The Use of Triangular Systems in a Dynamic Model with Time-of-Day Data: The Hourly Demand for Electricity

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ABSTRACT

To estimate econometric models of electricity demand with time-of-day data, a triangular system of simultaneous equations model is specified. Unlike previous empirical studies, this specification allows the noise components to have arbitrary between day and between hour correlation, despite the presence of lagged dependent variables. Problems associated with the dynamic specification due to hourly data, the nonlinearity of functional form, and the weekly cycle components are also considered. Although the model proposed has an ARMAX representation, it is more convenient to rely on the method of Three Stage Least-Squares or Instrumental Variables to obtain consistent estimates of the unknown parameters and their covariance matrix. The methodology is demonstrated in an empirical study.

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I. Introduction

Modeling the demand for energy has been an important task in empirical econometrics recently. Besides its practical value in energy economics, the undertaking is associated with many interesting issues of estimation, inference, and forecasting. See Dubin and McFadden (1984), Gallant and Koenker (1984), and Engle, Granger, Rice and Weiss (1986) for example. Early studies[1] in this area were based on data of aggregated consumption measured monthly or quarterly. Over the years, time-of-day and end-use data of energy consumption has become available in many databases collected by utility companies or forecasting firms. Since refined time series and cross section data are very useful to extract additional information, this type of data has broadened the subject of interest significantly. For instance, in order to determine optimal time-of-use pricing , it is useful to infer the demand pattern by time of day first from hourly consumption data. See Gallant and Koenker (1984) for a microeconomic model related to this issue. In contrast to time-of-day (TOD) data, end-use data makes it possible to study the joint determination of electric appliance choices, holdings, and energy consumption. Progress of this direction is succeded in Dubin and McFadden (1984). In many cases, the estimation problem involved is non-trivial, particularly when this is formulated in the context of nonparametric models as has been recently done by Engle, Granger, Rice and Weiss (1986).

On the other hand, the most important statistical problem faced by an utility is demand forecasting. In the very short run, decisions regarding the generating mix and the maintenance schedule as well as power transport with interconnected utilities must be made as instantaneous as possible. To this aim, some econometric models are solicited to compete with approaches originated from different disciplines, as is highlighted in the volume edited by Bunn and Farmer (1985). So far, the representative econometric model is the one proposed by Ramanathan, Granger, and Engle (1985); hereafter the reference is to

[1] See Taylor (1975) for an excellent survey.

be abbreviated as RGE. Their introductory section contains a complete description on the determination of electricity demand in the very short run. However the econometric specification of RGE, relying on the least-squares-with-dummy-variables, does not fully reflect what was done descriptively. To see this point, the following quote from RGE is necessary.

"The electricity usage at a given hour of the day is determined by two types of variables. The first is related to the day of the week and the time of the day. This will include both life-style effects as well as environmental determinants. Those resulting from the life-style of a household are: the time that a husband and/or wife is at work and the children are at school, the types of activities enjoyed in the evenings and weekends, etc. The environmental causes include the levels of temperature, windspeed, and humidity, the number of hours of sunlight, the occurrence of public holidays, time-of-day pricing, if any, etc. All these variables changes quickly from hour-to-hour or day-to-day.

The second type of variable determining the hourly load is that which changes slowly over time: income, family size and composition, the size of house, the type and number of appliance stocks, etc. If we are dealing with regional demand patterns rather than household demand, the list of variables would include population, income and employment levels industrial mix of the area, its demographic attributes, appliance saturation levels, and the price of electricity" (RGE 1985, p.132)

In light of this description, RGE then suggest a two-step methodology. First, a time series model with 24 equations and K parameters are estimated using quickly changing variables as regressors. The time horizon for this model would be a quarter or a month depending on the availability of the slowly varying variables. Hence, if there are N years, we will have $4N$ or $12N$ estimates for each parameter which is assumed to vary across quarters or months because of differences in the slowly varying variables. Second, a cross section model with

K equations are estimated using the slowly varying socio-economic variables as regressors. The dependent variables in the second step are the parameters of the first step.

The first-step estimator used in RGE is a fixed-effect model. Twenty four dummy variables are used to eliminate the hour-specific effects. They do not introduce lagged dependent variables into the model, while an AR(1) serial correlation is assumed. Besides, the slope coefficients are assumed to be constant across hours.[2]

In contrast to RGE, the present paper proposes an alternative specification using dynamic simultaneous equations models. It follows that we can start with a more general specification which allows state dependence, hourly-varying parameters, and heteroskedastic or serially correlated errors with unknown forms.

The paper is organized as follows. In section II, we first transpose the time-of-day (TOD) data into a dynamic model related to panel data. We then consider a variance components model with random day and random time effects. In this case, the likelihood function of a triangular systems with variance components is derived following Maddala (1971), and Bhargava and Sargan (1983). Throughout this process, we stress that there is no a priori information to assume constant slope coefficients over time. Thus a general model is suggested for our purpose. There are other issues related to the functional forms of the hourly demand functions. We discuss those aspects in section III. Section IV is an empirical study. We obtain consistent estimates of the unknown parameters and their covariance matrix by the method of instrumental variables. A 24 equations system with 192 parameters is estimated simultaneously. This approach is different from previous studies using time series models, including Box-Jenkins, Kalman, Wiener, Spectral, and Double

[2] The data set used in RGE is a sample of 400 Connecticut residents, half of them are subject to the experiments of time-of-use pricing. Unlike their data, the data we use is the aggregated consumption measured hourly.

Exponential Smoothing; see Parzen and Pagano (1979) and Bunn and Farmer (1985) for example. The final section is our concluding remarks about the TOD data.

II. Triangular Systems for Time-of-Day Data

In this paper the dependent variable of the TOD data is to be denoted by a double array $\{Y_{dt}, d = 1, 2, \cdots, D; t = 1, 2, \cdots, T\}$, where d refers to day and t is an index for time. The unit of time can be an hour, a half hour, or a quarter hour etc. Looking into the time path of energy consumption, a TOD data set provides refined dynamic information. Besides, when $T \to \infty$, the model approximates a continuous time model. Data of this type is very important and is increasingly available in business statistics. There is no doubt that soon TOD data will occupy an important subset of many databases of economics.[3] Without losing generality, it is assumed that the unit time interval is an hour. In this case the consumption is a flow variable which integrates the power demanded (stock variable) within a specific hour.

In general, one quick way to deal with the TOD data is to borrow the models used to deal with panel data. In its usual format, the data set available is $\{Y_{dt}, X_{dt}, W_d\}$ where X_{dt} is time-varying but W_d only varies between days. Apart from RGE's description, we may include the variables of lagged demands to construct a dynamic model. One earlier example is the study of Balestra and Nerlove (1966) on the demand for natural gas. Anderson and Hsiao (1982) refer it as a model including state dependence and investigate various dynamic specifications in this context. For the present study, we may interpret it as follows: a subset of the residential customers will continue their energy-demanded activities across hours. Therefore, we may assert a simple dynamic system by writing

$$Y_{dt} = \beta Y_{d,t-1} + X_{dt}\gamma + W_d\alpha + \zeta + U_{dt} \qquad (2-1)$$

[3] A leading example is the data set of energy consumption collected by the Data Resources Incorpration.

where β, γ, α, and ζ are fixed unknown parameters. The dimensions of X_{dt} and W_d are K_t and G, respectively. Assume that U_{dt} is K_t and W_d is G. Assume that U_{dt} is the noise component of the load system with mean zero. This notation implies an identity due to the reshaping of the original time series. It follows that the daily initial condition is the terminal demand of the previous day, i.e.

