Inventory Management Decisions: – Fuzzy Non-linear Goal Programming Approach

Abstract
This paper proposes an application of fuzzy non-linear goal programming (FNLGP) model for inventory management decisions. The model uses tolerance allowance fuzzy goal programming (TFGP) technique to determine the economic order quantity (EOQ) in a multi-item inventory problem. The proposed model takes into consideration fuzzy goals such as budget, warehouse space and amount of investment which are critical to a decision-making process. In TFGP modeling, the goals are converted in to deterministic goal constraints using their corresponding membership function values and the deterministic equivalent of the fuzzy model is derived. We have also presented sensitivity analysis to demonstrate effectiveness of the model using different amounts of budgets.

Keywords: Goal Programming; Fuzzy Goal Programming; Multi-Item Inventory, Nonlinear Goal Programming.

1. INTRODUCTION
Effective Inventory Management decisions are essential to support the company’s strategic plan and to meet the needs of market demand. Inventory generally refers to finished goods, raw material, spare purchased parts and supplies but this definition is not comprehensive and includes blood in a blood bank, cash on hand at a bank, and so forth. Inventory management involves making decisions on how to minimize total inventory costs, such as the cost to carry inventory, the cost to order inventory, and the item cost, while satisfying the demand for the items. Today’s business need for real-time or even daily inventory data to strengthen the financial health in a very highly competitive and technological driven world. Companies require immediate actions to change their Inventory Management process and approach to accommodate changes in market demand due to the new manufacturing technology and other economic factors including opportunities of worldwide marketing and manufacturing. Quantitative methods and models designed to help make effective inventory management decisions apply to independent demand items. Quantitative models help the process that requires information on how to - (i) efficiently manage the flow of materials and products, (ii) effectively utilize people and equipments, (iii) coordinate internal activities, and (iv) effectively communicate with customers. The role of Inventory Management and the activities of Inventory Control are to provide the information to Managers to make more precise and timely decisions in managing their operations. In business and industry, Operations managers are under pressure to develop models that help to estimate level of inventories that results in lower inventory investment costs, achieve higher productivity, and of course greater return on overall investments.

The general purposes for carrying inventory may include one or more—(a) forming the basis for doing business; (b) provide a favorable return on investment; (c) allow the buyer to take advantage of quantity discounts; and (d) protect against fluctuations in demand, delayed supply, and inflation. Here, our focus is to develop a model that helps determining an economic ordering quantity (EOQ). The EOQ concept was first proposed by Wilson (1934) in a model developed to calculate replenishment order size for a single item inventory system without space constraints. The basic EOQ model determines the order quantity considering the trade-off between order cost and inventory cost. Thus, as the number of orders increases, the order cost increases and the inventory cost decreases, and vice versa. The basic EOQ model has been extended to include the multi-item inventory problem under a storage space limit (Silver 1976, Zollar 1977). There is ample amount of literature reporting on inventory management control problems (see Clark (1972), Whitin (1953), Hadley and Whitin (1963), Raymond (1931) and Silver (1981). In most cases, the focus on the study was on of the theory of single-product and single-installation systems. In these systems, one item is stored in one location. The optimal solution is derived through the successful application of several well-known optimization techniques. However, a limited number of research studies have been conducted emphasizing the implementation of the theory of multiple items in multiple
locations. The concept of Goal programming that was first developed by Charnes and Cooper (1961) and later on extended by Lee (1972), Ignizio (1976), and other researchers. The goal programming has made it possible to solve real-world situations of multi-item inventory problems that have conflicting objectives such as minimization of inventory holding cost and lost customer goodwill due to inventory stock-outs and the cost functions are nonlinear in nature due to the associated set up cost. Recent research work that has used GP technique to incorporate multi-item inventory control systems for managing those conflicting objectives can be exhibited in the research work of Romero (1986), Golanly et al. (1991) and Basu et al. (1999).

