Figure 4(d),
Vedantam, Iyer, Lacourbe: Recycled Content Claims 60(0), pp. 000–000. © 2014 INFORMS

<table>
<thead>
<tr>
<th>( T = 1 )</th>
<th>( T = 2 )</th>
<th>( T = 3 )</th>
<th>( T = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_p^* )</td>
<td>( \Pi_p^* )</td>
<td>( r_p^* )</td>
<td>( \Pi_p^* )</td>
</tr>
<tr>
<td>( c_e = 2 )</td>
<td>( b_p = 10 )</td>
<td>0.21</td>
<td>94.5</td>
</tr>
<tr>
<td>( b_p = 15 )</td>
<td>0.21</td>
<td>94.5</td>
<td>0.22</td>
</tr>
<tr>
<td>( b_p = 20 )</td>
<td>0.21</td>
<td>94.5</td>
<td>0.22</td>
</tr>
<tr>
<td>( b_p = 40 )</td>
<td>0.21</td>
<td>94.5</td>
<td>0.23</td>
</tr>
<tr>
<td>( c_e = 0.2 )</td>
<td>( b_p = 10 )</td>
<td>0.33</td>
<td>95.2</td>
</tr>
<tr>
<td>( b_p = 11 )</td>
<td>0.33</td>
<td>95.2</td>
<td>0.34</td>
</tr>
<tr>
<td>( b_p = 12 )</td>
<td>0.33</td>
<td>95.2</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Table 3 Table showing the optimum PS & CW claim and profit for \( b_c = 10 \) and \( b_p = 10, 15, 20, 40 \) for \( c_e = 2 \) and \( b_p = 10, 11, 12 \) for \( c_e = 0.2 \). The other parameter values are: \( a = 100, s = 1, c_e = 0.1, c_m = 0.01, h = 0.1, \mu = 30, \sigma = 5 \).

<table>
<thead>
<tr>
<th>( T = 1 )</th>
<th>( T = 2 )</th>
<th>( T = 3 )</th>
<th>( T = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_p^* )</td>
<td>( \Pi_p^* )</td>
<td>( r_p^* )</td>
<td>( \Pi_p^* )</td>
</tr>
<tr>
<td>( c_e = 2 )</td>
<td>( \sigma = 5 )</td>
<td>0.21</td>
<td>94.4</td>
</tr>
<tr>
<td>( \sigma = 3 ) &amp; 0.24</td>
<td>94.8</td>
<td>0.22</td>
<td>94.5</td>
</tr>
<tr>
<td>( c_e = 0.2 )</td>
<td>( \sigma = 5 ) &amp; 0.33</td>
<td>95.1</td>
<td>0.34</td>
</tr>
<tr>
<td>( \sigma = 3 ) &amp; 0.31</td>
<td>95.2</td>
<td>0.32</td>
<td>95.18</td>
</tr>
</tbody>
</table>

Table 4 Table showing the effect of variability on optimum product specific claim and profits for \( \sigma = 5, 3 \) and \( b_p = 10 \). The other parameter values are: \( a = 100, s = 1, c_e = 0.1, c_m = 0.01, h = 0.1, \mu = 30 \).

<table>
<thead>
<tr>
<th>( T = 1 )</th>
<th>( T = 2 )</th>
<th>( T = 3 )</th>
<th>( T = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_p^* )</td>
<td>( \Pi_p^* )</td>
<td>( r_p^* )</td>
<td>( \Pi_p^* )</td>
</tr>
<tr>
<td>( c_e = 2 )</td>
<td>( b_1 = 10 )</td>
<td>0.67</td>
<td>103</td>
</tr>
<tr>
<td>( b_2 = 20 )</td>
<td>0.64</td>
<td>108.5</td>
<td>0.63</td>
</tr>
<tr>
<td>( c_e = 0.2 )</td>
<td>( b_1 = 10 )</td>
<td>0.77</td>
<td>103.8</td>
</tr>
<tr>
<td>( b_2 = 20 )</td>
<td>1.0</td>
<td>110.8</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 5 Table showing the effect of increasing demand benefit on optimum product specific claim and profits for \( a = 100, s = 1, c_e = 0.1, c_m = 0.01, h = 0.1, \mu = 80, \sigma = 5 \).

as the holding cost increases, the thresholds decrease i.e., as holding becomes more expensive the manufacturer carries over less for future claim requirements. Both of these figures illustrates the trade off between holding and purchasing costs to meet future claim requirements. Every unit of recycled input carried incurs a unit holding cost but possibly avoids a expensive future purchase. Figure 4(f) also illustrates that as the ex-ante recycled content claim \( r \) increases, it increases the minimum commitment for all periods, and the manufacturer accordingly carries over more each period.

Next, we report the optimum profits and recycled content for different lengths of the time horizon \( T \). Starting from a single “big” period \( T = 1 \) we increased \( T \) by splitting the original period into smaller and smaller time intervals. Correspondingly, we also split the demand and supply
appropriately each period. Thus, for $T = 1$ if the mean supply is $\mu$, standard deviation is $\sigma$ and demand is $D$ then for each period of the $T > 1$ period problem, the mean supply is $\frac{\mu}{T}$, standard deviation is $\frac{\sigma}{\sqrt{T}}$ and demand is $\frac{D}{T}$. This allows us to compare the optimum profits and claims as the frequency of audits increases.

In Figure 5, we plot the stage one expected profit $\Psi_p(r)$ with respect to $r$ for varying time horizons $T = 1, 2, 3, 4$. For small values of the recycled content claim, the revenue side benefit of increasing the recycled content dominates the marginal cost and the expected profit is increasing. As the recycled content increases, the operational cost of non local purchases and holding starts dominating the revenue and the expected profit decreases. As $T$ increases, the manufacturers optimum profits decrease since the audits are more frequent and the manufacturer needs to be compliant over more periods, thus incurring more compliance costs. We also note that commitment over a longer time horizon causes the manufacturer to make a greater recycled content claim. This is explained through Table 3 where optimum company wide and product specific claim and profits are tabulated for two values of non local purchase costs: a “high” cost $c_e = 2$ and a “low” cost $c_e = 0.2$. In Table 3, we increase the demand benefit $b_p$ keeping $b_c = 10$ constant, to check the impact of increasing the audit frequency on $r_p^*$ and conditions when the product specific claim is win win.

We first note that increasing the frequency of audits does not change the total commitment so the optimum company wide claim and profits do not change. These values are shown in first two columns after $b_p$. When $c_e$ is high ($c_e = 2$) increasing $T$ decreases the optimum claim $r_p^*$. When $c_e$ is ”low” ($c_e = 0.2$) the opposite effect is seen i.e., the $r_p^*$ is increasing. Both these together, support our intuition that a tighter supply constraint (higher $T$ albeit compliance over shorter periods) can increase the claim when the cost of compliance is not too large. The profits however decrease, which is expected since the number of periods requiring compliance increases. We show the product specific win-win conditions in bold. We observe here that when supply constraints are eased (low $c_e$) a smaller increase in demand is enough to create win win for all $T$‘s i.e., when $c_e = 0.2, b_p = 12$ is sufficient whereas for $c_e = 2$, a higher $b_p$ is required. Also note that higher frequency of audits could translate into a higher demand benefit from environmentally conscious consumers who value claims which are tighter (more stringent). The observation here is thus that manufacturers who make product specific claims that are audited more frequency i.e., a higher $T$, see a demand benefit (a higher $b_p$) and consequently, also choose a higher recycled content and generate higher profits.
Thus, greater standards of self compliance by manufacturers can pay off through higher profits and being more sustainable.

