Single versus multiple supplier sourcing strategies

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Abstract

Successful supply chain management necessitates an effective sourcing strategy to combat uncertainties in both supply and demand. In particular, supply disruption results in excessive downtime of production resources, upstream and downstream supply chain repercussions, and eventually a loss in the market value of the firm. In this paper we analyze single period, single product sourcing decisions under demand uncertainty. Our approach integrates product prices, supplier costs, supplier capacities, historical supplier reliabilities and firm specific inventory costs. A unique feature of our approach is the integration of a firm specific supplier diversification function. We also extend our analysis to examine the impact of minimum supplier order quantities. Our results indicate that single sourcing is a dominant strategy only when supplier capacities are large relative to the product demand and when the firm does not obtain diversification benefits. In other cases, we find that multiple sourcing is an optimal sourcing strategy. We also characterize a non-intuitive trade-off between supplier minimum order quantities, costs, and supplier reliabilities. Finally, we examine the robustness of our results through an extensive numerical analysis of the key parameters of our model.

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1. Introduction

Procurement activities in large part support a firm’s inbound logistics and are vital to value creation (Porter, 1985). Moreover, the long-term implications of poor supply management are far reaching, ultimately impacting both firm performance and market value. Since a typical manufacturing firm spends 55% of earned revenue on purchased materials (Leenders and Fearon, 1998), disruptions due to supply inadequacies could have a major impact on profitability. Hendricks and Singhal (2001) bear this out by showing in the 90 days prior and subsequent to a reported supply chain problem stemming from supplier glitches that the buying firm’s average shareholder return typically decreases by 12%. Clearly, a manufacturer’s operations strategy...
and financial livelihood rely on its chosen supplier pool and thus, decisions with regard to suppliers are fundamental to successful supply chain management.

A firm’s sourcing strategy is characterized by three key decisions (Burke and Vakharia, 2004): (a) criteria for establishing a supplier base; (b) criteria for selecting suppliers (a subset of the base) who will receive an order from the firm and (c) the quantity of goods to order from each supplier selected. Scoring models are typically used to evaluate suppliers for inclusion in the base. In general, this approach ranks each supplier in terms of objectives and then based on a relative weighting of each of the objectives, a total score for each potential supplier is derived. For example, Sun Microsystems ranks its suppliers with a “scorecard” based on quality, delivery, technology, and supplier support (Holloway et al., 1996).

From the approved supply base, the specific subset of suppliers which will actually receive an order to fill demand for a specific product must be determined. Since all suppliers in the base meet the quality, delivery, and other objectives of the firm, dominant industry practice appears to base this decision primarily on cost considerations. While the supplier’s price quote is important, buying firms also emphasize criteria related to robust delivery reliability capabilities. Once the selected set of suppliers (a subset of the base) is determined, the firm must allocate product(s) requirements among them. For the allocation decision, supplier yields (in terms of percentage of “good” units), order quantity policies, and transportation costs are typically considered.

The focus of this paper is on the latter two decisions of supplier selection and quantity allocation. Hence, we assume that the firm has already established an adequate supplier base. We believe that it is not uncommon for firms to make the supplier base specification and supplier selection/quantity allocation decisions hierarchically. For example, according to Metty et al. (2005), Motorola’s pre-qualification process is very time and labor intensive. However, Motorola allows suppliers which have not been pre-qualified to bid to supply particular items via its online auctions. Motorola then determines the number of suppliers to source from along with the order quantity for each of these suppliers. Of course, we also acknowledge that a firm may re-visit the supplier base specification decision periodically based on the actual performance of suppliers. Ultimately, our goal in analyzing the selection/allocation decisions is to characterize conditions under which a firm should choose a specialized (i.e., single supplier) or a generalized (i.e., multiple suppliers) strategic sourcing position while facing unreliable supply and stochastic demand.

Single-sourcing strategies strive for partnerships between buyers and suppliers to foster cooperation and achieve shared benefits. The tighter coordination between buyer and supplier(s) required for successful just-in-time (JIT) inventory initiatives encourage supplier alliances to streamline the supply network and tend to shift supply relations toward single sourcing. Managing more than one source is obviously more cumbersome than dealing with a single source. However, web-based SCM applications enable closer management of diverse suppliers, streamline supply chain processes and drive down procurement costs. For example, GM utilized internet tools to purchase more of its total budget online, which resulted in a streamlined procurement process and decreased lead times (Veverka, 2001). Other documented benefits of effectively utilizing web-based procurement tools include a reduction in price of materials, administrative costs, inventory costs, and purchase and fulfillment cycles. Consequently, firms that prefer single-sourcing for its ease of management can embrace multiple-sourcing via information technology-based SCM applications as a more viable strategy to capture risk-pooling benefits. In addition, firms can utilize these internet procurement tools for “pricing out” the total costs associated with sourcing from a particular supplier, thus creating a comprehensive cost measure which includes other supplier performance criteria. Teich et al. (2004) provide a comprehensive summary of the characteristics and benefits associated with various online procurement applications.

Single-sourcing dependency may also expose the buying firm to a greater risk of supply interruption. Toyota’s brake valve crisis in 1997 provides a recent example of realized supply risk resulting from a single sourcing strategy in a JIT inventory system. Operationally, multiple-sourcing provides greater assurance of timely delivery, and greater upside volume flexibility due to the diversification of the firm’s total requirements.

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1 In 1997, Toyota’s assembly plants were forced to shut down after a fire at Aisin’s main plant. This supplier’s particular facility provided 90% of all brake parts components and practically all brake valves for Toyota, before it was destroyed (Nishiguchi and Beaudet, 1998). It is estimated that the impact on Toyota’s net income from this single event at Aisin was a decrease of $300 million. Thereafter, Toyota sought at least two suppliers for each part (Treece, 1997).
Another shortcoming of single sourcing is that it exposes the buying firm to hold-up risk. Land Rover’s contractual problems with its only chassis supplier is an example of the operational difficulties this situation creates. Strategically, supplier power over the buyer is weakened when the firm splits its total requirements among multiple sources. Hence, multiple sourcing hedges the risks of creating a monopolistic (sole source) supply base and supplier forward integration (Newman, 1989).

Our general approach in examining the supplier selection and quantity allocation decisions is two fold. First, we develop an integrated model to address both of these decisions. Given uncertain product demand, we simultaneously consider supplier costs, supplier reliabilities, supplier capacities, manufacturer inventory costs, and manufacturer diversification benefits in making these integrated decisions. Second, using our model, we characterize conditions under which single-sourcing and multiple-sourcing strategies are optimal. A key feature of the model facilitating this analysis is an explicit treatment of supply diversification benefits.

The remainder of this paper is organized as follows. In the next section, we review the relevant literature focusing on supplier selection and quantity allocation. This is followed by the development of an integrated model for making these decisions in Section 3. In Section 4, we characterize the optimal sourcing strategies under various scenarios. In Section 5, we proceed to discuss extensions to our modeling effort and this is followed by an extensive numerical analysis in Section 6. Finally, the implications and directions for future research are discussed in Section 7.