$$Y_{d0} \equiv Y_{d-1,T}, \quad \text{for all } d. \tag{2 - 2}$$

We assume that the initial condition of the dependent variable Y_{10} is fixed in repeated samples. If we rewrite the model above using matrix notation, which is often used in the context of simultaneous equations models, then we have

$$YB + \tilde{Z}\tilde{\Gamma} = U \tag{2 - 3}$$

where $\tilde{Z}$ denotes the matrix of predetermined variables and $(B|\tilde{\Gamma})$ is a matrix of parameters subject to a set of constraints. Since the stochastic errors U_{dt} may be correlated across hours (equations), strictly speaking the model is not a recursive system. In fact, it is a special case of the triangular systems as the matrix B is an upper-triangular matrix. Properties of the triangular systems in simultaneous equations models are remarked in Lahiri and Schmidt (1979). The formulation has been applied to dynamic models with random effects in Bhargava and Sargan (1983). It is straightforward to write a more general model where the endogenous variable Y_{dt} depends on the path formed by $Y_{d,t-1}, \cdots, Y_{d,1}$ dated d. Hence,

$$\begin{aligned} Y_{d1} &= \beta Y_{d-1,T} + X_{d1}\gamma + W_d\alpha + \zeta + U_{d1} \\ Y_{d2} &= \beta Y_{d,1} + X_{d2}\gamma + W_d\alpha + \zeta + U_{d2} \\ Y_{d3} &= \beta Y_{d,2} + \beta(1)Y_{d1} + X_{d1}\gamma + W_d\alpha + \zeta + U_{d3} \\ &\vdots \\ Y_{dT} &= \beta Y_{d,T-1} + \sum_{j=1}^{T-2} \beta(j)Y_{dj} + X_{d1}\gamma + W_d\alpha + \zeta + U_{dT} \end{aligned} \tag{2 - 4}$$

where $\beta(j), j = 1 \cdots T-1$, are additional parameters associated with the lagged dependent variables Y_{dj}. It is worth emphasizing that these lagged variables now appeare as endogenous variables from the system point of view. Each individual equation of this model apparently resembles the demand equations estimated in Balestra and Nerlove (1967). As is pointed out in Bhargava and Sargan (1983), it is desirable to estimate this type of model using simultaneous equations techniques. To further see this point, consider a model with random time effects and random hour effects. Following the literature of variance components, we can write

$$U_{dt} = \tau_t + \mu_d + \nu_{dt} \qquad (2-5)$$

where μ_d are the random day effects which are normally idstributed with mean zero and variance σ_μ^2. Similarly, the random time effects τ_t are IN$(0,\sigma_\tau^2)$ and the residual effects ν_{dt} are IN$(0,\sigma_\nu^2)$. These error terms $(\mu_d, \tau_t, \nu_{dt})$ are assumed to be mutually and intertemporally independent. The normality assumption is innocuous since it is not required for the least squares approach.

In panel data, people often emphasize the latent individual effects and ignore the random time effects. For the present model, obviously, the time effects should not be neglected. To see the difference between the random effects model and the general simultaneous equations models, we first derive a concentrated likelihood function which has been discussed in Maddala (1971) and Bhargava and Sargan (1983) under different emphasis. Our goal is to see the restrictions imposed on the variance-covariance matrix of U due to the variance components assumptions in connection with Eq. (2-5).

Under the normality assumption, it is can be shown that

$$E[(\mathrm{vec}U')'\mathrm{vec}U'] = \sigma_\nu^2 I_{DT} + \sigma_\mu^2 I_D \otimes e_T e_T' + \sigma_\tau^2 I_T \otimes e_D e_D' \qquad (2-6)$$

where e_D and e_T are $D \times 1$ and $T \times 1$ column vectors, respectively, with all elements equal to one. Denote the variance matrix above by Ω. Then the log-likelihood function can be written as

$$-2\mathcal{L}_1 = \text{const.} + \log\det\Omega - 2\log|\det B| + (\text{vec}U')'\Omega^{-1}\text{vec}U', \qquad (2-7)$$

where the Jacobian term $\det(B)$ is equal to one as is implied by the triangular system. Now using the four eigenvalus derived in Nerlove (1971), we have

$$\log\det(\Omega) = (D-1)(T-1)\log\sigma_\nu^2 + (D-1)\log(\sigma_\nu^2 + T\sigma_\mu^2) +$$

$$(T-1)\log(\sigma_\nu^2 + D\sigma_\tau^2) + \log(\sigma_\nu^2 + T\sigma_\mu^2 + D\sigma_\tau^2). \qquad (2-8)$$

It is useful to define two ratios of variance components. Let us use $\theta_1 \equiv \sigma_\nu^2/(\sigma_\nu^2 + T\sigma_\mu^2)$, $\theta_2 \equiv \sigma_\nu^2/(\sigma_\nu^2 + D\sigma_\tau^2)$, and notice that θ_1, $\theta_2 \in (0,1]$. In the simplest case where $\theta_1 = 1$ and $\theta_2 = 1$, the solution of the likelihood equation is identical to the OLS estimate. In general, the maximum likelihood estimator is a weighted average of the between day, between hour , and within groups estimators; see Maddala (1971). Note that the weights can be parameterized differently as is shown in Bhargava and Sargan (1983). They cosider a case where ($\theta_1 = \sigma_\mu/\sigma_\nu$) and $\theta_2 \equiv 0$ (no time effects) to derive the concentrated likelihood function. To consider a model with random day and time effects, we prefer to use the former notation of θs. Denote an identity matrix with dimension N by I_N. Following Nerlove (1971), one can deduce the inverse matrix

$$\Omega^{-1} = \frac{1}{\sigma_\nu^2}[\Omega_0 + \theta_1\Omega_1 + \theta_2\Omega_2 + \frac{\theta_1\theta_2}{(\theta_1 + \theta_2 - \theta_1\theta_2)}\Omega_3] \qquad (2-9)$$

where

$$\Omega_0 = I_{DT} - I_D \otimes \frac{e_T e_T'}{T} - \frac{e_D e_D'}{D} \otimes I_T + \frac{e_{DT} e_{DT}'}{DT}$$

$$\Omega_1 = (I_D \otimes \frac{e_T e_T'}{T}) - \frac{e_{DT} e_{DT}'}{DT}$$

$$\Omega_2 = (\frac{e_D e_D'}{D} \otimes I_T) - \frac{e_{DT} e_{DT}'}{DT}$$

$$\Omega_3 = \frac{e_{DT} e_{DT}'}{DT}.$$

Concentrating $\mathcal{L}_1$ with respect to σ_ν^2 in equation (2-7), we obtain[4]

$$-2\mathcal{L}_1^*(B,\tilde{\Gamma},\theta_1,\theta_2) = \text{const.} + \log(\theta_1+\theta_2-\theta_1\theta_2) - T\log\theta_2 - D\log\theta_1 + DT\log\hat{\sigma}_\nu^2 \quad (2-10)$$

where

$$DT\hat{\sigma}_\nu^2 = \text{trace}(UU') + \frac{\theta_1-1}{T}\text{trace}(J_T U'U) + + \frac{\theta_2-1}{D}\text{trace}(UU'J_D) +$$

$$\frac{1}{DT}\left(\frac{\theta_1\theta_2}{\theta_1+\theta_2-\theta_1\theta_2} - \theta_1 - \theta_2 + 1\right)[\text{sum}(\text{vec}U')]^2.$$