In Table 4, we investigate the role of variability on the optimal product specific recycled content claim and profits. We fixed the parameter values as earlier and let $\sigma$ take the values $\sigma = 3, 5$ i.e., coefficient of variation $\rho = 0.1, 0.16$ respectively. At $c_e = 2$, decreasing variability increases the recycled content claim and expected profit. When $c_e = 0.2$, then decreasing variability, decreases the recycled content claim which confirms with our intuition from Proposition 7. In Table 5 we attempt to show that it is possible that a greater demand side benefit decreases the recycled content. To illustrate this we chose a higher value of $\mu = 80$ and increased $b_p$ from $b_1 = 10$ to $b_2 = 20$ for two different range of costs. At the low cost $c_e = 0.2$, doubling demand benefit immediately pushed the recycled content to 1, which is expected since the revenue side benefits far outweigh the costs. When $c_e$ is high however we see that increasing $b_p$ does not always increase the claim, in fact strictly decreases the recycled content claim for $T = 1, 2$, while keeping the recycled content claim unchanged for $T = 3$, even though profits are improved in all cases. The takeaway is that the federal government needs to be careful when encouraging “buy recycled” procurement: supply side constraints in terms of amount of municipal supply available and cost of non local purchases need to be considered in order to get win win situations.

8. Insights for Managers

Manufacturers incorporate recycled content in products to realize the associated production cost benefits and benefit from the demand side impact. We have seen examples where businesses and governments provide preferencial purchasing of products with environmental attributes like recycled content. Walmart, for example, ranks suppliers according to its sustainability index giving an indirect demand side benefit to suppliers of products with recycled content. The federal government procures sustainable products through its environmentally preferable purchasing program. But the choice of type of claim, company wide or product specific, impacts the associated supply required. We show that there are parameter regions where the supply constraints impact incentives and can actually increase recycled content claims.

We also find that though ‘buy recycled’ procurement always increases manufacturer profits, it might not always lead to greater recycled content. Indeed, when supply is constrained and external or non local purchases of recycled input are expensive, demand side incentives do not succeed in incentivizing the manufacturer to choose a greater recycled content in product design - in fact, the
manufacture lowers the recycled content. In the procurement of recycled glass, states like California that have legislation like bottle bills see recycle rates of 80% as compared to states like Indiana which see rates as low as 10% due to lack of such legislation. Our model allows us to say that in states like Indiana, where the cost of non local purchases is much higher, green procurement by businesses and government may not have a positive impact on recycled content use in products because the effect of supply constraints dominates that of any demand benefit. In fact, we find that manufacturer’s might lower the recycled content in products despite a demand boost when supply constraints are active. Hence, government should focus on easing supply constraints before giving demand incentives.

The next question we try to answer is whether there is value in reducing the variability of municipal supply. Recently, municipal agencies have moved towards consolidating the recyclables stream, called “single stream recycling”, by which consumers dispose recyclable plastics, paper, glass etc. in a single bin which is picked up by municipality. It is argued that by reducing the sorting effort for residents, the municipality can encourage residents to divert greater amount of recyclables from the regular waste stream (trash) that ends up in landfill. However, commingling of recyclables in a single bin also leads to significant contamination which increases downstream sorting costs and reduces the yield of the recycling process. Contrary to the expectation that lower recycled content variability should increase recycled content, we find that when cost of non local purchases is low, lowering variability, keeping the mean constant, actually decreases the recycled content in products i.e., higher variability increases the claim. These results suggest that regulators need to adapt strategies to encourage more environmentally friendly product design by ensuring manufacturer profitability and taking into account the supply constraints for individual industries. When the cost of non local purchases is high, lowering supply variability provides the desired environmental benefit. But, when the cost of non local purchase of recycled content is sufficiently low, the regulator should increase demand side benefit to recycled content use through, for example, ‘buy recycled’ procurement. This also suggests the menu of products that should be candidates in federal governments green purchasing - encourage products whose cost of non local purchases is low, and for products with high costs, ease supply constraints before influencing demand.

Concerted effort on easing supply constraints and creating demand side incentives for recycled content can also have another beneficial effect: manufacturers making company wide claim can be incentivized to make product specific claims while incorporate greater recycled content in each
product. In the context of beverage manufacturers that often make aggregate claims and not product level claims due to lack of availability of recycled PET, this means that ‘green purchasing’ rules that gives preference to higher recycled content claims on the product level, can motivate manufacturer to make product claims instead of average claims. This of course, needs that the supply constraints not be too tight i.e., the manufacturer’s cost of collecting recycled input should not be too high.

9. Conclusion
This paper looks at the impact of variable yield in the municipal supply stream on sustainable product design, modelled by the the fraction of recycled content incorporated by the manufacturer. We examine how the fraction of recycled content is influenced by supply constraints and demand for products with recycled content. We formulate and solve a two stage model, where in the first stage, the manufacturer chooses the recycled content \( r \) either on an aggregate level (averaged across a number of products) or on an individual product level. We called the latter a ‘product specific’ claim and the former a ‘company-wide claim’ and gave examples from industry of each type.

In the second stage, the manufacturer makes operational decisions regarding the amount of recycled input to purchase over and above the municipal supply and the amount of recycled input to use now versus carry over to future periods. The challenge with the non local purchases and holding decisions, are that they are dependent on the supply realizations i.e., constrained by the amount of supply available each period and the minimum commitments. We find that the optimal policy of the manufacturer making a product specific claim is a supply dependent threshold policy: when the supply is lower than a threshold , buy non local recycled content; if the supply of recycled material locally is higher, then carry over everything until a higher threshold, When supply is higher than the upper threshold carry a fixed amount that is dependent on \( r \). For the company-wide claim, the manufacturer’s optimal policy is a postponement policy: Use up municipal supply each period and postpone non local purchases as far out as possible.

After characterizing the optimal policy for each claim type, we solved the stage one problem of choosing the optimal claim and whether the claim is made on an individual or an aggregate basis. We find that by restricting the demand function be log-concave in the recycled content, we are able
to show that resulting objective has a unique optimum, irrespective of the supply distribution. This allows us to perform a numerical analysis comparing the different claim types and also performing sensitivity of the optimal recycled content claim. The broad results of our work are as follows:

1. There are regions where the optimal product recycled content claim is greater than the company wide claim. Thus, more stringent supply constraints may provide the incentive to increase recycled content claims.

2. We find that ‘buy recycled’ procurement that encourages increased use of recycled content generates win-win outcomes for the manufacturer and the environment when the cost of non local purchases is lower than a threshold. However, we find that when the cost is sufficiently ‘high’, a higher demand benefit can decrease the recycled content in products.

3. The effect of variability of supply on recycled content claims depends on the cost of non local purchases. When the cost is high, lowering variability of supply increases the recycled content decision and improves the profits. However, when cost is low the opposite effect is seen: lowering variability decreases the recycled content.

These observations suggest that regulators need to adopt calibrated strategies to encourage more environmentally friendly product design while ensuring manufacturer profitability, while taking into account supply constraints for recycled content. Thus, concerted effort to easing supply constraints through lowering cost of non local purchases and creating demand side incentives for recycled content can also have another beneficial effect: manufacturers making company wide claims can be incentivized to make product specific claims while incorporating greater recycled content in each product.

10. APPENDIX A

Proof of Lemma 1

In period $t$, if $x_t + \xi_t \leq rD$ then $x_t + \xi_t \leq rD \leq y_t$. From (3) we have,

$$\Pi_t(x_t, \xi_t) = sD - c_m \xi_t - c_e(rD - x_t - \xi_t) - c_v(D - rD) + \Pi_{t-1}(0)$$

(12)

where $y_t = rD$ since $c_e > c_v$ by assumption. If $x_t + \xi_t \geq rD$ and $y_t > x_t + \xi_t$ it is straightforward to see that the expected profit is decreasing in $y_t$. Thus, if $x_t + \xi_t \geq rD$ we must have $y_t < x_t + \xi_t$.