2. Relevant literature

Research on the number of sources to use to fulfill product requirements is somewhat controversial. At one extreme, we have empirical evidence of many firms shrinking their supplier base per item and ordering the majority of total units required from a single source (Spekman, 1988; Pilling and Zhang, 1992). Further, the documented benefits of single-sourcing are quantity discounts from order consolidation, reduced order lead times, and logistical cost reductions as a result of a scaled down supplier base (Hahn et al., 1986; Bozarth et al., 1998). Mohr and Spekman (1994) contend that single-sourcing performance benefits often outweigh the benefits of a price centric multiple-sourcing strategy.

In contrast, Bhote (1987) observed that relationship management costs, in terms of time and capital, may outweigh the performance-enhanced benefits of single sourcing. The primary rationale driving this argument is that single-sourcing requires the firm and the supplier to develop a partnered relationship based on trust. In line with this reasoning, McCutcheon and Stuart (2000) assert that the parties must achieve goodwill trust to have a successful partnership. Further, they conjecture that this level of trust is rarely attained. Elmaghraby (2000) contains an excellent overview of the literature related to this debate.

Analytical studies similar to ours address supplier selection and quantity allocation decisions and show that in certain cases, multiple-sourcing is preferable to single-sourcing. Horowitz (1986) provides an economic analysis of dual sourcing a single input at differing costs and shows that uncertainty in supply price and risk-aversion of the buyer motivate a firm to place positive orders from the high cost seller. Kelle and Silver (1990) investigate a continuous review inventory policy replenishment system for suppliers with stochastic delivery lead-times, and find that order-splitting among multiple sources reduces safety stock without increasing stockout probability. Ramasesh et al. (1991) also analyze a reorder point inventory model with stochastic supply lead-time, and find that in the presence of low ordering costs and highly variable lead-times, dual sourcing can be cost preferable. Gerchak and Parlar (1990) examine second-sourcing in an EOQ context to reduce the effective yield randomness of a buying firm’s purchase quantity. Agrawal and Nahmias (1997) examine a single period supplier selection and allocation problem with normally distributed supply and deterministic demand for a single product with fixed ordering costs. They are able to show that for two non-identical suppliers, the expected profit function is concave in the number of suppliers. This finding motivates our explicit diversification benefit function.

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2 This single supplier entered bankruptcy and required an additional $60 million above the contractually agreed amount to provide and deliver Land Rover’s chassis requirements (Lester, 2002).
Other analytical studies incorporate stochastic demand in the supplier selection and order allocation decisions. Gallego and Moon (1993) employ Scarf’s ordering rule for a distribution free optimal newsboy order quantity. They maximize profit against the worst possible distribution of demand with known mean and variance. Separate extensions incorporate a second purchasing opportunity, fixed ordering costs, random yields, and multiple items into the analysis. Bassok and Akella (1991) introduce the combined component ordering and production problem (CCOPP). The problem is one of selecting ordering and production levels of a component and a finished good for a single period with supply and demand uncertainty. Anupindi and Akella (1993) simultaneously determine ordering and production decisions for a two component assembly system facing random finished product demand and random yield from two suppliers, each providing a distinct component.

The two prior studies closest to our work are those of Pan (1989) and Parlar and Wang (1993). Pan (1989) proposes a linear programming model to optimally identify the number of suppliers and their respective quantity allocations to meet pre-specified product requirements. The overall objective is to minimize the price per unit per product as a weighted average of selected suppliers’ prices. It is assumed that product requirements are deterministic and supply is reliable and unlimited. Parlar and Wang (1993) compare the costs of single versus dual-sourcing for a firm assuming that the overall objective is to minimize purchasing and inventory related costs. Separately using an EOQ and newsboy based ordering policy, they are able to show that in certain cases dual-sourcing dominates single-sourcing when supplier yield is a random variable. Both of these studies ignore the supplier capacity issue in making supplier selection and quantity allocation decisions.

We build upon both of these studies by analyzing the simultaneous supplier selection and quantity allocation decisions for a single firm facing unreliable supply combined with demand uncertainty. Further, we incorporate an explicit benefit related to requirements diversification among the supplier base in identifying optimal sourcing strategies. In the next section, the integrated supplier selection and quantity allocation model which forms the basis of our analysis is described.

3. Integrated selection/allocation model

3.1. Preliminaries

Our examination of the supplier selection and quantity allocation decisions focuses on a single-period analysis of a two stage supply chain consisting of \( N \) suppliers \((i = 1, \ldots, N)\) and one buying firm. All \( N \) suppliers are assumed to have been pre-screened by the firm and are thus, included in the supplier base. The firm faces an uncertain single-period demand \( w \) (with \( f(w) \) and \( F(w) \) representing density and distribution functions, respectively) for the product requirements which it satisfies through procurement from the \( N \) suppliers. We assume that this product is being supplied to the next stage of the supply chain at a unit price \( p \). Excess inventory of the product is disposed of at the end of the single period by the firm which receives a price of \( s \) per unit while unsatisfied demand “costs” the firm \( u \) per unit. For each supplier \( i \) we assume that the firm has information on (a) the cost per unit \( c_i \); (b) the capacity (in units) \( y_i \) and (c) the reliability index \( r_i \) representing the historical percentage of “good” units (i.e., \( 0 < r_i \leq 1 \ \forall i \)) received from the supplier. Furthermore, we assume that the reliability index \( r_i \) is strictly greater than zero, as the firm would not wish to include a supplier for which the historical reliability is so poor. In line with the assumptions in single-period inventory models, we assume that \( p > c_i > s \) for all \( i \) (Silver et al., 1998). We assume zero fixed order placement costs for the firm since in current industrial settings where orders are issued online (through, for example, B2B exchanges), these costs are negligible (Nahmias, 2001).

A final component of our model characterizes the diversification benefit function \( d(\cdot) \). The motivation for incorporating this function stems from observed industry practices. For example, consider HPs sophisticated

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3 In contrast to Parlar and Wang (1993), we assume that the firm has exact knowledge on the reliability of each supplier. This is primarily due to the fact that the uncertainty in this parameter required Parlar and Wang (1993) to impose some very restrictive conditions on other parameters in order to show that dual-sourcing is preferred to single-sourcing. We do, however, analyze the impact of changes in this parameter on the resulting selection/allocation policies.
Procurement Risk Management program which was initially aimed at better managing the procurement risk of critical memory components. Incorporated in this program is a portfolio approach to assess and mitigate pricing and availability exposure to insure margin stability. HP expects the revenue contribution of this program to approach $1 billion as it expands its use throughout the company. This financial benefit stems from reduced unit costs and avoiding costs that arise when product delivery is delayed due to sourcing allocation. Additionally, as a result of this program, HP has learned that by segmenting its expected requirements it can target contracts to take advantage of a supplier’s particular strength(s). This results in more efficient supply chain practices that create shared savings for HP and its suppliers (Shah, 2002).

The issue of positive diversification benefits is well documented through anecdotal and/or case examples. In addition to HP, Unifine Richardson’s decision to change from a single honey supplier to multiple honey suppliers to meet more stringent regulation requirements and Wendy’s decision to find a second high-volume supplier of chicken given the increasing demand for its chicken products illustrates the importance of these diversification benefits (Prahinski, 2002; Lambert and Knemeyer, 2004). Toyota and Honda also have a policy of sourcing all components from a minimum of two or three suppliers (Liker and Choi, 2004).