We note that J is a matrix having all elements unity. The sum operator above is used to sum all of the elements of vecU'. Notice that the likelihood function (2-10) is a function of $B,\tilde{\Gamma},\theta_1$, and θ_2, because $U = YB + \tilde{Z}\tilde{\Gamma}$. Using this approach, an interesting fact is that the stochastic errors are now serially correlated because of the time-specific effects. It is easy to show that

$$E(U_{dt}U_{d't}) = \sigma_\tau^2, \text{ and } E(U_{dt}U_{ds}) = \sigma_\mu^2 \quad (2-11)$$

where $d' \neq d$ and $t \neq s$. However, the serial correlation appeared in the first item of Eq. (2-11) does not damp over time. It seems to be a strong assumption, as the error process in question is not even a member of the rich ARMA family. Second there is doubt as to the stability of parameters across hours. From this point of view, the variance component models should not be directly applied to the TOD data of interest. To solve this problem, we may resort to the specification based on the simultaneous equations models. If we assume that the error terms are not serially correlated, then under the normality assumption, the concentrated likelihood function is

$$-2\mathcal{L}_2^*(B,\tilde{\Gamma}) = \text{const.} + \log\det((\text{vec}U')'\text{vec}U') \quad (2-12)$$

Within this model, there is no need to assume equal slope coefficients across hours. In addition, the patterns of correlations of the errors are not so resticted such as Eq. (2-11). The

[4] Two lemmas related to the vec operator are used in the derivation. Suppose that the matrices A, B, and C are conformable for multiplication we can write $\text{vec}(ABC) = (C' \otimes A)\text{vec}B$ and $\text{trace}(AB) = (\text{vec}A')'\text{vec}B$.

weakness of this model is that, in empirical studies, we often encounter heteroskedasticity and/or autocorrelation with unknown forms. Note that the two likelihood functions above are belonging to two non-nested models. A comparison of these two likelihood functions suggests that different norms are used. While Eq. (2-11) invokes trace and sum operators, the second likelihood equation in (2-12) minimizes the determinant of a matrix formed by the squared error terms. Notice that in the simple version of the second model there is no serial correlation of error terms between days. However, the correlation matrix across equations is arbitrary. By contrast, the variance components model allows a particular type of serial correlation, but the between-hour correlation pattern of error terms is very restrictive. Obviously, it is desirable to consider a more general model which can encompass these two cases. We start off with the regularity condition employed in Gallant (1987 chapter 6 and 7). Within this context, consistent estimation and testing are still possible, even when the model is nonlinear and contains both lagged dependent variables and serially correlated errors. Furthermore, many specifications tests can be performed and a good model can be found through progressive simplification. This methodology has been advocated by many econometrians recently; see Sargan (1980) for a case of dynamic specification.

III. Hourly Demand for Electricity

The aim of this section is to provide a general specification to estimate the hourly demand for electricity. It is important to note that our model is not considered in the studies of RGE (1985), Woo et. al. (1986), and the references cited in Bunn and Farmer (1985) or Engle and Watson (1987). To proceed, we maintain a Markov assumption that only the nearest lagged variable influences the current demand. In other words, we assume that the effect due to $\beta(j), j = 1, 2 \ldots$ mentioned in section II are redundant. It follows that the equation system of interest can be written as

$$Y_{dt} = \beta_t Y_{d,t-1} + X_{dt}\gamma_t + W_d\alpha_t + \zeta_t + U_{dt} \qquad (3-1)$$

$$(d = 1, 2, \cdots, D; t = 1, 2, \cdots, T)$$

where the unknown parameters are allowed to be different across hours. Recall that the initial condition Y_{10} is assumed to be fixed in repeated damples. This assumption is reasonable as we have assumed that across seasons the parameters are changing. Of course, other treatments are possible but their value needs to be weighted againt the involved assumptions. The reader is referred to Anderson and Hsiao (1982) for the complexity arising from the assumptions related to the initial condition of panel data.

Using matrix notation, the system can be concisely expressed as

$$YB(L) = Z\Gamma + U \tag{3 - 2}$$

where Z is the matrix of exogenous variables in which the first column is a vector of one. We see that in this case equation for hour t plays the role of equation t in a system of T equations. Notice that L is the lag operator. In terms of this expression, the matrix $B(L)$ is not a upper-triangular matrix. In fact there is a nun-null element $-\beta_1 L$ appeared in the southwest corner of the matrix. Denote the adjoint matrix of $B(L)$ by $B^*(L)$ and note that $\det(B(L)) = 1 - \prod_{t=1}^{T} \beta_t L \equiv 1 - \beta_* L$. An useful account of this derivation is available in the appendix. Hence we can write the autoregressive final form as

$$(1 - \prod_{t=1}^{T} \beta_t L)Y = Z\Gamma \mathrm{Adj}(B(L)) + U\mathrm{Adj}(B(L)).$$

or

$$(1 - \beta^* L)Y = Z\Gamma B^*(L) + UB^*(L) \tag{3 - 3}$$

For each equation, we see that the coefficient associated with its own lagged dependent variable is a constant $\beta_* \equiv \prod_{t=1}^{T} \beta_t$. It can be shown that the necessary and sufficent condition of dynamic stability of the system is $|\beta_*| < 1$. This assumption is not necessary for consistent estimation, but it is important to keep this assumption for the purpose of statistical inferences. From this derivation, we also remark that there is no need to assume that every equation is stable. We will illustrate this point in our empirical study.

It is interesting to note that the equations in (3-3) has an ARMAX representation as follows:

$$Y_{dt} = \beta_{*} Y_{d-1,t} + X^{*}_{dt} + \eta_{dt}$$

where

$$\eta_{dt} = \sum_{\tau=1}^{T} u_{d\tau} B^{*}(\tau, t)$$

and

$$X^{*}_{dt} = \sum_{\tau=1}^{T} [X_{d\tau}\gamma + W_{d}\alpha + \zeta_{\tau}] B^{*}(\tau, t).$$

From Eq. (3-3) it follows that for an hourly model to be estimated with daily data the error terms are unlikely to be white noises.

Although the correlation structure of U_{dt} is not parameterized, consitent esstimation of the structural parameters is still possible. However, because of the nonparametric specification of error terms, efficient estimation using the likelihood approach is not appropriate here. As is well known, there is an inevitable trade-off to sacrify efficiency for robustness. On the other hand, since many nonlinear constraints need to be imposed, we make no effort to estimate the model using the ARMAX representation. The convenient approach to be adopted is the framework of simultaneous equations models with the method of instrumental variables. For the present study, the instruments to be used are measurements of temperature, humidity, and the sinusoidal components added to explain the weekly cyclical effects. Unlike most of macroeconomic variables, it seems that the instruments used are strictly exogenous. Otherwise, exogeniety tests should be performed to support this specification.

There are other issues associated with the specification of hourly demand for electricity. Previous studies generally ignore the possibility that both autocorelated errors and lagged dependent variables may enter the model. Instead, there are studies emphasizing the non-linear relation between electricity demand and temperature, see Engle et. al. (1986) for example. In this section, we argue that once the state dependence and serial correlation are taken into account, a quadratic form of the temperature variable will approximate that nonlinearship satisfactorily.

Due to life-style effect, it is quite possible that there is a weekly cycle component contributed to the determination of the electricity demand. For instance, during the weekend the electricity consumption seemingly to be higher especially in the evening. This situation recurrs weekly. From elementary spectral analysis, a time series with weekly periodicity can be extracted by using the periodigram $a_1 \sin(\omega d + a_2)$ with $\omega = 2\pi d/7$. Using a simple trigonometric identity, we can write

$$a_1 \sin(\omega d + a_2) = \alpha_1 \cos(\omega d) + \alpha_2 \sin(\omega d) \qquad (3-4)$$

where $a_1 = \sqrt{\alpha_1^2 + \alpha_2^2}$ and $a_2 = \arctan(\alpha_1/\alpha_2)$.

In case there are public holidays during the sampling period , we decide to use a dummy variable to absorb this intervention effect. Equilvalently, we assume that the holiday effect only influences the intercept terms.