The manufacturers problem (3) can then be written as,

$$\Pi_t(x_t, \xi_t) = \max_{y_t} \left[sD - c_m \xi_t - h(x_t + \xi_t - y_t) - c_v(D - y_t) + \Pi_{t-1}(x_t + \xi_t - y_t)\right]$$

(13)
subject to \( rD \leq y_t \leq D \). Let, \( z_t = x_t + \xi_t - y_t \) be the amount carried over. The problem can be rewritten as,

\[
\Pi_t(x_t, \xi_t) = \max_{z_t} \left[ sD - c_m \xi_t - h z_t - c_v (D - x_t - \xi_t + z_t) + \Pi_{t-1}(z_t) \right]
\]

subject to \((x_t + \xi_t - D)^+ \leq z_t \leq x_t + \xi_t - rD\). Thus, if \( x_t + \xi_t \geq rD \) the manufacturer solves the equivalent problem,

\[
\max_{z_t} G_t(z_t)
\]

where \( G_t(z_t) = -(h + c_v)z_t + \Pi_{t-1}(z_t) \),

subject to \((x_t + \xi_t - D)^+ \leq z_t \leq x_t + \xi_t - rD\) and carries over \( z^*_t = \text{arg max}_{z_t} G_t(z_t) \) to the next period. \( \square \)

Proof of Proposition 1

We first show that the result is true for period two. Then we assume that it true for period \( t \) and show that it holds in period \( t+1 \). By induction, the result is true for all \( t \). In the the final period the manufacturer will use up all the recycled material available and carry leftover recycled input to the end of the period to salvage at cost. The expected profit for an initial inventory \( x_1 \) is,

\[
\Pi_1(x_1) = \mathbb{E}_{x_1 + \xi_1 \leq rD} \left[ sD - c_m \xi_1 - c_v (rD - x_1 - \xi_1) - c_v (D - rD) \right] + \mathbb{E}_{x_1 + \xi_1 \leq D} \left[ sD - c_m \xi_1 - c_v (D - x_1 - \xi_1) \right]
\]

which gives,

\[
\Pi'_1(x_1) = c_v + (c_e - c_v) \Phi[rD - x_1] - (h + c_v - c_m) \Phi[D - x_1]
\]

Since \( \Pi'_1(x_1) \) is decreasing in \( x_1 \), \( \Pi_1(x_1) \) is concave. In period two, if \( x_2 + \xi_2 \leq rD \) then from Lemma 1, \( y_2^* = rD \). If \( x_2 + \xi_2 > rD \) the manufacturer solves (15). Using (17) we have,

\[
G'_t(z_2) = -h - c_v + \Pi'_1(z_2) = -h + (c_e - c_v) \Phi[rD - z_2] - (h + c_v - c_m) \Phi[D - z_2]
\]

Since \( G_t(z_2) \) is concave in \( z_2 \), the optimum can be obtained by setting (18) equal to 0 and subject to \( x_2 + \xi_2 - D \leq z_2 \leq x_2 + \xi_2 - rD \). Noting that (18) is independent to \( x_2 \) and \( \xi_2 \). Define \( z_2(r) \) as the unique positive root of (18). If (18) does not have a positive root then set \( z_2(r) = 0 \). Then we have,

\[
(y_2^*, z_2^*) = \begin{cases} (rD, 0) & \text{if } x_2 + \xi_2 < rD \\ (rD, x_2 + \xi_2 - rD) & \text{if } rD \leq x_2 + \xi_2 < rD + z_2(r) \\ (x_2 + \xi_2 - z_2(r), z_2(r)) & \text{if } rD + z_2(r) \leq x_2 + \xi_2 < D + z_2(r) \\ (D, x_2 + \xi_2 - D) & \text{if } x_2 + \xi_2 \geq D + z_2(r) \end{cases}
\]

This proves the proposition for the two period problem. Since nothing is carried over is period one we can say \( z_1(r) = 0 \). Let us assume that it true for any period \( t - 1 \) \( (t \geq 2) \) i.e., let
A1) $\Pi_{t-1}(x)$ is concave in $x$ with $\Pi'_{t-1}(x) \leq c_e$ for all $x$.

A2) $z_t(r) \geq z_{t-1}(r) \geq z_{t-1}(r) = 0$

We will use A1) and A2) to show,

B1) $$
(y^*_t, z^*_t) = \begin{cases} 
(rD, 0) & \text{if } x_t + \xi_t < rD \\
(rD, x_t + \xi_t - rD) & \text{if } rD \leq x_t + \xi_t < rD + z_t(r) \\
(x_t + \xi_t - z_t(r), z_t(r)) & \text{if } rD + z_t(r) \leq x_t + \xi_t < D + z_t(r) \\
(D, x_t + \xi_t - D) & \text{if } x_t + \xi_t \geq D + z_t(r)
\end{cases}
$$

and,

B2) $\Pi_t(x)$ is concave in $x$ with $\Pi'_t(x) \leq c_e$ for all $x$.

B3) $z_{t+1}(r) \geq z_t(r) \geq z_{t-1}(r) = 0$

Induction assumption (A1), implies concavity of $G_t(z_t)$ in (15) which proves (B1). We now show (B2) using (B1). The period-$t$ expected profit is,

$$
\Pi_t(x) = E_{x+\xi \leq D} \left[ sD - c_m\xi - c_e(rD - x - \xi) - c_v(D - rD) + \Pi_{t-1}(0) \right] \\
+ E_{rD \leq x+\xi \leq rD+z_t(r)} \left[ sD - c_m\xi - h(x + \xi - rD) - c_v(D - rD) + \Pi_{t-1}(x + \xi - rD) \right] \\
+ E_{rD+z_t(r) \leq x+\xi \leq D+z_t(r)} \left[ sD - c_m\xi - hz_t(r) - c_v(D - x - \xi + z_t(r)) + \Pi_{t-1}(z_t(r)) \right] \\
+ E_{x+\xi \geq D+z_t(r)} \left[ sD - c_m\xi - h(x + \xi - D) + \Pi_{t-1}(x + \xi - D) \right]
$$

which gives,

$$
\frac{d\Pi_t(x)}{dx} = E_{x+\xi \leq rD}(c_e) + E_{rD \leq x+\xi \leq rD+z_t(r)}(-h + \frac{d\Pi_{t-1}(x + \xi - rD)}{dx}) \\
+ E_{rD+z_t(r) \leq x+\xi \leq D+z_t(r)}(c_v) + E_{x+\xi \geq D+z_t(r)}(-h + \frac{d\Pi_{t-1}(x + \xi - D)}{dx})
$$

To show $\Pi_t(x)$ is concave, we will show that $\frac{d\Pi_t(x)}{dx}$ is decreasing in $x$ for any given $\xi$ i.e., $\Pi_t(x, \xi)$ is concave in $x$. This implies that $\Pi_t(x)$ is concave in $x$, since a convex combination of concave functions is concave. We show this for $x \leq rD$. The proof for other ranges of $x$ is similar. For a given $\xi$,

$$
\frac{d\Pi_t(x)}{dx} = \begin{cases} 
 c_e & \text{if } x + \xi < rD \\
-h + \frac{d\Pi_{t-1}(x + \xi - rD)}{dx} & \text{if } rD \leq x + \xi < rD + z_t(r) \\
c_v & \text{if } rD + z_t(r) \leq x + \xi < D + z_t(r) \\
-h + \frac{d\Pi_{t-1}(x + \xi - D)}{dx} & \text{if } x + \xi \geq D + z_t(r)
\end{cases}
$$

By (A1), $\Pi'_t(x) < c_e \implies -h + \frac{d\Pi_{t-1}(x + \xi - rD)}{dx} < c_e$. Since $\Pi'_{t-1}(x)$ is decreasing in $x$ we have $\Pi'_{t-1}(x + \xi - D) > \Pi'_{t-1}(z_t(r)) = h + c_v$ for $rD \leq x + \xi < rD + z_{t+1}(r)$, (from (15)). Thus,

$-h + \frac{d\Pi_{t-1}(x + \xi - D)}{dx} > c_v$. Similarly, if $x + \xi \geq D + z_t(r)$ then, $\Pi'_{t-1}(x + \xi - D) \leq \Pi'_{t-1}(z_t(r)) = h + c_v \implies -h + \frac{d\Pi_{t-1}(x + \xi - D)}{dx} \leq c_v$ for $x + \xi \geq D + z_t(r)$. Moreover, both $\Pi'_{t-1}(x + \xi - rD)$ and
Using (15), it is sufficient to show that the manufacturer purchases non local recycled content up to $\chi$ input to meet demand local purchases since the manufacturer cannot meet the commitment even with using only recycled supply.