In conventional GP, parameters of the problems need to be defined precisely. In most inventory problems, values of some parameters may not be known precisely. They are rather defined in a fuzzy sense. For successfully handling such problems, FGP techniques must be used. FGP technique provides better tools to represent a problem that contains fuzzy goals and objectives. Therefore, it is more realistic to consider estimated constraints or flexible technological coefficients. This type of constraints and objectives are assumed as fuzzy goals. Fuzzy goals and fuzzy constraints are regarded as fuzzy criteria. The use of fuzzy set theory in goal programming (GP) was first introduced by Narasimhan (1980). It was further developed by Hannan (1981 & 1982), Narasimhan (1981), Ignizio (1982), Rubin and Narasimhan (1984), Tiwari et al. (1986 & 1987), Chen (1994)) and others. Chen and Tsai (2001) presented an intensive review of FGP.

In this study, we have used Zimmermann’s (1985) approach to construct the membership function and Kim and Whang’s (1998) tolerance approach for the goals in a FGP for inventory management decision-making to obtain the economic order quantity (EOQ) in a multi-item inventory problem. This approach is especially useful for FGP problems having both unequal weights and unbalanced membership values. Also, sensitivity analysis on changes to values in the model can be conducted easily because of the simplified structure of the problem.

The remainder of the paper is organized as follows: Section 2 presents a brief review of literature. Section 3 defines the general models of the multi-item inventory problem. Section 4 explains the FGP models proposition. Section 5 demonstrates the model via an example. Section 6 analyses the result obtained from the example and section 7 presents concluding remarks.

2. REVIEW OF LITERATURE

Inventory control has always been an interesting research topic and researchers have developed models using the new computational tools and techniques. The study of literature reveals that Hadley and Whitin (1963) solved multiple-product EOQ using Lagrange multipliers. Kotchenberger (1971) used the geometric programming technique in developing his model to solve inventory problems. Worrall and Hall (1982) applied polynomial geometric programming to a multi-item inventory model with multiple items subject to multiple constraints. Cheng (1989a, 1989b) applied geometric programming in developing modified EOQ models and sensitivity analysis performance. Plenert (1990) derived the optimal solution of multiple-product constrained EOQ problem using a combination of geometric programming techniques and algebraic substitution. Golanly et al. (1991) developed an inventory model using goal programming (GP) technique and applied it to a large chemical plant for the purpose of accomplishing several conflicting objectives. Ben-Daya and Raouf (1993) developed a multi-item inventory model with stochastic demand with two constraints. Zhu et al. (1993) used the concept of GP for minimizing the number of production days in a continuous manufacturing system. Basu et al. (1997) developed a solution procedure using the nonlinear goal programming (NLGP) technique for solving multi-item inventory problems where the problem has the characteristic of dynamic programming, Panda et al. (2005) proposes how the system of penalties acts in a priority-based NLGP model to obtain the EOQ, where the market demands of all the items are uniform throughout the period. Junga and Klein (2006) analyzed three EOQ
based inventory models with an objective of profit maximization using geometric programming techniques.

3. MULTI-ITEM EOQ MODELS DESCRIPTION

3.1 Assumptions

A multi-item inventory model for obtaining the economic order quantity (EOQ) is based on the following assumptions:

i) The annual demand requirements are known
ii) The demand rate is reasonably constant
iii) The lead time does not vary
iv) Each order is received in a single delivery
v) There are no quantity discounts

The following notations will be used in the general models of the multi-item inventory problem and in the following sections:

3.2 Notations

- $M$: Index for the constraint $m = 1, 2, \ldots, M$
- $N$: Index for the inventory item $n = 1, 2, \ldots, N$
- $q_n$: Number of units for item $n$
- $C_n$: Sum of ordering and carrying costs for item $n$
- $S_n$: Setup costs for item $n$
- $Q$: Expected minimum inventory level
- $a_{mn}$: Average investment with item $n$ and constraint $m$
- $b_m$: Right hand side target associated with constraint $m$
- $G_n$: Investment target level for item $n$

3.3 Crisp EOQ model

The general model of the multi-item inventory problem can be defined as (Sharma et al. (2001)):

Find $q = (q_1, q_2, \ldots, q_N)$ so as to
min : \( \sum_{n=1}^{N} (C_n q_n + \frac{S_n}{q_n}) \)

subject to

\[
\sum_{n=1}^{N} a_{mn} q_n = b_m, \quad m = 1,2,\ldots, M
\] (3.1)

\[
\sum_{n=1}^{N} q_n \geq Q
\]

\[
q_n \geq 0; \quad n = 1,2,\ldots, N
\]