In summary, the following exogenous variables are introduced in our specification. The regressors include linear and quadratic term of temperature, linear term of humidity, cosine component, sine component, the holiday dummy variable, and a constant term associated with the intercept. Note that the last four regressors including a constant are hourly invariant. This specification is a member of the model specified in section II. This is so, because the model is liner in parameters, though we have included a quadratic approximation and a sinusoidal component. Within this context, we can jointly estimate the effects due to state dependence and other variables with nonlinear relations. Moreover, it avoids the use of computer-intensive techniques based on the methods of spline or switching regression models. To obtain tractible simulation and optimal forecasting, needless to say, a parametric model with linear structure is preferred at this time.

There are other advantages of using SEM to deal with the TOD data. For instance, we can estimate the impact of temperature over different time horizons. The notion of dynamic multipliers are suitable for this purpose. As is to be demonstrated in the next section, we can estimate the unknown switching point based on the matrix of the impact multiplier.

If we assume that U_{dt} are uncorrelated between days, then the asymptotical equivalence between Three-Stage Least Squares and Full-Information Maximum Likelihood is well known. Nevertheless, using FIML in large-scale models is so far not practical. Suppose that there are 8 parameters in each equation. While maximizing the likelihood function, we need to invert an 192×192 matrix for each iteration. From this concern, the 3SLS is more attractive, not mention that the normality assumption is not required.

Two cases need to be discussed. In the presence of serial correlation, the usual 3SLS estimator is no longer consistent. To overcome this problem, one only has to exclude the lagged endogenous variables Y_{d0} from the instruments set. For easy distinction, we denote the second estimator by IV, although it is known that the 3SLS is also a special case of the IV estimator. In the presence of unspecified autocorrelation, it is non-trivial to estimate the covariance matrix of the parameter estimates.[5] Recently, the problem has been treated in Gallant (1987) and Newey and West (1987). To apply their results in the present paper, it is useful to use an alternative expression of the system. We write

$$\begin{aligned} Y_t &= \beta_t Y_{t-1} + X_t \gamma_t + W \alpha_t + \zeta_t + U_t \\ &= Q_t \delta_t + U_t \end{aligned}$$

where Y_t is the tth column of the matrix Y and

$$Q_t = (Y_{t-1}, X_t, W, e_D)$$

$$\delta_t = (\beta_t, \gamma_t', \alpha_t', \zeta_t)'.$$

[5]Notice that in the presence of lagged dependent variables and autocorrelated errors, it is not legitimate to apply existing resampling techniques such as bootstraping here.

We then write this set of equations by stacking all of the equations,

$$Y_* = Q_*\delta_* + U_*$$

where $Q_* = \text{diag}(Q_1, Q_2, \cdots, Q_T)$. The 3SLS estimator in the present notation can be written as

$$\tilde{\delta}_* = \{Q_*'[\bar{\Sigma}_{\tilde{z}}^{-1} \otimes \tilde{Z}(\tilde{Z}'\tilde{Z})^{-1}\tilde{Z}'Q_*]\}^{-1}Q_*'[\bar{\Sigma}_{\tilde{z}}^{-1} \otimes \tilde{Z}(\tilde{Z}'\tilde{Z})^{-1}\tilde{Z}']Y_* \qquad (3-5)$$

where $\bar{\Sigma}_{\tilde{z}}$ is estimated from the residuals of 2SLS estimation using $\tilde{Z}$ as instruments. Note that the residuals are computed from the original equation not the purged equation. The covariance matrix of the 3SLS estimator can be consistent estimated by using

$$\{Q_*'[\bar{\Sigma}_{\tilde{z}}^{-1} \otimes \tilde{Z}(\tilde{Z}'\tilde{Z})^{-1}\tilde{Z}']Q_*\}^{-1} \qquad (3-6)$$

Note that $\tilde{Z}$ is the matrix of predetermined variables. If U_t are serially correlated then a consistent estimator of δ is to use $\tilde{Z}$ exclusive of Y_{d0} as instruments. In other words, we project Y and Y_{d0} on to the column space spanned by the exogenous variable Z. It follows that the IV estimator is

$$\hat{\delta}_* = \{Q_*'[\bar{\Sigma}_z^{-1} \otimes Z(Z'Z)^{-1}Z']Q_*\}^{-1}Q_*'[\bar{\Sigma}_z^{-1} \otimes Z(Z'Z)^{-1}Z']Y_* \qquad (3-7)$$

Although $\bar{\Sigma}_z$ is not necessarily a consistent estimator of $E(U_d'U_d)$, consistency of the IV estimator still follows. We assume that the model is identifiale in the sense of Hannan. Under general regularity conditions of IV approach, the asymptotic normality of $\hat{\delta}_*$ can be derived too. It is important to note that the following formula adopted from equation (3-6) is inconsistent in the presence of heteroskedasticity or autocorrelation.

$$\{Q_*'[\bar{\Sigma}_z^{-1} \otimes Z(Z'Z)^{-1}Z']Q_*\}^{-1} \qquad (3-8)$$

Following Gallant (1987), a consistent covariance matrix for the IV estimator is

$$\{Q_*'[\bar{\Sigma}_z^{-1} \otimes Z(Z'Z)^{-1}Z']Q_*\}^{-1}\{Q_*'[\bar{\Sigma}_z^{-1} \otimes Z(Z'Z)^{-1}]\hat{V}[\bar{\Sigma}_z^{-1} \otimes (Z'Z)^{-1}Z']Q_*\}$$

$$\{Q_*'[\bar{\Sigma}_z^{-1} \otimes Z(Z'Z)^{-1}Z']Q_*\}^{-1} \quad (3-9)$$

where $\hat{V}$ is to be computed using residuals and lagged residuals obtained from the 2SLS which employes Z as instruments to estimate the equations sequentially. While Newey and West (1987) use Bartlett wights, the formula proposed in Gallant (1987) adopts Parzen weights. In both cases, the number of the weights need to increase with the sample size with order $o(D^{\frac{1}{5}})$. In the next section where the sample size is 91, the appropriate formula of $\hat{V}$ is

$$\hat{V} = \hat{S}_{D0} + \frac{1}{4}\hat{S}_{D1} + \frac{1}{4}(\hat{S}_{D1})'$$

where

$$\hat{S}_{D\tau} = \sum_{d=1+\tau}^{D} [\bar{U}_d' \otimes Z_d'][\bar{U}_{d-\tau}' \otimes Z_{d-\tau}']' \quad \tau = 0, 1,$$

and $\bar{U}_d$ is the dth row of the residuals matrix $\bar{U}$ computed form the 2SLS estimation using Z as instruments. Notice that Z_d is the dth row of Z. Obviously, the 3SLS and IV estimators described above are two polar cases. In fact, if we have reasons to assume that the error terms is a vector autoregressive process, we can use Hatanaka's (1974) two-step residuals adjusted estimator, which is consistent and asymptotically efficient.

IV. Empirical Results

The TOD data set at hand was taken from a database of Florida Power Corp. The company expects to have an econometric model resulted from our investigation for the purposes of demand forecasting and rate-making. As is suggested initially, the econometric analysis should include the impact of temperature, season, and day of week. In addition, the company wanted to incorporate sequential hourly dependence into what was previously a straightforward pooling of time-series and cross section data—which assumes independence.

These objects can be achieved based on our specification proposed in Section III. Hence the remaining task is to actually estimate a system of 24 equations for demonstration. We again assume that the state dependence is of first order. Thus, only the nearest lagged demand enters each equation.

