Option Pricing
Beyond Black-Scholes

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Black-Scholes Formula (Historical Context)

- Produced a usable model where all inputs were easily observed
- Coincided with the introduction of exchange traded options
- Provided a methodology to create options
Black-Scholes Formula (Present)

- Standard by which all models are compared
- Market Convention
- Closed-Form Solution
  - Computationally fast
  - “Clean” Derivatives
- Traders prefer a simple model with a few approximated parameters to an exact model with many precise parameters that are difficult to calculate
Black-Scholes Pricing Assumptions

- Constant Interest Rates
- Exercise Only at Maturity
- No Dividends
- Continuous Hedging
- No Transaction Costs
- Equal Borrowing & Lending Interest Rates
- Constant Volatility
- Log-Normal Distribution of Underlyer Prices
Scope of Discussion

- **Plain-Vanilla** Options vs. Exotic Options

- Underlying Assets
  - *Equities* (Stocks, Stock Indexes, Funds)
  - Debt Instruments (Bonds, Rates, Swaps)
  - Commodities (Metals, Energy, Agriculture)
  - Currencies
  - Others (Real Estate, Inflation, Weather)

- *Arbitrage Pricing* vs. Relative Value
Two General Methods for Relaxing the Assumptions

- Numerical Approximation Models
  - Cox-Ross-Rubinstein/Binomial Models
  - Monte-Carlo Based Models

- Adjustments to the Black-Scholes Inputs
  - Although not theoretically correct, inputs can be adjusted to approximate reality creating a practical model
Cox-Ross-Rubinstein Model

- Flexible
- Intuitive
- Approximations yield results comparable to Black-Scholes formula
- Simple framework provides solutions for many exotic options
- Relatively computationally efficient
Cox-Ross-Rubinstein Model

Each Option Price is Expected Value of Future Option Prices

Stock Rises In the Future

Price Today

Stock Falls In the Future

Prices at Expiration

Stock Price

Time
Early Exercise

Check each option price for early exercise conditions

- Cox-Ross-Rubinstein Models can easily handle early exercise of calls and puts
Dividends

Tree “breaks” and drops by dividend amount

- Discrete future dividends are difficult to model
- Hard to predict across the distribution of prices
- Difficulty in placing dividends on a tree
Need to balance the cost of hedging against the risk that arises from not hedging.

Risk from not being perfectly hedged creates an option price bid-offer spread.

The larger the portfolio, the greater the diversification of single-name risk which reduces the need to frequently re-hedge.
Call option sales and put option purchases require a hedge with long assets and are priced using the borrow rate.
Borrowing and Lending Interest Rates

- **Borrow Transaction**
  - **Asset Lender**
  - **Trader**
  - **Market**
  - **Interest at Lending Rate (LIBOR -)**

- **Short Transaction**
  - **Asset Lender**
  - **Trader**
  - **Market**
  - **Interest at Lending Rate (LIBOR -)**

- **Call option purchases** and **put option sales** require a hedge with short assets and pricing using **lending rate**
Borrowing and Lending Interest Rates

Option Pricing Tree using Borrowing Rate (LIBOR+)

Option Pricing Tree using Lending Rate (LIBOR-)

- Leads to bid-offer spread where options are offered at a higher price and options are bid for at a lower price
Log-Normal Distribution of Underlyer Prices

5 Years of S&P 500 Index Closing Prices
Log-Normal Distribution of Underlyer Prices

5 Years of S&P 500 Daily Returns
Log-Normal Distribution of Underlyer Prices

Historical Return Distribution of Underlyer Prices

![Histogram showing the distribution of returns with standard deviation on the x-axis and probability on the y-axis. The histogram is overlayed with a normal distribution curve.]
Historical underlyer prices tend to exhibit “fat tailed” distributions when compared with log-normal.

Non log-normal distributions are difficult to describe in mathematical terms.

Traders compensate for this by introducing the concept of Skew.
Log-Normal Distribution of Underlyer Prices

- In practice options of different strikes are priced at different volatilities
- Inexact fix which causes other complications
- Tends to be the adjustment with the greatest influence on price
Final Comments

- Models are only approximations of market behavior and all have restrictive assumptions.
- As long as there is uncertainty in inputs, there will be uncertainty in outputs.
- Markets in many ways are irrational, it may not be possible to create a rational pricing model.
- Traders make money by both buying and selling, knowing the exact value is not a necessity.