Finally, in the last period, the manufacturer cannot postpone non local purchases, so if $\xi$ every unit of recycled input used up in the current period, goes towards meeting the commitment. in production. Note that the manufacturer never carries over recycled input to next period, since purchases until the last period. If average across periods, the manufacturer can maximize profits by deferring expensive non local

The proof follows by noting that since the company wide claim requires the claim to be met on average across periods, the manufacturer can maximize profits by deferring expensive non local purchases until the last period. If $\xi_t \leq \chi_t - (t-1)D$ however, the manufacturer cannot defer non local purchases since the manufacturer cannot meet the commitment even with using only recycled input to meet demand $D$ in each of the remaining $t-1$ periods. Thus, if $\xi_t \leq \chi_t - (t-1)D$ then the manufacturer’s purchases upto $y^*_t = \chi_t - (t-1)D$ and follows up by purchasing upto $D$ in each of the future periods; if $\xi_t > \chi_t - (t-1)D$ the manufacturer uses up all the municipal supply, $y^*_t = \xi_t$, in production. Note that the manufacturer never carries over recycled input to next period, since every unit of recycled input used up in the current period, goes towards meeting the commitment.

Finally, in the last period, the manufacturer cannot postpone non local purchases, so if $\xi_t \leq \chi_t$ the manufacturer purchases non local recycled content up to $\chi_t$, else it uses up all the municipal supply.

\[ \Pi'_{t-1}(x + \xi - D) \text{ are decreasing in } x. \] Thus, since we have shown for a given $\xi_t$, $\frac{d\Pi_t(x)}{dx}$ is decreasing in $x$ with $\Pi_t(x) < c_v$, which implies (B2).

Now, we use (A2) to show (B3). If we show that $0 = G'_{t+1}(z_{t+1}(r)) < G'_{t+1}(z_t(r))$, then we have $z_{t+1}(r) > z_t(r)$. Combining with (A2) would give (B3). But we know that $G'_t(z_t(r)) = 0$. Therefore, it is sufficient to show that $G'_{t+1}(z_t(r)) > G'_{t}(z_t(r)) = 0$

Using (15),

\[ G'_{t+1}(x) = -h - c_v + \frac{d\Pi_t(x)}{dx} = -h + E_{x+\xi \leq r}(c_v - c_v) + E_{r \leq x+\xi \leq r+D}(r-h-c_v + \frac{d\Pi_{t-1}(x+\xi - rD)}{dx}) \]

\[ + E_{x+\xi \geq D+z_t(r)}(-h - c_v + \frac{d\Pi_{t-1}(x+\xi - D)}{dx}) \]

(21)

\[ G'_t(x) = -h - c_v + \frac{d\Pi_{t-1}(x)}{dx} = -h + E_{x+\xi \leq r}(c_v - c_v) + E_{r \leq x+\xi \leq r+D}(r-h-c_v + \frac{d\Pi_{t-2}(x+\xi - rD)}{dx}) \]

\[ + E_{x+\xi \geq D+z_{t-1}(r)}(-h - c_v + \frac{d\Pi_{t-2}(x+\xi - D)}{dx}) \]

(22)

The first two terms are the same in both expressions. Since $z_t(r) > z_{t-1}(r)$ and $G'_t(z_t(r)) > G'_{t-1}(z_t(r))$. Also, at $x = z_t(r)$, the last term in the expression for $G'_{t+1}(x)$ is 0 since $\xi < A < D$ while the corresponding term in $G'_t(x)$ is negative. Putting everything together gives us $G'_{t+1}(z_t(r)) > G'_t(z_t(r))$ and completes the induction. \[ \square \]

**Proof of Proposition 2**

The proof follows by noting that since the company wide claim requires the claim to be met on average across periods, the manufacturer can maximize profits by deferring expensive non local purchases until the last period.
Proof of Proposition 3

We first derive the two period expected profit for the company wide claim. Note that the starting inventory \( x_2 = 0 \) in the analysis below. From (7) we have,

\[
\Psi_c(r) = \Pi_2(0, 2rD) = \mathbb{E}_{\xi_2} \Pi_2(0, 2rD, \xi_2)
\]

where \( \chi_2 = 2rD \). From Proposition 2, we have \( \bar{y}_2 = \max(0, \chi_2 - D) = \max(0, (2r - 1)D) \). Thus we take two cases: If (i) \( 0 \leq r \leq \frac{1}{2} \) then \( \bar{y}_2 = 0 \) and \( \bar{y}_2 = \xi_2 \). Substituting in (6) we get,

\[
\Pi_2(0, 2rD, \xi_2) = sD - c_m \xi_2 - c_v(D - \xi_2) + \Pi_1(0, 2rD - \xi_2)
\]

Similarly, if (ii) \( \frac{1}{2} \leq r \leq 1 \) then,

\[
\Pi_2(0, 2rD, \xi_2) = \begin{cases} 
  sD - c\xi_2 - c_v(2rD - D - \xi_2) - c_v(2D - 2rD) + \Pi_1(0, D) & \text{if } \xi_2 \leq (2r - 1)D \\
  sD - c\xi_2 - c_v(D - \xi_2) + \Pi_1(0, 2rD - \xi_2) & \text{if } \xi_2 > (2r - 1)D
\end{cases}
\]

Bu Assumption 3, we know that \( \xi_1 \leq \bar{A} < D \) which gives \( \Pi_1(0, D) = sD - c_m \xi_1 - c_v(D - \xi_1) \). Also, \( \Pi_1(0, 2rD - \xi_2) = sD - c_m \xi_1 - c_v(2rD - \xi_2 - \xi_1) - c_v(D - 2rD + \xi_2) \) if \( \xi_1 < 2rD - \xi_2 \) else, \( \Pi_1(0, 2rD - \xi_2) = sD - c_m \xi_1 - c_v(D - \xi_1) \). Combining, the two period expected profit and simplifying we get,

\[
\Psi_c(r) = \mathbb{E}_{\xi_1 + \xi_2 \leq 2rD}[2sD + (c_v - c_m)(\xi_1 + \xi_2) - c_v(2rD - \xi_2 - \xi_1) - c_v(2D - 2rD)] \\
+ \mathbb{E}_{\xi_1 + \xi_2 > 2rD}[2sD - c_m(\xi_1 + \xi_2) - c_v(2D - \xi_2 - \xi_1)]
\]

Thus, the two period profit of the company wide claim can be written as,

\[
\Psi_c(r) = 2(s - c_v)D + 2(c_v - c_m)\mu - (c_v - c_e)\mathbb{E}_{\xi_1 + \xi_2 \leq 2rD}[2rD - \xi_2 - \xi_1]
\]

which gives,

\[
\frac{d\Psi_c(r)}{dr} = 2(s - c_v)D' - 2(c_v - c_e)(rD')P[\xi_1 + \xi_2 \leq 2rD]
\]

Writing the above as,

\[
\frac{d\Psi_c(r)}{dr} = 2(c_v - c_e)D' \left[ \frac{s - c_v}{c_v - c_e} - (r + \frac{D'}{D}) \Phi_{12}[2rD] \right]
\]