Consequently, we can write the system to be actually estimated as

$$Y_{dt} = \beta_t Y_{d,t-1} + \sum_{k=1}^{3} \gamma_{kt} x_{kdt} + \sum_{g=1}^{3} w_{gd} \alpha_{gt} + \zeta_t + u_{dt} \qquad (4-1)$$

where Y_{dt} is the residential demand in Kilowatts per customer.[6] The time varying variables x_{kdt} contains linear and quadratic terms of temperature and the percentage of relative humidity. There is no time-of-day pricing; and the variables of wind-speed and the number of hours of sunlights are not available. Otherwise, these variables should be included too. Note that the temperature is the dry bulb temperature divided by 100. Including the humidity variable, these weather variables are computed originally by the utility using system weighted method over the whole serving territory. Next, the time-invariant variables include the cosine and sine components, and a dummy variable to absorb the effects of public holidays. Given this situation, the functional form to be estimated is

$$Y_{dt} = f(\text{LAGY}, \text{TEMP}, \text{TEMPSQ}, \text{HUMID}, \text{COS}, \text{SIN}, \text{HOLIDAY}) + U_{dt}$$

The expected signs of the coefficients are listed as follows:

$$\beta_t > 0,\ \gamma_{1t} < 0,\ \gamma_{2t} > 0,\ \gamma_{3t} > 0,\ \alpha_{1t}(?),\ \alpha_{2t}(?),\ \alpha_{3t}(?),\ \zeta_t > 0.$$

The expected sign of β_t is positive for this is a common pattern of consumption data. Nevertheless, there is no guarantee that these coefficients are uniformly less than 1. To see

[6] There are 7 classes of customers: (a) residential (b) commercial (c) industrial (d) street lighting (e) public authorities (f) rural electric (7) municipals. Only the first class, which contains 0.8 million customers, is estimated in this study. For a standard load curve of this territory see Figure I in the appendix.

this point, consider the consumption in the evening at 17 and 18 o'clock ,the second time series is likely to be greater most of time due to dinner cooking. This pattern can happen in the morning too. This observation suggests that the coefficients are different across hours. In addition, it is possible to see a subsystem of nonstationary time series even when the system is stable.

Coming next to the coefficients γ_{t1} and γ_{t2}, as these two parameters are associated with the quadratic form used to approximate a V-shape function, we expect to have positive signs on the quadratic terms. To see that the coefficients of linear terms are negative, we rewrite equation (4-1) in its final form

$$Y_{dt} = \sum_{k=1}^{3} \gamma_{kt} x_{kdt} + \sum_{g=1}^{3} w_{gd} \alpha_{gt} + \zeta_t + u_{dt} + R_t \qquad (4-2)$$

where the remainder R_t is a function of the initial condition Y_{10} and a sequence of the exogenous variables and stochastic errors; see eq. (A-2) in the appendix. Invoking conditional expectation on Y_{dt} and taking partial derivative with respect to the temperature variable, we obtain

$$\frac{\partial E(Y_{dt})}{\partial x_{1dt}} = \gamma_{1t} + 2\gamma_{2t} x_{1dt} = 0 \qquad (4-3)$$

Solving the first order condition above, we can estimate the expected minimum of the V-shape function. Notice that the solution is corresponding to the switching point between the heating regime and cooling regime. In previous studies, the switching points are treated differently. Often the number is assumed to be equal across hours, and to be known in the neighborhood of the 65 degree point. The nonlinear relation is illustrated in Figure II in the Appendix. We notice that the pattern is quite strong no matter which time scale is used. Consequently, the sign of γ_{1t} should be negative to ensure a switching point with positive number.

As to the humidity variable, we expect to have a positive coefficient. If we are estimating the model of industrial demand, the chance of having positive estimates is even higher. This is so, because many production processes require strict control of the humidity level. The signs of the sine and cosine components are uncertain. In fact, these two coefficients are related to the periodogram of the dependent variables Y_{dt}. The introduction of the state dependence and the sinusoidal components at the same time makes it difficult to predict the signs of these two coefficients.

It is important to note that our sample ran from 09/01/84 (Saturday) to 11/30/84 (Friday); so $D = 91$. The selection of this period as a quarter is not arbitrarily. We try to classify the load data into four regimes (quarters) such that within each quarter the model has an invariant structure.[7] There were two public holidays dated on 09/03/84 (Labor Day) and 11/22/84 (Thanksgiving). Depending on the hour of the holiday, the coefficient may or may not be positive. Since the socio-economic variables are not gathered in the data set currently, we can not perform the second-step estimation as is shown in REG (1986) and Woo. et. al. (1986).

The estimation procedure is tedious but straighforward if we make use of the IV interpretation to code the system estimators. The instruments we used include all of the exogenous variables defined in equation (4-1). There are 72 hourly varying instruments and 3 hour-invariant instruments. Plus the constant term, the number of instruments is 76. In case the number of instruments is more than D, the number of days, one can use the principal components of the instrumental variables to do the job.

The estimation results are discussed in the following. There are two points related to the preliminary analysis worth mentioning. First, to see whether Y_{d0} is exogenous, we estimate the first equation using the ordinarl least-squares and the single equation IV estimator, respectively. Then a Hausman test is performed. The test statistic is 7.9944 which is significant at 5% level. Thus, the null hypothesis that the lagged dependent variable Y_{d0} is

[7] To support this point, see Figure III in the Appendix.

exogenous is rejected. Second, we use Godfrey's (1976) π test to test the existence of AR(1) errors for each equation. It is important to note that Durbin's h test and his modefied cumulative periodiogram test are not appropiate in this case. The π tests are rejected in equations of hours 6, 11, 16, 17, 18, and 19. This evidence suggests that the system noises are not independent between days, and the 3SLS is not appropriate here. Guided by the above evidence, we decide to estimate the model using the full-information IV estimator and Hatanaks's two-step estimator. We also assume that the intercepts are equal across equations. Without this assumption some estimates of the intercept terms are negative thereby making interpretation difficult.[8]

Table I presents the coefficient estimates using the IV estimator. The table also provides information of statistical significance for individual coefficient. Several implications are worth emphasizing. First, the constant term is 0.9265 (Kilowatts). Approximately, this is the fixed amount of electricity consumed per customer within an hour (not during the public holidays). Second, there are six equations in which the estimates associated with the lagged dependent variables are greater than 1. These equations are at hours 6, 13, 15, 16, 17, and 18. However, since $\prod_{t=1}^{24} \hat{\beta}_t \doteq 0.15408 < 1$, the system is dynamic stable, not surprisingly.

Third, except at hours 7 and 18, the linear terms of temperature are all significant with negative coefficients. It is also true that, except at hours 7 and 18, the quadratic term are all significant and positive. The exception makes sense. Because in the morning at hour 7 and in the evening at hour 18 the demand for electricity is of central importance to the households. This fact makes the demand relatively insensitive to the square term of the temperature variable and explains why γ_2 is not significant at these two hours. Notice that the inference is based on the formula of Eq. (3-9). Otherwise, since the standard errors computed by (3-8) is underestimated, the conclusion will be different. See Table I

[8] Perhaps, the problem is due to the fact that the unconstrained model resembles a fixed-effect model in which the dummy variable tends to elliminate a significant portion of the variation of the dependent variables and the regressors. Consequently, the convergence rate is slower and the estimates may fluctuate more.