The diversification function \( d(X) \) reflects buyer specific supply chain efficiency savings and strategic positioning benefits. In general, this function captures the net benefits of choosing to source product requirements from multiple suppliers and is analogous to the risk-averse expected utility function maximized in the portfolio selection problem (Gerchak and Parlar, 1990). By choosing multiple suppliers the firm can reduce the risk associated with selecting a single supplier (Ramasesh et al., 1991). Consequently, the diversification function is essentially insurance against supply disruptions attributable to the size of the supply base for a specified part.

We also recognize, that the there is a potential decline in diversification benefits if the number of selected suppliers is too large due to excessive costs associated with maintaining a large supplier base. We incorporate this by assuming that \( d(X) \) is piece-wise concave in \( X \) where \( X \) represents the number of suppliers selected by the firm. To support this specific functional form, consider the findings of Agrawal and Nahmias (1997). These authors show that expected firm profits are concave in the number of suppliers selected in a setting characterized by stochastic supplier quantity reliability. Further, the notion of such benefits being piece-wise concave is documented in prior work (e.g., Gerchak and Parlar, 1990; Ramasesh et al., 1991). Thus, our \( d(X) \) function could be regarded as a proxy for diversification benefits associated with uncertainty in the quantities delivered by each supplier.\(^4\)

The key decision variables for our selection/allocation model are both the number of suppliers and the order quantity for each supplier. We define the binary decision variable \( x_i \) to be 1 if we choose to source from supplier \( i \), and 0 otherwise and the related allocation quantity \( q_i \) (in units) procured from supplier \( i \).

### 3.2. Model development

Let us start by expressing the profit function without diversification benefits as:\(^5\):

\[
\Pi = \begin{cases} 
    pw - \sum_{i=1}^{N} c_i r_i q_i + s \left[ \sum_{i=1}^{N} r_i q_i - w \right] & \text{if } w < \sum_{i=1}^{N} r_i q_i, \\
    p \sum_{i=1}^{N} r_i q_i - \sum_{i=1}^{N} c_i r_i q_i - u \left[ w - \sum_{i=1}^{N} r_i q_i \right] & \text{if } w \geq \sum_{i=1}^{N} r_i q_i.
\end{cases}
\]

\(^4\) While the diversification function may be difficult to quantify, we have recommended a procedure that can be used from a value/utility function perspective and this is described in Appendix 2. The process outlined is based on established work in utility theory and decision making under uncertainty, and is an adaptation of the reference gamble procedure. Furthermore, it utilizes results from Theorem 1 where no diversification benefits are realized. The outcome of this process is to determine a “value” for \( d(i) \) for \( i = 2, \ldots, n \) (\( d(1) = 0 \) since no diversification benefits are realized by the firm with a single supplier).

\(^5\) Similar to Agrawal and Nahmias (1997), this profit realization characterizes a situation where the firm compensates suppliers for only the “good” units supplied (i.e., \( r_g \)). Later in the paper, we consider the impact of the firm compensating the supplier for the complete order quantity (i.e., \( q_i \)).
Since this profit is uncertain and depends on the exact realization of demand \( w \), we use the traditional newsboy analysis for this profit function \((Silver \ et \ al., \ 1998)\), and determine the expected profits as

\[
E(\Pi) = (p - s)\mu - \sum_{i=1}^{N} c_i q_i r_i + s \sum_{i=1}^{N} r_i q_i - (p - s + u)ES,
\]

where

\[
\begin{align*}
\mu &= \text{mean demand}, \\
ES &= \text{expected number of units short} \\
&= \int_{(\sum_{i=1}^{N} q_i r_i)}^{\infty} \left[ w - \left( \sum_{i=1}^{N} q_i r_i \right) \right] f(w) \, dw
\end{align*}
\]

and depending upon the distribution of demand, we can specify the expected shortage \( ES \).\(^6\)

Based on this, the firm’s expected profit (including the diversification benefit) maximization sourcing model can be defined as follows:

Maximize \( Z_{q_i, x_i} = E(\Pi) + d(X) \)

\[
= (p - s)\mu - \sum_{i=1}^{N} c_i q_i r_i + s \sum_{i=1}^{N} q_i r_i - (p - s + u)ES + d(X)
\]

subject to

\[
\begin{align*}
q_i &\leq y_i x_i \quad \forall i, \\
X &= \sum_{i=1}^{N} x_i, \\
q_i &\geq 0 \quad \forall i, \\
x_i &= \{0, 1\} \quad \forall i,
\end{align*}
\]

where constraint set (2) integrates capacity limitations when \( x_i = 1 \) (or supplier \( i \) is selected) or forces the quantity allocation decision \( q_i \) to be 0, if \( x_i = 0 \) (or when supplier \( i \) is not selected), constraint (3) determines the total number of suppliers chosen for sourcing total product requirements, and Eqs. (4) and (5) are the non-negativity and binary restrictions on the decision variables \( q_i \) and \( x_i \), respectively. In the next section, we proceed to analyze this model and characterize the optimal solutions and sourcing strategies for several cases.

4. Analysis

The focus of our paper is on investigating sourcing strategies for the supply chain. Primarily we are interested in identifying when it is optimal for the manufacturer to use multiple suppliers versus a single supplier. We start our analysis by first assuming a zero diversification benefit. These results will serve as a base case in our analysis.

4.1. No diversification benefit

Assuming that the firm does not obtain any explicit diversification benefit for choosing to source from more than one supplier, let us examine the structure of the optimal sourcing policies. When the diversification benefit \( d(X) = 0 \), our integrated sourcing model can be formulated as follows:

\(^6\) For example, if demand is normally distributed with mean \( \mu \) and standard deviation \( \sigma \), then \( ES = \sigma G(Z) \) where \( G(\cdot) \) is the loss function and \( Z \) is the standard normal deviate (i.e., \( Z = \left( \frac{\sum_{i=1}^{N} q_i r_i - \mu}{\sigma} \right) \)).
Maximize \[ Z_{q_i} = (p - s)\mu - \sum_{i=1}^{N} c_i q_i r_i + s \sum_{i=1}^{N} q_i r_i - (p - s + u)ES \]
subject to
\[ q_i \leq y_i \quad \forall i, \quad (6) \]
\[ q_i \geq 0 \quad \forall i. \quad (7) \]

Let us first analyze this problem assuming that supplier capacity is not a significant issue. Note that this scenario may be relevant to smaller manufacturing firms with larger suppliers. In this case, it is relatively easy to show that the firm commits all its requirements to a single supplier and this supplier is, as would be expected, the one which offers the lowest per unit cost to the firm. The theorem given below formalizes this result.