If \( 0 \leq rD \leq A \), clearly the last term is 0 and \( \frac{d\Psi_c(r)}{dr} = 2(s - c_v)D' > 0 \). For \( rD > A \), \( (r + \frac{D'}{D}) \Phi_{12}[2rD] \) is strictly increasing in \( r \) if \( \frac{D'}{D} \) is increasing in \( r \) i.e., \( D \) is log-concave in \( r \). Further, if at \( r = 1 \),
(r + \frac{D'}{D})\Phi_{12}[^2rD] = 1 + \frac{D'}{D(1)} > \frac{s-c_v}{c_v-c_e} \text{ then there exists an } r^* \in [r\Delta, 1] \text{ that is a unique root of (8).} \hspace{1cm} \Box

\textbf{Proof of Lemma 2}

Similar to the earlier proof we set the starting inventory \(x_2 = 0\). From the proof of Lemma 1 we know that if \(x_2 + \xi_2 \leq rD\) i.e., \(\xi_2 \leq rD\) then \(z_2^* = (x_t + \xi_t - y_t^*)^+ = 0\). If \(\xi_2 > rD\) then the optimum carry over can be obtained by solving (15)-(18) gives the corresponding first order condition:

\[ G_2'(z_2) = -h - c_v + \Pi'(z_2) = -h + (c_e - c_v)\Phi[rD - z_2] - (h + c_v - c_m)\Phi[D - z_2] = 0 \]

subject to \(0 \leq z_2 \leq \xi_2 - rD\) (since \(\xi_2 \leq \bar{A} < D\) by assumption). The expression above is strictly decreasing in \(z_2\), therefore it crosses 0 at a unique point. If \(h < (c_e - c_v)\Phi[rD]\) then the root either belongs to the feasible region or is to the right of it. Accordingly, if \(z_2(r)\) is this unique root, then if \(z_2(r) \geq \xi_2 - rD > 0\) then \(z_2^* = \xi_2 - rD\). If \(0 \leq z_2(r) \leq \xi_2 - rD\) then \(z_2^* = z_2(r)\). If \(h > (c_e - c_v)\Phi[rD]\) then the root is to the left of the feasible region (\(\Phi[D] = 0\)) we have \(z_2^* = 0\). Define \(z_2(r) = 0\) for \(h > (c_e - c_v)\Phi[rD]\) gives us the result. \hspace{1cm} \Box

\textbf{Proof of Lemma 3}

From Lemma (3), if \(0 \leq r < \bar{r}_1\) then \(z_2^* = z_2(r) = 0\). Consider following regions for \(z_2\) in (18):

1. If \(rD - z_2 \leq A\) then \(G_2'(z_2) = -h - (h + c_v - c_m)\Phi[D - z_2] < 0\). Thus, the first order condition cannot cross zero in this region.

2. If \(D - z_2 \geq \bar{A}\) then,

\[ G_2'(z_2) = -h + (c_e - c_v)\Phi[rD - z_2] = 0 \implies z_2 = rD - \Phi^{-1}[\frac{h}{c_e - c_v}] \]

If \(rD - \Phi^{-1}[\frac{h}{c_e - c_v}] \leq D - \bar{A}\) then \(z_2(r) = rD - \Phi^{-1}[\frac{h}{c_e - c_v}]\) which is the region \(\bar{r}_1 \leq r < \bar{r}_2\). For \(\bar{r}_2 \leq r \leq 1\), \(z_2(r)\) is the unique positive root of (18) but it cannot be expressed in closed form. Note that in this region \(A < rD - z_2(r) \leq D - z_2(r) < \bar{A}\). Taking the derivative of \(z_2(r)\) with respect to \(r\) for the regions \([0, \bar{r}_1), (\bar{r}_1, \bar{r}_2)\) and \((\bar{r}_2, 1]\) gives,

\[ \frac{dz_2(r)}{dr} = \begin{cases} 0 & \text{if } 0 \leq r < \bar{r}_1 \\ \frac{(rD')}{(c_e - c_v)(rD')\phi(rD - z_2(r)) + (h + c_v - c_m)D'\phi(D - z_2(r))} & \text{if } \bar{r}_1 < r < \bar{r}_2 \\ \frac{(c_e - c_v)(rD')\phi(rD - z_2(r)) + (h + c_v - c_m)D'\phi(D - z_2(r))}{(c_e - c_v)\phi(rD - z_2(r))} & \text{if } \bar{r}_2 < r \leq 1 \end{cases} \]

The denominator is > 0 for all \(r\) since in this region \(rD - z_2(r) \geq A\) and \(D - z_2(r) \leq \bar{A}\) and \(\phi(x) > 0\) for all \(x \in [A, \bar{A}]\) by assumption. Thus \(z_2(r)\) is continuous in \(r\) and is differentiable everywhere except at the break points \(r = \bar{r}_1, \bar{r}_2\). Also, note that \(z_2(r)\) is non decreasing in \(r\).
Similarly it is straightforward to show that \( z_2(r) \) is non increasing in \( c_e \) and non decreasing in \( h \).

\[ \square \]

**Proof of Lemma 4**

In the first period of stage two the manufacturer purchases from non local sources only if \( \xi_2 < rD \) i.e., with probability \( \Phi[rD] \). From Lemma 2 the manufacturer carries over 0, \( \xi_2 - rD \) and \( z_2(r) \) if \( \xi_2 \leq rD \) (Region \( R_1 \)), \( rD < \xi_2 \leq rD + z_2(r) \) (Region \( R_2 \)) and \( \xi_2 > rD + z_2(r) \) (Region \( R_3 \)) respectively. Accordingly the probability that the manufacturer purchases from non local sources in the last period of stage two is \( \Phi[rD] \) if \( \xi_2 \in R_1 \), \( \Phi[2rD - \xi_2] \) if \( \xi_2 \in R_2 \) and \( \Phi[rD - z_2(r)] \) if \( \xi_2 \in R_3 \). Summing gives the probability that the manufacturer makes non local purchases in stage two. We expand the sum by writing the integrals when \( A \leq rD \) and \( rD + z_2(r) \leq A \). Other cases are similar.

\[
p_e(r) = \Phi[rD] + \Phi^2[rD] + \int_{rD}^{rD+z_2(r)} \Phi[2rD-\xi_2] \phi(\xi_2) d\xi_2 + \int_{rD+z_2(r)}^{A} \Phi[rD-z_2(r)] \phi(\xi_2) d\xi_2 \tag{28}\]

Using Lemma 3 and applying Leibnitz rule for taking derivatives of integrals gives,

\[
\frac{dp_e(r)}{dr} = \phi(rD)(rD)' + \Phi[rD]f(rD)(rD)' + \int_{rD}^{rD+z_2(r)} 2(rD)' \phi(2rD-\xi_2) \phi(\xi_2) d\xi_2 \\
+ \int_{rD+z_2(r)}^{A} \phi(rD-z_2(r)) \left[ (rD)' - \frac{d\phi(2rD)(rD)'}{dr} \right] \phi(\xi_2) d\xi_2 \\
= \phi(rD)(rD)' + \Phi[rD]\phi(rD)(rD)' + \int_{rD}^{rD+z_2(r)} 2(rD)' \phi(2rD-\xi_2) \phi(\xi_2) d\xi_2 \\
+ \int_{rD+z_2(r)}^{A} \frac{(h+c_v-c_m)\phi(rD-z_2(r))\phi(D-z_2(\xi_2))((rD)'-D')}{(c_e-c_v)\phi(D-z_2(\xi_2)) + (h+c_v-c_m)\phi(D-z_2(\xi_2))} \phi(\xi_2) d\xi_2
\]

Since \( (rD)' > D' \) then \( p_e' > 0 \).

\[ \square \]

**Proof of Lemma 5**

The proof follows in exactly the same way as Lemma 4.