3.4 Fuzzy EOQ model

If the amount of investment for each item, available storage area and maximum number of orders become fuzzy, then the EOQ model of the multi-item inventory problem can be written as follows:

Find \( q = (q_1, q_2, \ldots, q_N) \) so as to

\[
\min : (C_n q_n + \frac{S_n}{q_n}) \leq G_n, \quad n = 1,2,\ldots, N
\] (3.2)

Subject to

\[
\sum_{n=1}^{N} a_{mn} q_n \leq b_m, \quad m = 1,2,\ldots, M
\] (3.3)

\[
\sum_{n=1}^{N} q_n \geq Q
\]

\[
q_n \geq 0; \quad n = 1,2,\ldots, N
\] (3.5)

4. FGP MODEL PROPOSITIONS

In model (3.2)-(3.5), the investment goal for each item, constraint for warehouse space and number of ordered quantities are defined fuzzily. In this section, we propose equivalent crisp models for the fuzzy multi-item EOQ model. We utilized tolerance allowance technique as well as linear membership function to obtain the crisp equivalent model. Let, \( I^*_n \), \( S^*_m \) and \( O^* \) are the respective tolerance limits for cost goal, investment goal and inventory goal. Then according to the specified tolerance limits, linear membership functions can be constructed as follows:
Next, we develop the FGP model based on tolerance allowance for the above defined membership functions.

### 4.1 Tolerance Allowance FGP Model

Here, we present the concept of tolerance allowance FGP model. Kim and Whang (1998) used this concept for the first time to convert a FGP model to a single objective LP problem for linear membership functions. In the FNLGP model of the EOQ inventory problem, the ordering and setup cost goal (4.1), and investment goal (4.2) are of type \( z_k(x) < b_k \). Whereas the inventory level goal is of type \( z_k(x) > b_k \). If all three goals are achieved completely then their corresponding membership function values must be unity and upper tolerance limit values \( I_n^u \) and \( S_m^u \) are exactly equal to zero. If the goals are partially achieved then \( I_n^u, S_m^u \in (0,1) \) and hence \( \mu_{t_n}^l(q), \mu_{i_m}^s(q) \in (0,1) \). Similar argument holds for the other type of goal, namely the inventory level goal. We then write fuzzy goals corresponding to the membership functions and formulate the following model:

The fuzzy goals corresponding to membership functions defined in (4.1)-(4.3) are written as:

\[
(C_n q_n + S_n / q_n) + \mu_{t_n}^l(q) I_n^u \leq G_n + I_n^u, \quad n=1,2,\cdots, N
\]
\[
\sum_{n=1}^{N} a_{mn} q_n + \mu_{l_n}^{S}(q) S_n^u \leq b_m + S_m^u, \quad m = 1, 2, \ldots, M
\]
\[
\sum_{n=1}^{N} q_n + \mu_{l_n}^{O}(q) O^L \geq Q - O^L
\]

We now introduce tolerance allowance variables \( \theta_{l_n}^{I} = 1 - \mu_{l_n}(q) \), \( \theta_{l_n}^{S} = 1 - \mu_{l_n}^{S}(q) \) and \( \theta_{l_n}^{O} = 1 - \mu_{l_n}^{O}(q) \). In Narasimhan’s (1980) FGP model, the overall membership values of all goals are maximized. In our formulation, tolerance allowance variables corresponding to investment, storage area and number of orders are to be minimized to obtain maximum membership function values. In particular, if all the tolerance allowance variables are zero then all goals will be satisfied completely and hence the membership grades of all goals will be unity. Therefore, the TAM can be formulated as:

\[
\begin{align*}
\min & : \sum_{n=1}^{N} \theta_{l_n}^{I} + \sum_{m=1}^{M} \theta_{l_n}^{S} + \theta_{l_n}^{O} \\
\text{subject to} & \\
(C_n q_n + S_n / q_n) - \theta_{l_n}^{I} f_n^u & \leq G_n, \quad n = 1, 2, \ldots, N \\
\sum_{n=1}^{N} a_{mn} q_n - \theta_{l_n}^{S} S_n^u & \leq b_m + S_m^u, \quad m = 1, 2, \ldots, M \\
\sum_{n=1}^{N} q_n + \theta_{l_n}^{O} O^L & \geq Q \\
0 & \leq \theta_{l_n}^{I}, \theta_{l_n}^{S}, \theta_{l_n}^{O} \leq 1 \\
q_n & \geq 0; \quad n = 1, 2, \ldots, N
\end{align*}
\]

The following section discusses the application of the model using multi-item inventory problem.