**Theorem 1.** When the suppliers are uncapacitated and there are no diversification benefits and there is a unique least cost supplier, then it is optimal for the firm to order its total requirements from the least cost supplier.\(^7\) Under this scenario, the total usable quantity ordered from the least cost supplier (i.e., \( q_{y[1]} \)) is determined such that:

\[ F(q_{y[1]}) = \frac{p - c_{y[1]} + u}{p - s + u}, \]

where \( c_{y[1]} \) is the cost per unit charged by the lowest cost supplier. If multiple suppliers have the same lowest cost, then the total order may be split amongst all of the lowest cost suppliers such that the total usable quantity ordered still satisfies the above critical ratio.

**Proof.** See Appendix 1. □

One surprising result of this single sourcing strategy is that supplier reliabilities do not impact the supplier choice (i.e., the supplier choice is based strictly on cost considerations regardless of the quantity reliability parameter \( r_i \)). On further investigation, we find that this result is solely due to the fact that the manufacturer only incurs the purchasing cost for “good” units (i.e., incurs cost \( c_i \) per unit for \( r_i q_i \) units). In certain situations, the cost of defective units in a delivery may need to be absorbed by the manufacturer. To reflect this scenario, the uncapacitated supplier model without diversification benefits can be reformulated as

Maximize \[ E(\Pi)^c = (p - s)\mu - \sum_{i=1}^{N} c_i q_i + s \sum_{i=1}^{N} q_i r_i - (p - s + u) \int_{q_{y[1]}}^{\infty} \left[ w - \sum_{i=1}^{N} q_i r_i \right] f(w) dw \]
subject to \[ q_i \geq 0 \quad \forall i. \]

It is easy to show that \( E(\Pi)^c \) is strictly concave in \( q_i \), and thus, the FOC are necessary and sufficient to identify a global optimal solution to this model. Essentially, the manufacturer determines the total “good” quantity received from all suppliers (i.e., \( \sum_{i=1}^{N} q_i r_i \)) such that:

\[ F\left( \sum_{i=1}^{N} q_i r_i \right) = \frac{p - (c_i / r_i) + u}{p - s + u}. \]

The issue, of course, is which supplier’s reliability adjusted unit cost (i.e., \( c_i / r_i \)) is relevant in determining this total quantity. The optimal policy, which can be easily verified, is for the manufacturer to place an order for

\(^7\) While this result is analytically trivial, it is driven by the fact that for all suppliers \( (j = 1, \ldots, N) \), \( c_j r_j \) is a constant. However, it points to the fact that when the firm uses historical information of supplier reliabilities and only compensates a supplier for the “good” units received, then only cost considerations play a role in determining which supplier will be chosen to receive the complete order. This result also forms the basis for how the firm’s single supplier selection decision is moderated should it decide to compensate suppliers for all units ordered (i.e., the firm absorbs the complete costs of defective units which could occur in situations where suppliers hold more “power” in the channel). In this case, we show that the ratio of costs to reliability drives the choice of the single supplier who will receive the entire order from the firm.
the total quantity from a single supplier with the lowest cost/reliability ratio. Hence, if suppliers are indexed in order of decreasing cost/reliability ratio such that:

\[ \frac{c[1]}{r[1]} \leq \frac{c[2]}{r[2]} \leq \cdots \leq \frac{c[N]}{r[N]} \]  

then the manufacturer determines the quantity to purchase from supplier [1] such that:

\[ F(q[1], r[1]) = \frac{p - (c[1]/r[1]) + u}{p - s + u} \]  

and orders zero units from all other suppliers. Thus, even in this case, it is optimal for the manufacturer to adopt a single sourcing strategy except that the choice of the supplier is based on the lowest cost/reliable unit.

Given that eliminating capacity constraints results in single sourcing, let us now proceed to examine how this solution changes if supplier capacity constraints do not necessarily permit the firm to place orders for all requirements with the least cost supplier. The theorem below characterizes the optimal supplier selection and quantity allocation policy with capacitated suppliers.

**Theorem 2.** When suppliers are capacitated and there are no diversification benefits, then the optimal number of suppliers selected and the corresponding quantity allocated to each supplier can be determined as follows.

1. **Step 1:** Index all suppliers in increasing order of cost per unit (i.e., \( c[1] \leq c[2] \leq c[3] \cdots \leq c[N] \)).
2. **Step 2:** For each supplier \( [i] \) (\( i = 1, \ldots, N \)), determine \( Q[i] \) such that:

   \[ F(Q[i]) = \frac{p - c[i]/r[i] + u}{p - s + u} \]

   and based on this determine:

   For \( i = 1 \), \( t[i] = Q[i] \). Otherwise, let \( t[i] = Q[i] - \sum_{j=1}^{i-1} y[j] r[j] \).

3. **Step 3:** The optimal number of suppliers selected (\( k \)) is \( \max\{1 \leq k \leq N | t[k] \geq 0\} \).
4. **Step 4:** The quantities allocated to supplier \( j = 1, \ldots, k - 1 \) are determined such that \( q[j] = y[j] \), and the quantity allocated to supplier \( k \) is \( q[k] = \min\{t[k], y[k]\} \). The total quantity ordered by the firm from all suppliers can be determined as \( \min\{Q[k], \sum_{j=1}^{k} y[j]\} \).

**Proof.** See Appendix 3. □

An interesting observation based on these results is that the firm’s optimal total order quantity when suppliers are capacity constrained is always lower than the optimal total order quantity when the lowest cost supplier’s capacity is not binding. This leads to the general result that expected profits for the firm dealing with capacitated suppliers are never higher than the profits realized by a firm dealing with uncapacitated suppliers. This serves as a rationale for the observed industry practice of firms expending resources to encourage lower cost suppliers to increase capacity.

### 4.2. Diversification benefit

In this section, we analyze our complete model which includes explicit benefits derived from the size of the selected supplier pool. To start with assume that suppliers are uncapacitated. In this case, our sourcing model is

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8 All of the analysis in the remainder of this paper reflects the original assumption that suppliers are compensated by the firm for only the “good” units received by the firm rather than all the units supplied.
Maximize \[ Z_{q,x_i} = (p - s)u - \sum_{i=1}^{N} c_{qi}x_i + s \sum_{i=1}^{N} q_ir_i - (p - s + u)ES + d(X) \] \[(11)\]

subject to

\[ X = \sum_{i=1}^{N} x_i, \]
\[(12)\]

\[ q_i \geq 0 \quad \forall i, \]
\[(13)\]

\[ x_i = \{0, 1\} \quad \forall i. \]
\[(14)\]

The optimal solution to this problem is characterized in Theorem 3 below.

**Theorem 3.** If the diversification benefits are positive and suppliers are uncapacitated, then the optimal number of suppliers to source from is \( v^* \), where \( 1 \leq v^* \leq N \), and is determined such that \( v^* \) maximizes the diversification benefit function \( d(X) \).

The proof of this theorem is a direct extension of Theorem 1. Note that based on Theorem 1, we choose to source the entire quantity from the lowest cost supplier. If the diversification benefit function is maximized when we choose \( v^* \) suppliers where \( 1 < v^* \leq N \), then we order the total requirements from the lowest cost supplier and simply include the others (i.e., suppliers 2, \ldots, \( v^* \); indexed in order of increasing unit costs) in the supplier selection set.