**Proof of Proposition 4**

Similiar to Proposition 3 we first derive the two period expected profit for the product specific claim. The steps of the derivation is provided in Appendix B. We directly state the first derivative and give conditions under which there exists a unique optimum. We have,

\[
\frac{d\Psi_p(r)}{dr} = 2(s-c_v)D' - (c_e-c_v)(rD)'p_e(r) + (h+c_v-c_m)D'p_e(r) - (rD)'k(r) \tag{29}\]

where \( p_e(r) \) and \( p_e'(r) \) are as in Lemmas 4 and 5 and \( k(r) = \int_{\xi_2 \in R_2} G_2(\xi_2-rD)\phi(\xi_2) d\xi_2 \).

The first order condition can then be written as,

\[
\frac{d\Psi_p(r)}{dr} = D' \left[ 2(s-c_v) - \left( \frac{(c_e-c_v)(rD)'p_e(r) + (rD)'k(r) - (h+c_v-c_m)p_e(r)D'}{f(r)} \right) \right] \tag{30}\]
where by assumption $D' > 0$ for all $r$. We first note that $f(r = 0) = 0 < 2(s - c_v)$ (since $z_2(r) = 0$ at $r = 0$). Also if $1 + \frac{D(1)}{D'(1)} > \frac{s - c_v}{c_e - c_v}$ then $f(r = 1) > 2(s - c_v)$. We will find conditions so that $f(r)$ is increasing in $r$, so that $\Psi_p(r)$ is quasiconcave and has a unique maximum. The proof uses the following:

1. $p'_e(r) \geq 0$
2. $p'_c(r) \leq 0$
3. $(c_e - c_v)p'_c(r) + k'(r) \geq 0$

(1) and (2) follow from Lemmas 4 and 5. To see (3) note that $k(r) > 0$ since $G'_2(\xi_2 - rD) > 0$ when $\xi_2 \in R_2$. $k(r)$ an be written as,

\[
k(r) = \int_{rD}^{D(1+r)-A} k_1(r, \xi_2)\phi(\xi_2)d\xi_2 + \int_{D(1+r)-A}^{rD+z_2(r)} k_2(r, \xi_2)\phi(\xi_2)d\xi_2 \quad (31)
\]

and $k_1(r, \xi_2) = -h + (c_e - c_v)\Phi[2rD - \xi_2], k_2(r, \xi_2) = -h + (c_e - c_v)\Phi[2rD - \xi_2] - (h + c_e - c_m)\Phi[D(1 + r) - \xi_2]$ Taking the derivative with respect to $r$,

\[
dk(r) = -(rD)' \left(-h + (c_e - c_v)\Phi[rD]\right)\phi(rD) + \int_{rD}^{D(1+r)-A} \frac{\partial k_1(r, \xi_2)}{\partial r} \phi(\xi_2)d\xi_2 + \int_{D(1+r)-A}^{rD+z_2(r)} \frac{\partial k_2(r, \xi_2)}{\partial r} \phi(\xi_2)d\xi_2
\]

Cancelling the term $-(rD)'(c_e - c_v)\Phi[rD]\phi(rD)$ with the corresponding term in expression for $p_e(r)$ and noting that the partials are each non negative we have $(c_e - c_v)p'_c(r) + k'(r) > 0$, which shows (3). Now we can find conditions so that $f(r)$ is strictly increasing in $r$ as follows:

\[
f'(r) = \frac{[D'(rD)'' - D''(rD)'](c_e - c_v)p_c(r) + k(r)]}{(D')^2} + \frac{(c_e - c_v)(rD)'p'_c(r) + (rD)'k'(r)}{(D')} - (h + c_e - c_v)p'_c(r)
\]

By (1), (2) and (3) above the last two terms are non negative. If $D'(rD)'' - D''(rD)' > 0$ i.e., $D(r)$ is log concave then since $p_c(r), k(r) > 0$, we have that $f(r)$ is strictly increasing in $r$ and $\Psi_p(r)$ is strictly quasiconcave and has a unique optimum. □

**Proof of Proposition 5**

The first order condition is for the company wide claim is,

\[
\frac{d\Psi_c(r)}{dr} = 2(s - c_v)b_c - 2(c_e - c_v)(a + 2b_c r)\Phi_{12}[2r(a + b_c r)] 
\]

For, $2r(a + b_c r) > 2A$ the condition simplifies to,

\[
\frac{d\Psi_c(r)}{dr} = 2(s - c_v)b_c - 2(c_e - c_v)(a + 2b_c r)
\]

Equating above expression to 0 gives $r = \frac{s - c_v}{2(c_e - c_v)} - \frac{a}{2b_c}$. Substituting into the inequality above
and simplifying we get that if \( c_e < c_v + \frac{b_c(s - c_v)}{\sqrt{a^2 + 4b_cA}} \) then \( r^*_e = \frac{s - c_v}{2(c_v - c_e)} - \frac{a}{2b_c} \). The maximum expected profit for the company wide claim is,

\[
\Psi^*_c = 2(s - c_v)(a + b_e r^*_e) + 2(c_v - c_m)\mu - (c_e - c_v) \int_{\xi_1}^{\xi_2} (r^*_e(a + b_e r^*_e) - \xi_1 - \xi_2) \phi(\xi_1)\phi(\xi_2)d\xi_2d\xi_1 \\
= \frac{(s - c_v)^2b_c}{2(c_v - c_e)} + \frac{a^2(c_e - c_v)}{2b_c} + a(s - c_v) + 2\mu(c_e - c_m)
\]

where we have used \( 2r^*_e(a + b_e r^*_e) > 2\bar{A} \). It can be verified that \( \Psi^*_c \) is increasing in \( b_e \). Similarly for the product specific claim we have,

\[
\frac{d\Psi^*_p(r)}{dr} = 2(s - c_v)b_p - 2(c_v - c_e)(a + 2b_pr)\Phi[r(a + b_pr)]
\]

If \( b_p > b_e \) then we have \( c_e < c_v + \frac{b_c(s - c_v)}{\sqrt{a^2 + 4b_cA}} < c_v + \frac{b_p(s - c_v)}{\sqrt{a^2 + 4b_pA}} \) so that, and \( r^*_p = \frac{s - c_v}{2(c_v - c_e)} - \frac{a}{2b_p} \)

\[
r^*_e = \frac{s - c_v}{2(c_v - c_e)} - \frac{a}{2b_c}.
\]

The expected profit at the optimum is,

\[
\Psi^*_p = 2(s - c_v)(a + b_p r^*_p) + 2(c_v - c_m)\mu - (c_e - c_v) \int_{\xi_1}^{\xi_2} (r^*_p(a + b_p r^*_p) - \xi_1) \phi(\xi_1)d\xi_1 \\
- (c_e - c_v) \int_{\xi_1}^{\xi_2} (r^*_p(a + b_p r^*_p) - \xi_2) \phi(\xi_2)d\xi_2 = \frac{(s - c_v)^2b_p}{2(c_e - c_v)} + \frac{a^2(c_e - c_v)}{2b_p} + a(s - c_v) + 2\mu(c_e - c_m)
\]

since \( r^*_p(a + b_p r^*_p) > A \). Thus, if \( b_p > b_e \) and \( c_e < c_v + \frac{b_c(s - c_v)}{\sqrt{a^2 + 4b_cA}} \) then \( r^*_p > r^*_e \) and \( \Psi^*_p > \Psi^*_e \). \( \square \)

**Proof of Proposition 6**

By the Implicit Function Theorem,

\[
\frac{d\Psi^*_c}{db_e} = \frac{\partial\Psi^*_c(r)}{\partial c_e} \bigg|_{r=r^*_e} = 2(s - c_v)r^*_e - 2(c_v - c_e)(r^*_e)^2\Phi_12[2r^*_e(a + br^*_e)]
\]

\[
= 2r^*_e[(s - c_v) - (c_e - c_v)r^*_e\Phi_12[2r^*_e(a + br^*_e)]] = \frac{2(p - c_e)r^*_e(a + br^*_e)}{a + 2b_e r^*_e} > 0
\]