5. APPLICATION OF THE MODEL

The application of tolerance allowance goal programming to a car dealer problem will be described in this section. The dealer sells a total of twenty two types of different cars, trucks, SUV, and Vans listed in Table 1. The estimated annual total cost of vehicles purchased is $11.5 million dollars (with a tolerance of $0.7 million) and the warehouse is capable of holding a 650 or more vehicles (with a tolerance 45 vehicles). The desire of the dealer is to meet the demand of local customer for each type of vehicle. Due to the warehouse space and other possible constraints, the dealer seeks the optimal inventory mix of all types of vehicles. The average investment (\( a_{mn} \)), total ordering and car carrying cost (\( C_n \)), setup cost (\( S_n \)), and total investment (\( G_n \)) for each vehicle type are provided in Table 1 below.

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>( a_{mn} )</th>
<th>( S_n )</th>
<th>( C_n )</th>
<th>( G_n )</th>
<th>Tolerance</th>
</tr>
</thead>
</table>


| q1 | Small Car 01 | $95 | $80 | $19 | $95 | $50 |
| q2 | Small Car 02 | $110 | $85 | $22 | $92.4 | $48 |
| q3 | Small Car 03 | $135 | $95 | $27 | $102.6 | $52 |
| q4 | Medium Car 01 | $143 | $97 | $29 | $105 | $53 |
| q5 | Medium Car 02 | $155 | $98 | $31 | $87 | $44 |
| q6 | Medium Car 03 | $163 | $105 | $33 | $99 | $50 |
| q7 | Medium Car 04 | $165 | $110 | $33 | $100 | $50 |
| q8 | Large Car 01 | $175 | $125 | $35 | $112 | $56 |
| q9 | Large Car 02 | $185 | $130 | $37 | $111 | $56 |
| q10 | Large Car 03 | $190 | $145 | $38 | $126 | $63 |
| q11 | Large Car 04 | $215 | $160 | $43 | $108 | $54 |
| q12 | Truck 01 | $135 | $135 | $27 | $103 | $52 |
| q13 | Truck 02 | $145 | $140 | $29 | $99 | $50 |
| q14 | Truck 03 | $150 | $145 | $30 | $93 | $47 |
| q15 | Truck 04 | $162 | $155 | $32 | $87 | $44 |
| q16 | Truck 05 | $175 | $160 | $35 | $88 | $44 |
| q17 | SUV 01 | $220 | $185 | $44 | $101 | $51 |
| q18 | SUV 02 | $255 | $190 | $51 | $102 | $52 |
| q19 | SUV 03 | $295 | $197 | $59 | $118 | $60 |
| q20 | Van 01 | $200 | $165 | $40 | $140 | $70 |
| q21 | Van 02 | $225 | $155 | $45 | $126 | $63 |
| q22 | Van 03 | $285 | $165 | $57 | $103 | $52 |

In the form of NLGP, the goal constraints and the objective function of the above problem are formulated as follows:

The TAM model:

$$\min: \sum_{n=1}^{22} \theta^I_{t_n} + \theta^S_{t_n} + \theta^O_{t_n}$$

subject to

$$95q_1 + 110q_2 + 135q_3 + 143q_4 + 155q_5 + 163q_6 + 165q_7 + 175q_8 + 185q_9 + 190q_{10} + 215q_{11} + 135q_{12} + 145q_{13} + 150q_{14} + 162q_{15} + 175q_{16} + 220q_{17} + 255q_{18} + 295q_{19} + 200q_{20} + 255q_{21} + 285q_{22} - 7000 \theta^S_{t_n} \leq 115000;$$

$$q_1 + q_2 + q_3 + q_4 + q_5 + q_6 + q_7 + q_8 + q_9 + q_{10} + q_{11} + q_{12} + q_{13} + q_{14} + q_{15} + q_{16} + q_{17} + q_{18} + q_{19} + q_{20} + q_{21} + q_{22} \geq 650 - 45\theta^O_{t_n};$$