In this setting the diversification benefits drive the choice of the optimal number of suppliers. In particular, if it is relatively costless to source from additional suppliers at a negligible level, then the firm can reap the \( v^* \) selected supply pool diversification benefits without incurring additional costs. When the marginal benefits of sourcing from an additional supplier are positive, we can always find an order quantity that is sufficiently small enough to warrant selecting that supplier. Qualitatively, the supplementary suppliers (2, \ldots, \( v^* \)) are those the firm would source from in an emergency.

For the capacitated supplier case, the structural insights into the optimal sourcing strategy are characterized in the following theorem. The proof follows directly from Theorem 3 and is omitted.

**Theorem 4.** If the suppliers are capacitated and diversification benefits are positive, identify the number of suppliers \( v^* \) where \( 1 \leq v^* \leq N \), such that \( v^* \) maximizes the diversification benefit function \( d(X) \). Using the results of Theorem 2, identify the optimal number of suppliers \( k^* \) and optimal order quantities for the capacitated suppliers problem without diversification benefits. If \( v^* \geq k^* \geq 1 \), then allocate a minimal quantity \( \epsilon > 0 \) to suppliers \( k^* + 1, \ldots, v^* \) (assuming that suppliers are indexed in increasing order of unit costs).

The implications of Theorems 3 and 4 for decision making concerning an appropriate supplier base are clear. Recall from the introduction that there are three interrelated decisions with regards to a firms sourcing strategy (Burke and Vakharia, 2004) (a) criteria for establishing a supplier base; (b) criteria for selecting suppliers (a subset of the base) who will receive an order from the firm and (c) the quantity of goods to order from each supplier selected. Theorem 3 offers direct managerial guidance concerning the second decision, or the number of suppliers who will receive an order from the firm. Furthermore, as a consequence of the positive diversification benefits, the total order quantity will slightly exceed that of the original solution for the capacitated suppliers. Finally, there may also be benefits beyond those directly captured by the model for ordering at a negligible level from some of the suppliers. Depending upon the particular contracts negotiated with these suppliers concerning upside order flexibility, the manufacturing firm could potentially place larger orders with these suppliers in the event that lower cost suppliers cannot deliver good units.

Note that if \( 1 \leq v^* < k^* \), then the optimal \( v^* \) that maximizes the diversification benefit function \( d(X) \) is actually smaller than the number of suppliers needed to satisfy total requirements due to capacity limitations. In this case the firm foregoes profit from product sales for a greater strategic benefit of a smaller selected supplier pool. Analytically, the optimal strategy would be to source from any number of suppliers \( m^* \) such that \( v^* \leq m^* \leq k^* \). We now turn to describing an extension to our modeling effort.

---

9 Obviously if \( v^* = 1 \), then we would simply source the entire quantity from a single lowest cost supplier.
5. Model extensions

In this section, we extend our model to examine the impact of including a minimum order quantity when sourcing from a supplier. In certain situations, firms may have to commit to ordering some minimum order quantity from each supplier in order to reap the benefits of diversification. Suppose that we add an additional constraint which reflects the minimum order quantity for each supplier as shown below

\[ q_i \geq z_i x_i \quad \forall i. \]  

While the complete solution algorithm for this model is fairly complicated, we can easily obtain boundaries for the optimal number of suppliers. As compared to Theorems 3 and 4 in the previous section, we now have non-negligible costs associated with including additional suppliers in our supplier base. In particular, these costs are associated with shifting enough units from a lower cost supplier to a higher cost supplier to meet that higher cost supplier's minimum order quantity. The intuition developed in Theorems 3 and 4 as to the appropriate number of suppliers is still relevant. In particular, we would want to consider all candidate solutions for the number of suppliers where the marginal diversification benefits are non-negative. Therefore, it's likely that the optimal number of suppliers will be less than or equal to the number determined in Theorem 3.

Diversification benefits may be difficult to quantify precisely. However, the firm can analyze the marginal costs associated with a re-allocation strategy to evaluate the diversification benefits. To illustrate, if the order quantity for the \( i+1 \)th supplier is fairly low, and the cost per unit of the \( i+1 \)th supplier is only slightly higher than for the \( i \)th supplier, then it may be worthwhile to source from \( i+1 \) suppliers. Recall the Toyota dilemma discussed in the introduction where Toyota was forced to shut down an assembly plant because of a problem with its sole supplier (Nishiguchi and Beaudet, 1998). In this case, a secondary supplier hedges against the potential costs incurred from problems associated with a single supplier strategy. The marginal costs of including a secondary supplier at a minimum required level can be utilized as a proxy for the “insurance” premium necessary to reap the benefits of a larger pool of suppliers. Likewise, the implicit risk premium for single sourcing can be determined by comparing procurement costs for single versus multiple supplier allocation strategies.

If we assume that the firm has already determined the subset of suppliers that will receive orders, then Theorem 5 below specifies the structure of a simple algorithm which can be used to determine optimal order quantities.

**Theorem 5.** When each supplier has both maximum and minimum limitations placed on the size of the order, then the optimal quantity allocated to each supplier can be determined as follows:

**Step 1:** Index all chosen suppliers in increasing order of cost per unit (i.e., \( c[1] \leq c[2] \leq c[3] \cdots \leq c[X] \)).

**Step 2:** For each supplier \([i]\) \((i = 1, \ldots, X)\), determine \( Q[i] \) such that:

\[ F(Q[i]) = \frac{p - c[i]}{p - s + u} \]

and based on this determine:

\[ t[i] = Q[i] - \sum_{j=1}^{i-1} y[j] r[j] - \sum_{j=i+1}^{X} z[j] r[j]. \]

**Step 3:** The quantity allocated to supplier \( i \) is \( q[i] = \min\{\max\{t[i], z[i] r[i]\}, y[i] r[i]\} \) and the total quantity ordered by the firm from all suppliers can be determined as \( \sum_{i=1}^{X} q[i] \).

**Proof.** See Appendix 4. \( \square \)

From Theorem 5, we know that at most one of the chosen suppliers will be unconstrained. Suppose that supplier \([i]\) is unconstrained (i.e., \( q[i] = t[i] \)). Then, the optimal order quantity for the lower cost suppliers \((j = 1, \ldots, i-1)\) is determined by the capacity constraint for each supplier. Similarly, the optimal order
quantity for the higher cost suppliers \((j = i + 1, \ldots, X)\) is determined by the minimum order quantity dictated by each supplier. Interestingly, the total order quantity in this situation (i.e., sum of all orders placed to the subset of suppliers) is determined by the cost of the unconstrained supplier \([i]\) and is such that \(F(Q_f) = \frac{p-c_s+u}{p-c_s+u}\).

However, because the generalized structure of this problem has a wide range of supplier options corresponding to alternate cost levels with differing combinations of minimum and maximum order quantities, a simple solution algorithm cannot be easily derived. This leads us to the problem of determining the optimal subset of potential suppliers that will receive an order from the firm, in addition to allocating appropriate order quantities. Branch and bound methodologies can be utilized based on Theorem 5 to enumerate all possible subsets of suppliers for an optimal solution. In addition to the optimal mathematical solution, firms may want to consider different qualitative evaluation measures in determining an appropriate subset of suppliers to source from. For example, in an international sourcing context, firms may wish to pick a subset of suppliers in a variety of countries, thereby hedging against country specific risks such as changing political climate and/or exchange rates.