Therefore, \( \Psi^*_c \) is increasing in \( b_e \). The first order condition was given by,

\[
\frac{d\Psi^*_c(r)}{dr} = (s - c_v)b_e - (c_e - c_v)(a + 2b_e r)\Phi_12[2r(a + b_e r)]
\]

Clearly if \( r(a + b_e)r \leq A \) then \( \frac{d\Psi^*_c(r)}{dr} = (s - c_v)b_e > 0 \), so the optimum lies in the region \( r(a + b_e)r > A \). At \( r(a + b_e)r = \bar{A} \), i.e., \( r = r_A(b_e) \), the first order condition,

\[
\frac{d\Psi^*_c(r)}{dr} \bigg|_{r=r_A(b_e)} = (s - c_v)b_e - (c_e - c_v)\sqrt{a^2 + 4b_eA}
\]
Thus, if \(c_e - c_v < \frac{b_e(s - c_v)}{a^2 + 4b_e A} \) then \(r_e^* = \min\left(\frac{s - c_v}{2(c_e - c_v) - \frac{a}{2b_e}}, 1\right)\). Now, if \(c_e - c_v \geq \frac{b_e(s - c_v)}{\sqrt{a^2 + 4b_e A}}\) then the optimum lies in the region \(A \leq r(a + b_r) \leq \bar{A}\). To find conditions, when \(r_e^*\) decreases in \(b_e\), consider two values of \(b_e = b_1, b_2\) such that \(b_2 > b_1\) and \(c_e - c_v \geq \frac{b_2(s - c_v)}{\sqrt{a^2 + 4b_2 A}}\), i.e., both the minimizers, say \(r_1\) and \(r_2\), lie in the region above. Then both minimizers satisfy the first order conditions i.e.,

\[
\Psi'_e(r_1, b_1) = (s - c_v)b_1 - (c_e - c_v)(a + 2b_1r_1)\Phi_{12}[2r_1(a + b_1r_1)] = 0
\]

\[
\Psi'_e(r_2, b_2) = (s - c_v)b_2 - (c_e - c_v)(a + 2b_2r_2)\Phi_{12}[2r_2(a + b_2r_2)] = 0
\]

A sufficient condition for \(r_2 < r_1\) is \(\Psi'_e(r_1, b_2) < \Psi'_e(r_1, b_1) = 0\). Simplifying we need,

\[
s - c_v < \frac{(a + 2b_2r_2)\Phi_{12}[2r_1(a + b_2r_1)] - (a + 2b_1r_1)\Phi_{12}[2r_1(a + b_1r_1)]}{b_2 - b_1}
\]

Now from the first equation above as \(c_e \to \infty\), \(r_1(a + b_1r_1) \to A\) (i.e., \(r_1 \to r_A(b_1)\)). Thus as \(c_e \to \infty\), the left hand side of the inequality tends to 0 while the right hand side tends to \(\frac{(a + 2b_2r_2)(b_1)}{b_2 - b_1} \Phi_{12}[2r_2(b_1)] > 0\) since \(\Phi_{12}[2r_1(a + b_1r_1)] \to \Phi_{12}[2\bar{A}] = 0\). Thus, there must exists a value of \(c_e = \bar{c}_e(b_1, b_2)\) such that \(r_2 < r_1\) for all \(c_e > \bar{c}_e(b_1, b_2)\).

**Proof of Proposition 7**

The result follows from writing the first order condition for the company wide claim as follows,

\[
\frac{d\Psi_e(r)}{dr} = (s - c_v)b_e - (c_e - c_v)(a + 2b_e r)\Phi_{12}[2r(a + b_e r)]
\]

\[
= (s - c_v)b_e - (c_e - c_v)(a + 2b_e r)P[X_\alpha = \alpha X + (1 - \alpha)\mu = r(a + b_e r)]
\]

\[
= (s - c_v)b_e - (c_e - c_v)(a + 2b_e r)P[X \leq \frac{r(a + b_e r) - \mu}{\alpha} + \mu]
\]

Thus increasing \(\alpha\) increases the claim when the optimum lies in the region \(r(a + b_e r) > \mu\) and decreases it when the optimum lies in the region \(r(a + b_e r) > \mu\). Setting \(r(a + b_e r) \geq \mu\) in (41) gives the required result.

**11. APPENDIX B**

11.1. Derivation of two period expected profit for \(0 \leq r \leq \tilde{r}\)

We have, \(z^*_r = z_2(r) = 0\). Since no recycled input is carried over the two period problem decouple into two separate single period problems. The two period expected profit for the product specific claim is,

\[
\Psi_p(r) = 2(s - c_v)D + 2(c_v - c_m)\mu - 2(c_e - c_v)\int_A^{r^D}(rD - \xi_2)\phi(\xi_2)d\xi_2
\]

\[
\Rightarrow \Psi'_p(r) = 2(s - c_v)D' - 2(c_e - c_v)(rD')\Phi[rD]
\]
To see the equivalence with Proposition 4, note that since $z_2(r) = 0$, $R_2 = \{\emptyset\}$. From Lemma 4, $p_c(r) = \Phi[rD] + \Phi^2[rD] + \Phi[rD]\Phi[rD] = 2\Phi[rD]$ and $p_c(r), k(r) = 0$ which gives the expression above.

11.2. Derivation of two period expected profit for $\bar{r}_2 \leq r \leq 1$

We know from Lemma 3 that in this region $\bar{A} < rD - z_2(r) \leq D - z_2(r) < \bar{A}$. We write the expected profit over $\xi_1$ for each $\xi_2$ as follows: Let $\Psi(r, \xi_2, \xi_1) =$ Two period profit for given $r, \xi_2, \xi_1$. Then,

$$
\Psi_p(r, \xi_2|\xi_2 \in R_1) = \mathbb{E}_{\xi_1}[\Psi(r, \xi_2, \xi_1)|\xi_2 \in R_1] = sD - c_m\xi_2 - c_e(rD - \xi_2) - c_v(D - rD)
+ \int_{\bar{A}}^{A} (sD - c_m\xi_1 - c_e(rD - \xi_1) - c_v(D - rD))\phi(\xi_1)d\xi_1
+ \int_{rD}^{A} (sD - c_m\xi_1 - c_v(D - \xi_1))\phi(\xi_1)d\xi_1
$$

To write in integral form when $\xi_2 \in R_2$, we need to consider two cases: (i) $\bar{A} + \xi_2 - rD \leq D$ (Region $R_{21}$) and (ii) $\bar{A} + \xi_2 - rD > D$ (Region $R_{22}$). Noting that $R_{21} \cup R_{22} = R_2$ we have,

$$
\Psi_p(r, \xi_2|\xi_2 \in R_{21}) = \mathbb{E}_{\xi_1}[\Psi(r, \xi_2, \xi_1)|\xi_2 \in R_{21}] = sD - c_m\xi_2 - c_e(rD - rD) - h(\xi_2 - rD)
+ \int_{\bar{A}}^{A} (sD - c_m\xi_1 - c_e(2rD - \xi_1 - \xi_2) - c_v(D - rD))\phi(\xi_1)d\xi_1
+ \int_{2rD - \xi_2}^{A} (sD - c_m\xi_1 - c_v(D - \xi_1 - \xi_2 + rD))\phi(\xi_1)d\xi_1
$$

$$
\Psi_p(r, \xi_2|\xi_2 \in R_{22}) = \mathbb{E}_{\xi_1}[\Psi(r, \xi_2, \xi_1)|\xi_2 \in R_{22}] = sD - c_m\xi_2 - c_e(rD - rD) - h(\xi_2 - rD)
+ \int_{\bar{A}}^{A} (sD - c_m\xi_1 - c_e(2rD - \xi_1 - \xi_2) - c_v(D - rD))\phi(\xi_1)d\xi_1
+ \int_{2rD - \xi_2}^{A} (sD - c_m\xi_1 - c_v(D - \xi_1 - \xi_2 + D(1+r)))\phi(\xi_1)d\xi_1
$$

$$
\Psi_p(r, \xi_2|\xi_2 \in R_3) = \mathbb{E}_{\xi_1}[\Psi(r, \xi_2, \xi_1)|\xi_2 \in R_3] = sD - c_m\xi_2 - c_e(D - \xi_2 + z_2(r)) - hz_2(r)
+ \int_{\bar{A}}^{A} (sD - c_m\xi_1 - c_e(rD - \xi_1 - z_2(r)) - c_v(D - rD))\phi(\xi_1)d\xi_1
+ \int_{rD - z_2(r)}^{A} (sD - c_m\xi_1 - c_v(D - \xi_1 - z_2(r)))\phi(\xi_1)d\xi_1
+ \int_{D - z_2(r)}^{A} (sD - c_m\xi_1 - h(\xi_1 + z_2(r) - D))\phi(\xi_1)d\xi_1
$$