TABLE I. Parameter Estimates of the Hourly Demand for Electricity (IV Estimator)

EQU	INTERCEPT	LAGY	TEMP	TEMPSQ	HUMID	COS	SIN	HOLIDAY	SWITCH
1	0.9266**	0.85992**	-3.2335**	2.36276**	0.16594	0.00689#	0.01121	-0.03991#	68.43
2	0.9266**	0.85392**	-2.6092**	1.97712**	-0.04657#	-0.00001#	0.00495#	0.06243	65.99
3	0.9266**	0.91423**	-2.7368**	2.02707**	0.00424#	0.00660#	-0.00510#	-0.03309#	67.51
4	0.9266**	0.87554**	-2.7888**	2.09321**	0.06400#	0.00024#	-0.00338#	0.00498#	66.62
5	0.9266**	0.90693**	-2.7923**	1.97795**	0.13681	-0.00011#	-0.00150#	-0.00064#	70.59
6	0.9266**	1.03584**	-2.8282**	2.03101**	0.13143#	0.00781#	-0.03725**	-0.06429	69.63
7	0.9266**	0.99462**	-1.8119**	1.04120	-0.02937#	-0.01560	-0.07388**	-0.10535	NA
8	0.9266**	0.94984**	-2.4338**	1.51505**	0.22776	-0.00212#	-0.00336#	0.05420#	80.32
9	0.9266**	0.78320**	-2.6515**	1.89590**	0.30568	0.02075#	0.06880**	0.33221**	69.93
10	0.9266**	0.89346**	-3.4795**	2 88634**	0.15958	0.01416#	0.09239**	0.25875**	60.27
11	0.9266**	0.91652**	-3.6089**	3.03439**	0.21181	0.00258#	0.02011	0.13260	59.47
12	0.9266**	0.94288**	-3.6947**	3.13282**	0.17019	-0.01765	0.00401#	0.06051#	58.97
13	0.9266**	1.02263**	-3.4640**	2.72817**	0.16482	0.00194#	-0.00791#	-0.09092	63.49
14	0.9266**	0.92478**	-3.3795**	2.88588**	0.00599#	0.01849	0.01223#	0.04178#	58.55
15	0.9266**	1.03624**	-3.2157**	2.48344**	0.06045#	-0.02421	-0.01703	-0.09610	64.74
16	0.9266**	1.00700**	-2.8593**	2.19526**	0.01739#	-0.00627#	-0.03673	-0.11380	65.13
17	0.9266**	1.02731**	-2.2570**	1.56035**	-0.03334#	-0.01177#	-0.04144	-0.09278#	72.32
18	0.9266**	1.09282**	-1.2158**	0.29603#	-0.26725	-0.02895	-0.04123	-0.19780	NA
19	0.9266**	0.91968**	-2.2018**	1.35733**	0.17314	-0.02179	-0.01296#	0.00097#	81.11
20	0.9266**	0.89628**	-2.7773**	2.05339**	0.10275#	-0.01369#	-0.00459#	0.02895#	67.63
21	0.9266**	0.88135**	-2.7221**	2.08095**	0.02023#	0.00599#	-0.01837	0.09670#	65.41
22	0.9266**	0.85999**	-3.0338**	2.31757**	0.13265#	0.00787#	0.00301#	0.03101#	65.45
23	0.9266**	0.88136**	-2.4756**	1.66242**	-0.03953#	0.02060	-0.00415#	-0.00795#	74.46
24	0.9266**	0.80070**	-2.8248**	2.12727**	0.04783#	0.02181	0.01280#	0.01908#	66.40

NOTES: 1. #Not Significant at 0.05 level using formula (3-8)
2. **Significant at 0.05 level using formula (3-9)

for a straightforward comparison. For example, if the inferences are drawn based on the incorrect standard errors, we see that the humidity variable is significant at hours 1, 5, 8-13, 18, 19. For these equations with significant estimates, the signs are all positive except that of hour 18. Similarly, we observe many significant coefficients associated with the holiday variable and the sinusoidal components.

Using the formula

$$x_t^* = -\gamma_{1t}/2\gamma_{2t}$$

We are able to estimate the switching point for each equation except at hour 7 and 18. In these two exceptional equations, the estimated coefficients of γ_2 seem to be zero. Consequently, the estimated switching points are not reliable. Further explanations are in order. At hours 7 and 18, it is expected that most of the appliances are used. So, even the heat increases the second-order effect of increasing demand is not significant.

For other equations, the estimates of the switching points are reported in the last column of Table I. It is interesting to note that the switching points are close to the values used in many previous studies. Nevertheless, the estimates obtained here avoids the pitfall to choose a fixed switching point first. Overall, if we use the correct covariance matrix of the IV estimator, the coefficients of the hourly invariant regressors are not important.

Table II presents estimates based on the two-step estimator. The column of ρ provides the autocorelation estimates of equations 6, 11, 16, 17, 18, and 19. For the equations with insignificant π test statistics, the estimates are set to be zero for the sake of asymptotical efficiency of the two-step estimator. We note that the autoregressive process used in the particular equation does not include lagged errors of other equations. The assumption is plausible in that *the noise at hour t is unlikely to be correlated with the noise at hour τ of the previous day, where $\tau \neq t$.* The estimates of INTERCEPT, LAGY, TEMP, TEMPSQ, and SWITCH of Table II is similar to that of Table I. By contrast, many coefficents of the

TABLE II. Parameter Estimates of the Hourly Demand for Electricity (Two-Step Estimator)

EQU	INTERCEPT	LAGY	TEMP	TEMPSQ	HUMID	COS	SIN	HOLIDAY	RHO	SWITCH
1	0.9066**	0.8614**	-3.1568**	2.3042**	0.1589**	-0.0384	0.0069	0.0112**	0.0000	68.501
2	0.9066**	0.8553**	-2.5607**	1.9358**	-0.0406	0.0596**	-0.0001	0.0048	0.0000	66.141
3	0.9066**	0.9157**	-2.6821**	1.9825**	0.0072	-0.0337	0.0066	-0.0052	0.0000	67.644
4	0.9066**	0.8774**	-2.7369**	2.0486**	0.0686	0.0047	0.0001	-0.0034	0.0000	66.799
5	0.9066**	0.9092**	-2.7356**	1.9290**	0.1397	-0.0014	-0.0002	-0.0016	0.0000	70.907
6	0.9066**	1.0464**	-2.2202**	1.5206**	0.1351	-0.0740**	0.0072	-0.0383**	-0.2095**	73.004
7	0.9066**	0.9954**	-1.8282**	1.0491**	-0.0001	-0.1093**	-0.0158	-0.0742**	0.0000	87.132
8	0.9066**	0.9526**	-2.4109**	1.5043**	0.2357**	0.0548	-0.0021	-0.0033	0.0000	80.134
9	0.9066**	0.7873**	-2.5814**	1.8478**	0.2930**	0.3392**	0.0207	0.0697**	0.0000	69.851
10	0.9066**	0.8970**	-3.3938**	2.8247**	0.1405**	0.2696**	0.0143	0.0931**	0.0000	60.074
11	0.9066**	0.9033**	-4.0460**	3.3042**	0.1991**	0.1317**	0.0031	0.0215**	0.1368**	61.225
12	0.9066**	0.9479**	-3.6627**	3.1143**	0.1716**	0.0613	-0.0178**	0.0034	0.0000	58.805
13	0.9066**	1.0250**	-3.4100**	2.6890**	0.1640**	-0.0911	0.0019	-0.0081	0.0000	63.406
14	0.9066**	0.9266**	-3.3171**	2.8404**	0.0017	0.0396	0.0185**	0.0122	0.0000	58.391
15	0.9066**	1.0366**	-3.1532**	2.4346**	0.0628	-0.1002**	-0.0243**	-0.0168**	0.0000	64.758
16	0.9066**	0.9939**	-3.3073**	2.5121**	0.0439	-0.1094**	-0.0066	-0.0349**	0.1945**	65.827
17	0.9066**	1.0177**	-2.8818**	2.0036**	-0.0197	-0.0852	-0.0113	-0.0403**	0.1957**	71.916
18	0.9066**	1.0849**	-1.7310**	0.7045**	-0.2459**	-0.1924**	-0.0289**	-0.0406**	0.1485**	122.853
19	0.9066**	0.8954**	-2.9025**	1.9345**	0.1499	0.0158	-0.0223**	-0.0136	0.0516**	75.019
20	0.9066**	0.8957**	-2.6896**	1.9842**	0.0949	0.0320	-0.0136	-0.0044	0.0000	67.775
21	0.9066**	0.8791**	-2.6418**	2.0200**	0.0170	0.1000**	0.0058	-0.0183**	0.0000	65.391
22	0.9066**	0.8603**	-2.9845**	2.2842**	0.1347	0.0289	0.0078	0.0028	0.0000	65.329
23	0.9066**	0.8849**	-2.4223**	1.6163**	-0.0383	-0.0099	0.0207**	-0.0041	0.0000	74.933
24	0.9066**	0.8028**	-2.8037**	2.1094**	0.0615	0.0182	0.0218**	0.0127	0.0000	66.457

NOTES: 1. **Significant at 0.05 level
2. The column of rho is partially based on the pi test. When the null hypothesis can not be rejected, the autocorrelation is set to be zero. When the hypothesis is rejected, we then take it into account while using the full-information version of Hatanaka's two-step residuals adjusted estimator.

hourly invariant regressors are significant in Table II. To explain the abnormal switching point occurs at hour 18, again we argue that the appliances are under maximum use at this hour. Consequently, the second order effect of temperature is less important so that the derived minimum is not appropriate.