We now turn to an extensive numerical analysis in order to illustrate some of our results and explore the sensitivity of these results for key parameters in our analysis.

6. Numerical analysis

Analytic results have been presented offering insights concerning the optimal choice of suppliers and appropriate order quantities for a manufacturer. In this section, we present the results of a numerical study to illustrate several key cases for which analytical insights cannot be obtained. We also examine the sensitivity of our results based on changes in the key input parameters. Our intention is to show an overview of these examples which offer insights concerning the relative impact of these factors on a firm’s sourcing strategy.

6.1. Experimental design

The parameters and functions were chosen to capture the underlying assumptions outlined in Section 3. The explicit numerical parameters selected for the base case example reflect those shown in Jucker and Rosenblatt (1985). For the manufacturer: (a) price/unit \((p) = 19\); (b) salvage value \((s) = 2\); (c) lost sales cost \((u) = 6\); and (d) demand is assumed to be uniformly distributed with parameters \([300, 700]\). Our supplier base consists of 5 suppliers (i.e., \(i = 1, \ldots, 5\)) with identical reliabilities \((r_i = 0.9)\), minimum order quantities \((z_i = 200 \forall i)\), and capacities \((y_i = 300 \forall i)\). Suppliers are assumed to be heterogeneous with respect to costs (i.e., \(c_i\)) and these parameter settings are \(c_1 = 6.5\); \(c_2 = 7\); \(c_3 = 8\); \(c_4 = 9\) and \(c_5 = 10\). While the diversification benefit function is discrete, we assume that it is roughly quadratic in the number of suppliers \((X)\) and when \(X = 1, \ldots, 5\), this function is defined as \(d(X) = d_1 - d_2(d_3 - X)^2\) and when \(X = 0\), \(d(0) = 0\). This functional form of \(d(X)\) was chosen since the single parameter \(d_1\) represents the optimal number of suppliers which maximizes this function. All the results discussed next were obtained using LINGO optimization software.

6.2. Results

Table 1 summarizes the results of a set of numerical examples which show sensitivity of the optimal supplier strategy to changes in parameter values. Model A represents the case where both diversification benefits and supplier minimum order quantities are included. For the remaining examples in Table 1, the parameter changes are specified in the variable range column. Not surprisingly, models B and C in Table 1 show that an increase in the price or salvage value of the items increases the total quantity ordered, the total number of suppliers, but decreases the total profit earned.

Model E is intended to illustrate the impact of changes in the diversification benefit function on the optimal sourcing strategy. In this model, the peak in the magnitude of diversification benefits earned (i.e., \(d_1\)) is varied between $250 and $2000 (it is set to $1000 for the base case example). In response to an increase in the peak value of the diversification function, the total order quantity and the optimal number of suppliers remains the same, while the profit increases. This would indicate that the optimal sourcing policy is fairly robust in that it is not sensitive to large increases in the peak diversification value for this example.
Models F and G illustrate how the optimal sourcing policy changes with alterations in the first supplier’s cost and reliability. While small increases in the first supplier’s cost do not change the optimal number of suppliers, it does decrease the total quantity ordered and the total profit. Similarly, an increase in the first supplier’s reliability decreases the total number of units ordered and increases profit. In general, the firm simply compensates for small changes in reliability by ordering proportionately more items since it does not pay for the bad units.

Next we show the impact of the minimum order quantity constraints on the optimal sourcing strategy as described in Section 5. In this case, we choose three scenarios to illustrate our results. In all three scenarios, the results are generated assuming capacitated suppliers. Table 2 contains the results for these three scenarios in the following manner: model A is the same as in Table 1; model H represents the case where diversification benefits are included without supplier minimum order quantities and model I represents the case where supplier minimum order quantities are incorporated in the absence of diversification benefits. For models A and H, the diversification benefit function parameters were \( d_1 = 1000, \ d_2 = 62.5 \) and \( d_3 = 4 \).

Comparing models A and H, the optimal number of suppliers is reduced in the presence of the minimum order quantity constraints. In this case, the marginal benefits of diversification from including an additional supplier do not outweigh the marginal costs of placing a minimal order of 200 units from the fourth supplier. In model H, note that the optimal number of suppliers is four with the fourth supplier actually receiving an order of zero. This occurs due to the absence of a minimum order quantity, and reflects the situation where the buying firm would optimally qualify the fourth supplier. Ideally, a contractual arrangement would be negotiated with this supplier facilitating an agreement whereby the buying firm could place an actual order in an emergency situation. Comparing models A and I, the optimal number of suppliers is reduced when the diversification benefits are equal to zero. More specifically, when no diversification benefits exist, then the marginal cost of placing an order with the third supplier at the minimal level of 200 units is not economical.

A third set of numerical examples shown in Table 3 illustrates further interesting interactions that can occur between the minimum order quantities and the reliability factors for the suppliers. Diversification benefits, minimum order quantities, and capacity constraints are included for both of these models. In both of these examples, the supplier costs are \( c_1 = 6.5; \ c_2 = 7; \ c_3 = 8; \ c_4 = 9; \) and \( c_5 = 10 \). In model A, the reliabilities of the individual suppliers are homogenous and equal to 0.90. In model J, the reliability factor of the fourth supplier is set to equal half of that of other suppliers, (i.e., 0.45). The impact of the lower reliability factor on the optimal solution is that supplier four receives an order while supplier three does not. Moreover, the buying firm orders more units and makes more profit. This result is somewhat counter-intuitive, in that lower reliability leads to higher profit and total order quantities. Moreover, profit increases by sourcing from a higher cost

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter range</th>
<th>Firm profit ($)</th>
<th>Optimal # of suppliers</th>
<th>Total qty. ordered</th>
<th>Quantity allocations q1 q2 q3 q4 q5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>NA</td>
<td>6103.59</td>
<td>3</td>
<td>691</td>
<td>291 200 200 0 0</td>
</tr>
<tr>
<td>B</td>
<td>( p = [10,25] )</td>
<td>[1737,9065]</td>
<td>[2,3]</td>
<td>[600,701]</td>
<td>[300,300] [300,201] [0,200] 0 0</td>
</tr>
<tr>
<td>C</td>
<td>( s = [-6,6] )</td>
<td>[5375,6703]</td>
<td>[2,3]</td>
<td>[591,704]</td>
<td>[300,300] [291,254] [0,200] 0 0</td>
</tr>
<tr>
<td>D</td>
<td>( \alpha = [0,12] )</td>
<td>[6166,6065]</td>
<td>[2,3]</td>
<td>[660,701]</td>
<td>[260,300] [200,201] [200,200] 0 0</td>
</tr>
<tr>
<td>E</td>
<td>( d_1 = [250,2000] )</td>
<td>[5354,7103]</td>
<td>[3,3]</td>
<td>[691,691]</td>
<td>[291,291] [200,200] [200,200] 0 0</td>
</tr>
<tr>
<td>F</td>
<td>( c_1 = [6.25,6.75] )</td>
<td>[6170,6039]</td>
<td>[3,3]</td>
<td>[667,658]</td>
<td>[296,286] [200,200] [200,200] 0 0</td>
</tr>
<tr>
<td>G</td>
<td>( r_1 = [0.5,1] )</td>
<td>[6045,6104]</td>
<td>[3,3]</td>
<td>[700,662]</td>
<td>[300,262] [200,200] [200,200] 0 0</td>
</tr>
</tbody>
</table>