Now we can write the expected profit $\Psi_p(r)$ as,

$$
\Psi_p(r) = \int_{\bar{A}}^{A} \int_{\bar{A}}^{A} \Psi_p(r, \xi_1, \xi_2)\phi(\xi_2)\phi(\xi_1)d\xi_1d\xi_2 =
$$
\[ \int_{\Delta} \Psi(r, \xi_2 | \xi_2 \in R_1) \phi(\xi_2) d\xi_2 + \int_{r_D}^{D(1+r)-\bar{A}} \Psi(r, \xi_2 | \xi_2 \in R_21) \phi(\xi_2) d\xi_2 \\
+ \int_{D(1+r)-\bar{A}}^{D(1+r)+z_2(r)} \Psi(r, \xi_2 | \xi_2 \in R_2) \phi(\xi_2) d\xi_2 + \int_{r_D+z_2(r)}^{\bar{A}} \Psi(r, \xi_2 | \xi_2 \in R_3) \phi(\xi_2) d\xi_2 \]

We can write the first term as it is since \( rD \geq \bar{A} \). If \( rD + z_2(r) > \bar{A} \) then the last term is 0 and the upper limit of the integral is \( \bar{A} \). \( \Psi(r, \xi_2) \) is continuous in \( \xi_2 \) and for every given \( \xi_2 \), \( \Psi(r, \xi_2, \xi_1 | \xi_2 \in R_i) \) is continuous in \( \xi_1 \) for every \( i = \{1, 2, 3, 4\} \). Another way of saying this is that \( \Psi(r, \xi_2, \xi_1) \) is continuous in \( \xi_1, \xi_2 \) which lets us apply Leibnitz rule to write,

\[ \frac{d \Psi_p(r)}{dr} = \int_{\bar{A}}^{r_D} \frac{d \Psi(r, \xi_2 | \xi_2 \in R_1)}{dr} \phi(\xi_2) d\xi_2 + \int_{r_D}^{D(1+r)-\bar{A}} \frac{d \Psi(r, \xi_2 | \xi_2 \in R_2)}{dr} \phi(\xi_2) d\xi_2 \\
+ \int_{D(1+r)-\bar{A}}^{D(1+r)+z_2(r)} \frac{d \Psi(r, \xi_2 | \xi_2 \in R_3)}{dr} \phi(\xi_2) d\xi_2 + \int_{r_D+z_2(r)}^{\bar{A}} \frac{d \Psi(r, \xi_2 | \xi_2 \in R_4)}{dr} \phi(\xi_2) d\xi_2 \]

if \( rD + z_2(r) \leq \bar{A} \) else,

\[ \frac{d \Psi_p(r)}{dr} = \int_{\bar{A}}^{r_D} \frac{d \Psi(r, \xi_2 | \xi_2 \in R_1)}{dr} \phi(\xi_2) d\xi_2 + \int_{r_D}^{D(1+r)-\bar{A}} \frac{d \Psi(r, \xi_2 | \xi_2 \in R_2)}{dr} \phi(\xi_2) d\xi_2 \\
+ \int_{D(1+r)-\bar{A}}^{D(1+r)+z_2(r)} \frac{d \Psi(r, \xi_2 | \xi_2 \in R_3)}{dr} \phi(\xi_2) d\xi_2 \]

We evaluate each derivative separately and combine to arrive at the expression for \( \frac{d \Psi_p(r)}{dr} \). Let \( D' = \frac{d D(r)}{dr} \) and \( \frac{d(D(r))}{dr} = r D' + D \). Then,

\[ \frac{d \Psi(r, \xi_2 | \xi_2 \in R_1)}{dr} = 2(s - c_v)D' - (c_e - c_v)(rD)'(1 + \Phi[rD]) \]

\[ \frac{d \Psi(r, \xi_2 | \xi_2 \in R_21)}{dr} = 2(s - c_v)D' + h(rD)' - 2(c_e - c_v)(rD)' \Phi[2rD - \xi_2] \\
= 2(p - c_v)D' - (rD)' \left[ -h + (c_e - c_v) \Phi[2rD - \xi_2] \right] - (c_e - c_v)(rD)' \Phi [2rD - \xi_2] \]

\[ G'_{2}(\xi_2 - rD) \text{ for } \xi_2 \in R_{21} \]

\[ \frac{d \Psi(r, \xi_2 | \xi_2 \in R_22)}{dr} = 2(s - c_v)D' + h(rD)' - 2(c_e - c_v)(rD)' \Phi[2rD - \xi_2] \\
+ (h + c_v - c)(rD)' \Phi[D(1 + r) - \xi_2] \\
= 2(p - c_v)D' - (rD)' \left[ -h + (c_e - c_v) \Phi[2rD - \xi_2] - (h + c_v - c_m) \Phi[D(1 + r) - \xi_2] \right] \]

\[ G'_{2}(\xi_2 - rD) \text{ for } \xi_2 \in R_{22} \]

Note that \( G'_{2}(\xi_2 - rD) \) for \( \xi_2 \in R_{2} \).
\[
\frac{d\Psi(r, \xi_2|\xi_2 \in R_3)}{dr} = 2(s - c_v)D' + \left(h - (c_e - c_v)\Phi[rD - z_2(r)] + (h + c_v - c_m)\Phi[D - z_2(r)]\right) \frac{d\xi_2}{dr}
\]

Note that \(\frac{d\Psi(r, \xi_2)}{dr}\) remains continuous in \(\xi_2\) between regions and at the boundaries. Combining and collecting the terms involving the parameters \(s - c_v, c_e - c_v\) and \(h + c_v - c\) gives,

\[
\frac{d\Psi_p(r)}{dr} = 2(s - c_v)D' + h(rD)' \int_{rD}^{rD+z_2(r)} \phi(\xi_2)d\xi_2
\]

\[
- (c_e - c_v)(rD)' \left(\Phi[rD] + \Phi^2[rD] + 2 \int_{rD}^{rD+z_2(r)} \Phi[2rD - \xi_2] \phi(\xi_2)d\xi_2 + \int_{rD+z_2(r)}^{A} \Phi[rD - z_2(r)] \phi(\xi_2)d\xi_2\right)
\]

\[
+ (h + c_v - c_m) \left(\int_{D(1+r) - A}^{rD+z_2(r)} \Phi[D(1+r) - \xi_2] + \int_{rD+z_2(r)}^{A} D' \Phi[D - z_2(r)]\right)
\]

\[
= 2(s - c_v)D' - (c_e - c_v)(rD)' p_e(r) + (h + c_v - c)D' p_e(r) - (rD) k(r) \tag{43}
\]

where \(p_e(r), p_e(r)\) and \(k(r)\) are as defined. The derivation for the region \(\tilde{r}_1 \leq r \leq \tilde{r}_2\) is similar.

References