$$1900q_1 + 80q_1 - 5000 \theta^I_{t_n} \leq 95000;$$
2200q_2 + 85/q_2 - 4800 \theta_{q_2}^I \leq 92400;
2700q_3 + 95/q_3 - 5200 \theta_{q_3}^I \leq 102600;
2900q_4 + 97/q_4 - 5300 \theta_{q_4}^I \leq 105000;
3100q_5 + 98/q_5 - 4400 \theta_{q_5}^I \leq 87000;
3300q_6 + 105/q_6 - 5000 \theta_{q_6}^I \leq 99000;
3300q_7 + 110/q_7 - 5000 \theta_{q_7}^I \leq 100000;
3500q_8 + 125/q_8 - 5600 \theta_{q_8}^I \leq 112000;
3700q_9 + 130/q_9 - 5600 \theta_{q_9}^I \leq 111000;
3800q_{10} + 145/q_{10} - 6300 \theta_{q_{10}}^I \leq 126000;
4300q_{11} + 160/q_{11} - 5400 \theta_{q_{11}}^I \leq 108000;
2700q_{12} + 135/q_{12} - 5200 \theta_{q_{12}}^I \leq 103000;
2900q_{13} + 140/q_{13} - 5000 \theta_{q_{13}}^I \leq 99000;
3000q_{14} + 145/q_{14} - 4700 \theta_{q_{14}}^I \leq 93000;
3200q_{15} + 155/q_{15} - 4400 \theta_{q_{15}}^I \leq 87000;
3500q_{16} + 160/q_{16} - 4400 \theta_{q_{16}}^I \leq 88000;
4400q_{17} + 185/q_{17} - 5100 \theta_{q_{17}}^I \leq 101000;
5100q_{18} + 190/q_{18} - 5200 \theta_{q_{18}}^I \leq 102000;
5900q_{19} + 197/q_{19} - 6000 \theta_{q_{19}}^I \leq 118000;
4000q_{20} + 165/q_{20} - 7000 \theta_{q_{20}}^I \leq 140000;
4500q_{21} + 155/q_{21} - 6300 \theta_{q_{21}}^I \leq 126000;
5700q_{22} + 165/q_{22} - 5200 \theta_{q_{22}}^I \leq 103000;
0 \leq \theta_{q_n}^I \leq 1, n = 1,2,\cdots,22; \ 0 \leq \theta_{q_n}^I, \theta_{q_n}^O \leq 1, q_n \geq 0; n = 1,2,\cdots,22 \ \& \text{are integers.}

6. Results Analysis

The above problem is executed using LINGO 5.0 software. Solutions for different available budget are presented in Table 3. The sensitivity analysis is performed for the available budget of 11.00, 11.5 and 12.00 million. The sensitivity results provide the optimal inventory in each category that minimizes costs for the given budget amount.

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>Optimal Inventories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Budget = $11.0 million</td>
<td>Budget = $11.5 million</td>
</tr>
<tr>
<td>q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11}, q_{12}, q_{13}, q_{14}, q_{15}, q_{16}, q_{17}, q_{18}, q_{19}, q_{20}, q_{21}, q_{22}</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Sensitivity Analysis
7. Conclusion

In this study, we have presented a tolerance allowance fuzzy goal programming (TFGP) approach to solve inventory management decision problems. We have applied Zimmermann’s (1985) approach to construct the membership function, and Kim and Whang’s (1998) tolerance approach for the goals in a FGP for inventory management decision-making to obtain the economic order quantity (EOQ) in a multi-item inventory problem. This approach is especially useful for FGP problems having both unequal weights and unbalanced membership values. Also, sensitivity analysis on changes to values in the model can be conducted easily because of the simplified structure of the problem.

The model incorporates fuzzy goals such as budget, warehouse space and amount of investment. The model developed provides the optimal solution subject to the given constraints. Sensitivity analysis considering three different budget structures of the goals has been performed to see the adaptability of the proposed model. The model developed is flexible enough to accommodate other situation-specific constraints. The sensitivity analysis of the result suggests that as the available budget decreases total number of cars also decreases. The model may be used as an analytical tool for appropriately handling similar type of inventory management problems.

References