Overall, we may conclude that the data is explained well by the proposed model including the following regressors: the intercept, the state dependence, and the temperature variables. For the IV estimator, about 97.2% of the variation of the system load can be accounted by our specification. For the two-step estimator the number is 97.4%. Ultimately, a good criterion to evaluate this model is the accuracy of the out-of-sample forecasting. We intend to report some comparable results of forecasting elsewhere.

V. Concluding Remarks

In this paper, we have studied in some detail several problems that arise in the estimation of hourly residential demand for electricity using time-of-day data. The proposed specification seems to be a potential approach to deal with the TOD data increasingly available in public utilities and financial markets. Further investigation of theoretical models and empirical work are worthwhile.

Keywords. Simultaneous Equations Model, Triangular System, Time-of-Day Demand

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Appendix

In this appendix, we write the matrices used in the text in detail for completeness. The notation is widely accepted in the literature of econometrics. Since there are potential readers from other disciplines, it is helpful to present the notation here. Recall that there are D days and T hours in the context of the TOD data. We decide to use T in the following, though it is equal to 24 in our empirical study. Also, we assume that there are K_t hourly varing explanatory variables appeared in equation t, and there are G hourly invariant regressors. Including the constant term to fit the intercepts, there are $\sum_1^T K_t + G + 1$ exogenous variables. Thus, Y is a $D \times T$ matrix of the dependent variables, and

$$\tilde{Z} = (Y_0 | X | W | e_D),$$

is a $D \times (\sum_1^T K_t + G + 1)$ matrix of the predetermined variables.

For clarity, we note that

$$Y = \begin{pmatrix} Y_{11} & Y_{12} & \cdots & Y_{1T} \\ Y_{21} & Y_{22} & \cdots & Y_{2T} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{D1} & Y_{D2} & \cdots & Y_{DT} \end{pmatrix}$$

$$X = \begin{pmatrix} X_{11} & X_{12} & \cdots & X_{1T} \\ X_{21} & X_{22} & \cdots & X_{2T} \\ \vdots & \vdots & \ddots & \vdots \\ X_{D1} & X_{D2} & \cdots & X_{DT} \end{pmatrix}$$

$$Y_o = (Y_{10} Y_{20} \dots Y_{D0})'$$

$$W = \begin{pmatrix} W_1 \\ W_2 \\ \vdots \\ W_D \end{pmatrix}.$$

Next we recall that B is a $T \times T$ matrix, which is an upper-triangular matrix with determinant 1. It is useful to write the matrix $\tilde{\Gamma}$ explicitly. We have

$$\tilde{\Gamma} = \begin{pmatrix} \beta \iota_T' \\ I_T \otimes \gamma \\ e_T' \otimes \alpha \\ \zeta e_T' \end{pmatrix}$$

where ι_T is the first column of an identity matrix of dimension T which is customarily denoted by I_T.

Turning now to the general model we proposed. We can write

$$B(L) = \begin{pmatrix} 1 & -\beta_2 & 0 & \dots & 0 \\ 0 & 1 & -\beta_3 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & -\beta_T \\ -\beta_1 L & 0 & 0 & \dots & 1 \end{pmatrix}.$$

Unlike the B matrix, the matrix $B(L)$ involves the lag operator L. It is interesting to find the adjoint matrix to see the dynamic relation expressed by the final equation mentioned in the text. We deduce that $\det(B(L)) = 1 - \prod_1^T \beta_t L$ and the adjoint matrix is

$$B^* = \begin{pmatrix} 1 & \beta_2 & \beta_2\beta_3 & \dots & \prod_2^{T-1}\beta_t & \prod_2^T \beta_t \\ \prod_3^T \beta_t(\beta_1 L) & 1 & \beta_3 & \dots & \prod_3^{T-1}\beta_t & \prod_3^T \beta_t \\ \vdots & \vdots & \vdots & \ddots & \vdots & \\ \beta_T(\beta_1 L) & \beta_T\beta_2(\beta_1 L) & \beta_T\beta_2\beta_3(\beta_1 L) & \dots & 1 & \beta_T \\ (\beta_1 L) & \beta_2(\beta_1 L) & \beta_2\beta_3(\beta_1 L) & \dots & \prod_1^{T-1}\beta_t L & 1 \end{pmatrix}$$

To see the matrix Γ, we note that it is slightly different from $\tilde{\Gamma}$. First we don't have to include the parameters associated with the lagged dependent variables Y_0. Second, the coefficients are not constrained to be equal. Therefore, we have

$$\Gamma = \begin{pmatrix} \gamma 1 & 0 & 0 & \dots & 0 \\ 0 & \gamma 2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \gamma_T \\ \alpha 1 & \alpha 2 & \alpha 3 & \dots & \alpha_T \\ \zeta 1 & \zeta 2 & \zeta 3 & \dots & \zeta_T \end{pmatrix}$$

Finally, it is straightforward to rewrite the system as follows.

$$Y_{dt} = \beta_* Y_{d-1,t} + X^*_{dt} + \eta_{dt} \qquad (A-1)$$

where

$$\eta_{dt} = \sum_{\tau=1}^{T} u_{d\tau} B^*(\tau, t)$$

and

$$X^{*}_{dt} = \sum_{\tau=1}^{T} [X_{d\tau}\gamma + W_d\alpha + \zeta_\tau] B^{*}(\tau, t).$$

Obviously, the error term η_{dt} is a moving average process. In this sense, the system also has an ARMAX representation.

Return to equation (3-3), we can derive the final form immediately. Note that $\beta_* \equiv \prod_1^T \beta_t$ and

$$\frac{1}{1 - \beta_* L} = (1 + \beta_* L + \beta_*^2 L^2 + \cdots).$$

It follows that the final form is

$$Y_d = (Z_d\Gamma + U_d) B^{*}(L)(1 + \beta_* L + \beta_*^2 L^2 + \cdots). \qquad (A-2)$$

Now we can write the final form of equation t as

$$Y_{dt} = (Z_d\Gamma + U_d) B^{*}(t, t) + R_t$$

where the remainder is

$$R_t = \sum_{\tau=1, \tau \neq t}^{T} (Z_d\Gamma + U_d) B^{*}(\tau, t)(\beta_* L + \beta_*^2 L^2 + \cdots).$$

Since the diagonal of $B^{*}(L)$ is a unit vector, we see that, for equation t, the impact multipliers associated with the included exogenous variables at time t is the tth column of Γ. In Section IV, we have used this fact to estimate the impact of temperature on the demand for electricity. In addition, the switching points can be estimated based on the estimated coefficients of γ_{1dt} and γ_{2dt}.

Figure I.