Table 2
Impact of minimum order quantity on the sourcing strategy

<table>
<thead>
<tr>
<th>Model</th>
<th>Firm profit ($)</th>
<th>Optimal # of suppliers</th>
<th>Total qty. ordered</th>
<th>Quantity allocations q1 q2 q3 q4 q5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5166.09</td>
<td>937.50</td>
<td>6103.59</td>
<td>3</td>
</tr>
<tr>
<td>H</td>
<td>5288.04</td>
<td>1000.00</td>
<td>6288.04</td>
<td>4</td>
</tr>
<tr>
<td>I</td>
<td>5199.00</td>
<td>0.00</td>
<td>5199.00</td>
<td>2</td>
</tr>
</tbody>
</table>
Recall first that in the original model from Section 3.2, we assume that the buying firm pays only for good units delivered. The net effect of this assumption is that the minimum order quantity in terms of the good units delivered is much lower for the lower reliability supplier. Also, note that the total quantity delivered is actually lower for model J than for model A due to the lower reliability factor. Therefore, there may be situations where it is optimal to source more units from a lower reliability yet higher cost supplier. We recognize that this result is a direct consequence of the assumption that no additional costs are incurred for the bad units that are delivered. In contrast, if we assume that the firm must pay for all units delivered regardless of the quality, then the result no longer holds.

Finally, based on model A and adjusted capacities of 700 for each supplier, we examine cases where the three lowest cost suppliers also have differing reliabilities and minimum order quantities. Each of the three lowest cost suppliers is ranked as best (B), middle (M), or worst (W) for cost, reliability and minimum order quantity. Notationally, instance BWM signifies that the lowest cost supplier is worst in regard to reliability and middle in regard to minimum order quantity. Table 4 summarizes the optimal sourcing strategies for selected instances. A general insight from this experiment is that the total usable quantity is determined by the lowest cost supplier who is not optimally allocated an amount equal to its minimum order quantity or its capacity (confirming the intuitive result developed in Theorem 5). Additionally, it is typically preferable to have higher cost suppliers with lower reliabilities. This situation effectively lowers the minimum order quantities of higher cost suppliers and requires shifting fewer units from lower cost suppliers to gain incremental diversification benefits.

### Table 3
Interactions between minimum order quantities and reliabilities

<table>
<thead>
<tr>
<th>Model</th>
<th>Firm profit ($)</th>
<th>Optimal # of suppliers</th>
<th>Total qty. ordered</th>
<th>Quantity allocations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sales</td>
<td>Diversification</td>
<td>Total</td>
<td>$q_1$</td>
</tr>
<tr>
<td>A</td>
<td>5166.09</td>
<td>937.50</td>
<td>6103.59</td>
<td>3</td>
</tr>
<tr>
<td>J</td>
<td>5142.39</td>
<td>937.50</td>
<td>6079.89</td>
<td>3</td>
</tr>
</tbody>
</table>

### Table 4
Heterogeneous low cost suppliers (capacity adjusted model A)

<table>
<thead>
<tr>
<th>Supplier characteristic vector ($c, r, z$)</th>
<th>Firm profit ($)</th>
<th>Optimal quantity allocations</th>
<th>Total quantity, $Q$</th>
<th>“Good” units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supp. 1</td>
<td>Supp. 2</td>
<td>Supp. 3</td>
<td>Sales</td>
<td>Divers.</td>
</tr>
<tr>
<td>BBB</td>
<td>MMM</td>
<td>WWW</td>
<td>5466.09</td>
<td>750.00</td>
</tr>
<tr>
<td>BBB</td>
<td>MMW</td>
<td>WWM</td>
<td>5288.59</td>
<td>937.50</td>
</tr>
<tr>
<td>BBW</td>
<td>MMM</td>
<td>WWB</td>
<td>5361.09</td>
<td>937.50</td>
</tr>
<tr>
<td>BMB</td>
<td>MBW</td>
<td>WWM</td>
<td>5278.59</td>
<td>937.50</td>
</tr>
<tr>
<td>BMB</td>
<td>MWW</td>
<td>WBM</td>
<td>5456.09</td>
<td>750.00</td>
</tr>
<tr>
<td>BWB</td>
<td>MBW</td>
<td>WMM</td>
<td>5256.09</td>
<td>937.50</td>
</tr>
<tr>
<td>BWB</td>
<td>MMM</td>
<td>WBB</td>
<td>5462.39</td>
<td>750.00</td>
</tr>
<tr>
<td>BWM</td>
<td>MBB</td>
<td>WMW</td>
<td>5462.39</td>
<td>750.00</td>
</tr>
<tr>
<td>BWM</td>
<td>MBW</td>
<td>WMB</td>
<td>5316.09</td>
<td>937.50</td>
</tr>
</tbody>
</table>

7. Conclusions and implications

In this paper, we have incorporated strategic diversification considerations within the traditional news-vendor framework to determine the optimal number of suppliers to place an order with and the corresponding quantities of those orders. Through the introduction of the diversification benefit function, we explicitly account for buyer specific supply base management benefits based on the size of the selected supplier pool. In essence, this function could be construed as capturing the monetized utility of supply insurance provided by the size of the selected supplier pool. This consideration highlights the need for a purchasing manager to align sourcing strategies with the firm’s operations and broader corporate strategy.
Analytic and numerical analysis of our model provide several managerial insights. To start with, consider the situation where supplier minimum order quantities are not considered and when the firm does not obtain any explicit benefits by diversifying its supplier base. First, the industry practice of single sourcing is only optimal when supplier capacities are relatively large as compared to product demand. In such a case, the firm's optimal choice is to source all its requirements from the least cost supplier. Interestingly, supplier reliabilities do not moderate the choice of the supplier unless the firm is required to compensate suppliers for all units ordered rather than simply the “good” units received. In the latter case, the ratio of costs to reliabilities is relevant in determining the supplier from which all demand is sourced. Second, we show that when supplier capacities are relevant, the optimal strategy for the firm is to source from multiple suppliers. Under this scenario we find that the firm’s total order quantity (across all suppliers) and expected profits are both lower than that compared to the scenario where suppliers are uncapacitated. The difference in profits could be regarded as the value to the firm which could be realized if the lowest cost supplier could be motivated to increase his/her capacity.

When positive net diversification benefits are incorporated (without supplier minimum order quantities), the key results are as follows. If suppliers are uncapacitated, then multiple supplier sourcing strategies are always optimal where the number of suppliers is determined by the diversification benefit function. Managerially, this implies that the firm should determine the total order quantity based on the least cost supplier. However, in placing orders, it should order the required amount from the least cost supplier and order marginal quantities from all the other selected suppliers. When suppliers are capacitated, a similar simple decision rule can be used by the firm when the number of suppliers which optimizes the diversification benefits is larger than the number which are selected without such a benefit.