THE VARIATION OF ELECTRICITY DEMAND

Figure II. THE RELATION BETWEEN ELECTRICITY DEMAND AND TEMPERATURE

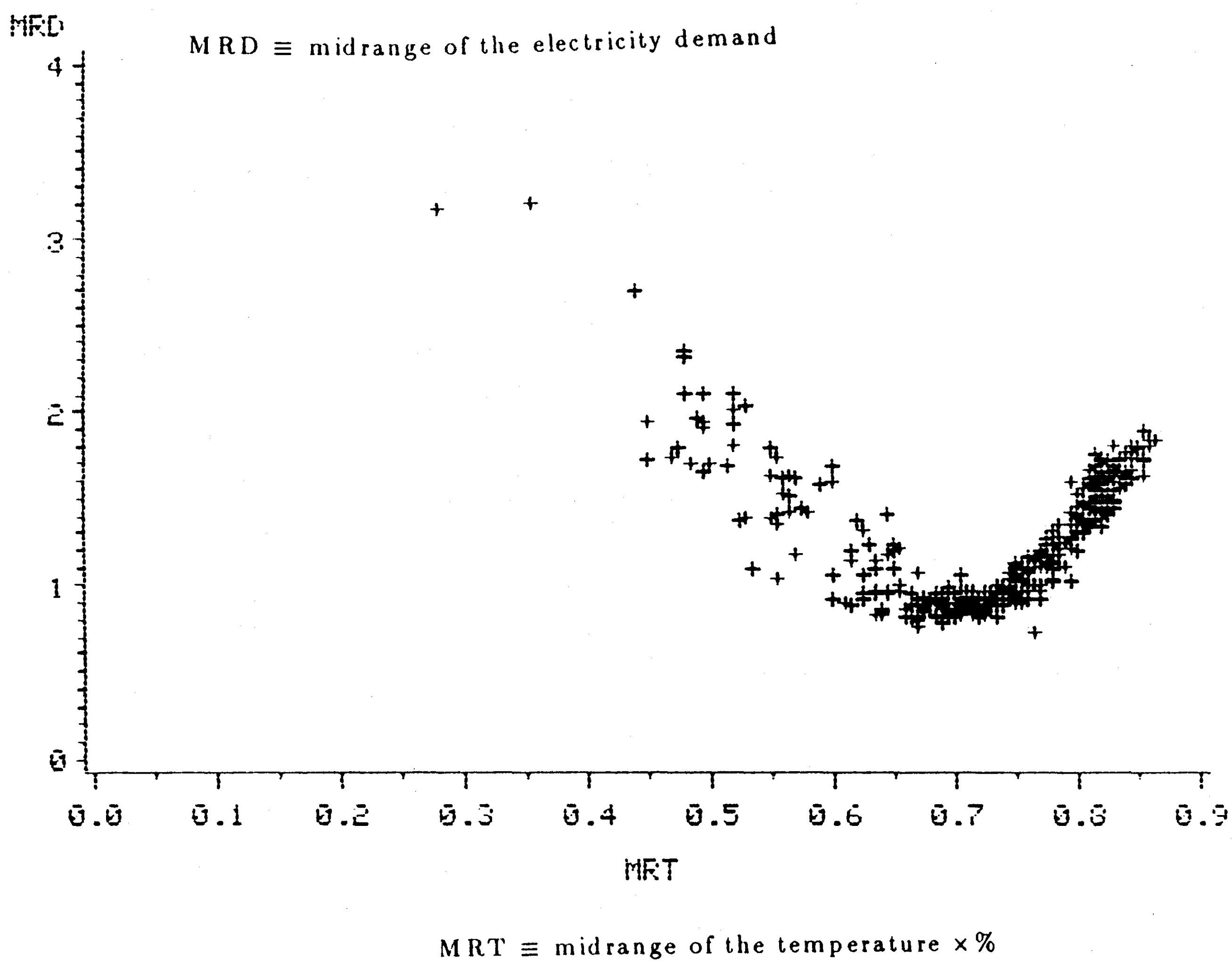
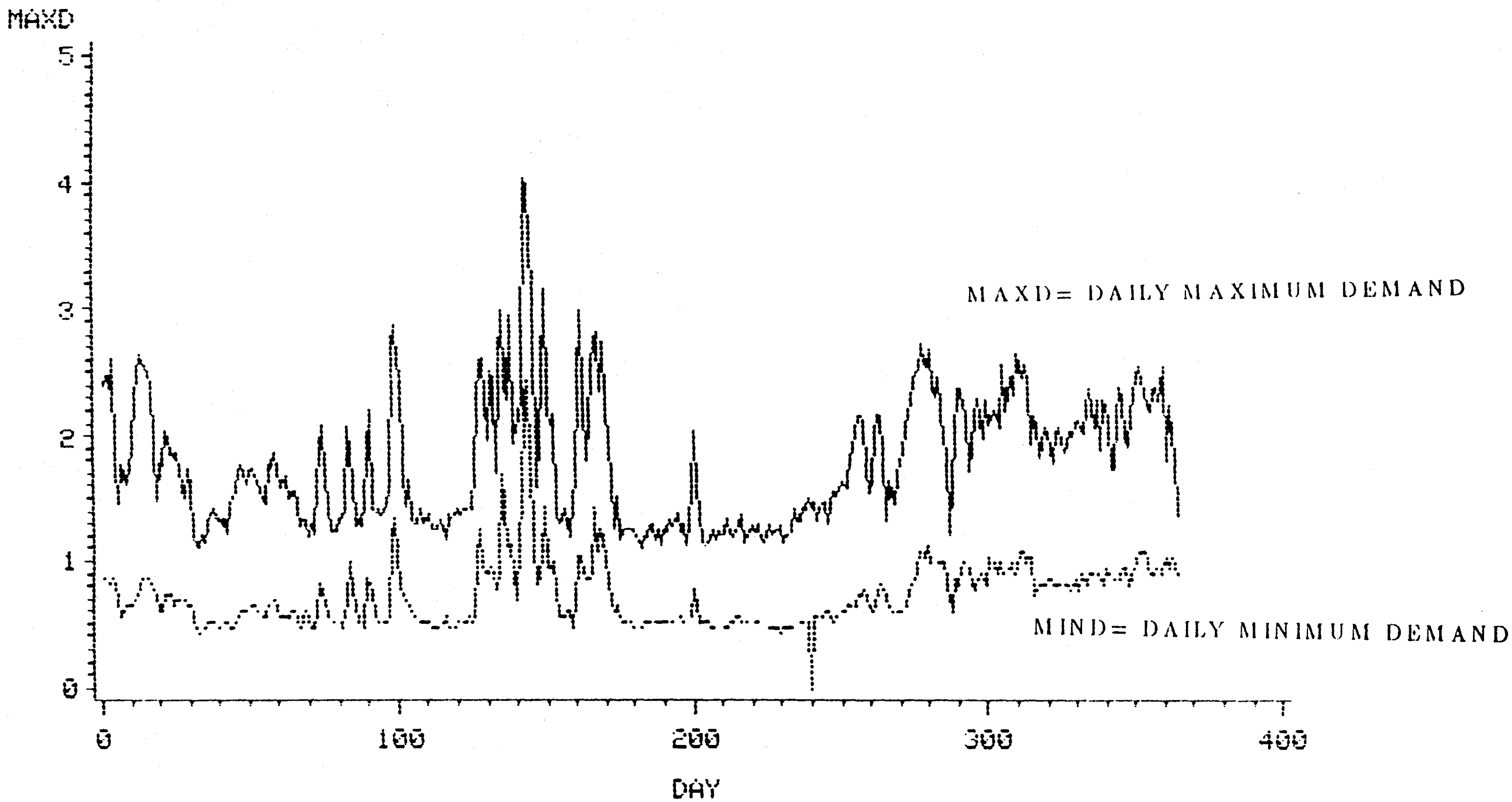


Figure III.

THE BOUND OF DEMAND OVER ONE YEAR



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