Through an extensive numerical analysis we also examine the robustness of our results when supplier minimum order quantities are relevant in making a firm’s sourcing decisions. A counter-intuitive insight we obtain for this case is that there is an interaction between reliabilities, costs, and minimum order quantities. For example, we show that in certain cases, it may be optimal to source from a higher cost, lower reliability supplier as compared to a lower cost, higher reliability supplier. This is generally the case when a lower effective minimum order quantity is economically preferable. The insight here is that the flexibility of a supplier may have greater bearing on selection than unit cost.

There are several directions for future research that stem from our paper. First, an extension of this model that incorporates stochastic supplier reliability in determining the diversification benefits function would yield more insights on the benefits of risk pooling. One direct extension would be to examine distinct supply base configurations with alternative power positions held in the buyer-supplier relationship. In such a case, certain issues considered in our analysis could be reconfigured depending upon the partner who initiates/controls the sourcing process. In addition, the situation where suppliers can bid on multiple products for a single buying firm is an interesting problem that many firms are encountering. Finally, another study could focus on extending our analysis for a single-capacity constrained supplier in making distribution decisions for a set of distributors under the assumption of marketing related diversification benefits for the supplier.

Acknowledgement

We would like to thank the editors and referee for their insightful comments on the original manuscript which have led to a substantially improved paper.

Appendix 1. Proof of Theorem 1

Proof. Before proving the result in this theorem, we first characterize the optimal solution to the uncapacitated suppliers problem with no diversification benefit. This problem can be formalized as follows:

Maximize \[ Z_{q_i \geq 0} = (p - s)\mu - \sum_{i=1}^{N} c_i r_i q_i + s \sum_{i=1}^{N} r_i q_i - (p - s + u)ES. \] (16)
Adding the non-negativity constraints to this objective, the corresponding KKT conditions are

\[-c_ir_i + sr_i + (p - s + u)r_i \left(1 - F\left(\sum_{i=1}^{N} r_i q_i\right)\right) + \lambda_i = 0 \quad \forall i\]  

\[q_i \lambda_i = 0 \quad \forall i,\]  

\[\lambda_i \geq 0 \quad \forall i.\]  

We also note that \(\frac{\partial^2 Z}{\partial q_i} = -(p - s + u)r_i \frac{\partial F}{\partial q_i} < 0\) for all \(i\). Thus the objective is concave (i.e., the Hessian is negative semidefinite), and the KKT conditions are necessary and sufficient to obtain a global optimum solution to Eq. (16). Assuming that \(q_i > 0\) for any supplier \(i\), the FOC results in the following relationship:

\[F\left(\sum_{i=1}^{N} r_i q_i\right) = \frac{p - c_i + u}{p - s + u}.\]  

If the costs per unit \((c_i)\) are not equal among suppliers, then this relationship can only hold for any one supplier \(i\). Further, for all other suppliers \(j (j \neq i)\), it is obvious that \(q_j = 0\). Thus, Eq. (20) provides the following result for supplier \(i\):

\[F(r_i q_i) = \frac{p - c_i + u}{p - s + u}.\]  

Using this result, suppose that \(q_i > 0\) for some supplier \(i\), and \(q_j = 0 \forall j \neq i\). Then, for all suppliers \(j\) we have the following relationships:

\[F(r_i q_i) = \frac{(p - c_j + u)}{(p - s + u)} + \frac{\lambda_j}{r_j(p - s + u)},\]  

\[\lambda_j = r_j(p - s + u)F(r_i q_i) - r_j(p - c_j + u),\]  

\[\lambda_j = r_j(c_j - c_i).\]  

In order to satisfy the KKT conditions, then \(\lambda_j\) must be non-negative \(\forall j \neq i\). This can only occur when the \(i\)th supplier is the lowest cost supplier such that \(c_j > c_{(1)}\), where \(c_{(1)}\) is the cost per unit charged by the lowest cost supplier. For the situation where more than one supplier has the lowest cost, then it is obvious from the KKT conditions of optimality that the total usable quantity ordered must still satisfy the above critical ratio in Eq. (17). This concludes our proof. \(\square\)

Appendix 2. Method for quantifying diversification benefits \(d(X)\)

1. To start with assume that diversification benefits are not included. Then using the result of Theorem 1, determine the total quantity ordered by the firm \(MQ_{nc} = q_{(1)}\) and the corresponding expected firm level profits be \(MZ_{nc}\).

2. Set \(j = 2\).

3. Pose the following question to the firm level sourcing manager:
   
   Selecting \(j - 1\) suppliers, you will order \(MQ_{nc}\) units and realize average profits of \(MZ_{nc}\). Assuming that you decide to select \(j\) suppliers for ordering \(MQ_{nc}\) units, your expected profits will decline for certain by \$ \((c_j - c_{j-1})\) per unit ordered from supplier \(j\). What \$ amount would compensate you for the decline in expected profits assuming that you decide to: (a) source exactly \(MQ_{nc}\) units from all \(j\) suppliers and (b) order at least 1 unit from supplier \(j\)?
   
   Record this \$ amount as \(d(j)\).

4. If \(j = N\) stop else set \(j = j + 1\) and repeat Step (c).
Appendix 3. Proof of Theorem 2

**Proof.** Recall the capacitated suppliers problem with no diversification benefit is as follows:

Maximize \[ Z_{q_i} = (p - s)\mu - \sum_{i=1}^{N} c_i r_i q_i + s \sum_{i=1}^{N} r_i q_i - (p - s + u)ES \]

subject to

\[ q_i \leq y_i \quad \forall i, \]  
\[ q_i \geq 0 \quad \forall i, \]  
\[ q_i \leq z_i \quad \forall i. \]  

Based on the proof of Theorem 1, we know that \( Z \) is strictly concave in \( q_i \) and the constraints are all linear. Thus, by noting the concavity of the Lagrange function \( LL = Z + \sum_{i=1}^{N} \lambda_i (y_i - q_i) \), we know that Karush–Kuhn–Tucker (KKT) conditions are necessary and sufficient to identify the optimal solution to this problem. The KKT conditions for the Lagrangean are

\[ q_i \left[ \frac{\partial L}{\partial q_i} \right] = r_i q_i \left[ -c_i + s - (p - s + u) \left( 1 - F \left( \sum_{i=1}^{N} r_i q_i \right) \right) - \lambda_i \right] \quad \forall i, \]  
\[ \lambda_i \left[ \frac{\partial L}{\partial \lambda_i} \right] = \lambda_i [y_i - q_i] \quad \forall i. \]

To start with assume that there is some amount \( Q \) which will be sourced from all the suppliers. Then it is obvious that using the underlying logic of Theorem 1, we would choose to source the maximum amount possible from the lowest cost suppliers. Thus, our optimal algorithm (given \( Q \)) has the following structure:

1. Index suppliers in increasing order of the costs \( c_i \) such that \( c_{[1]} \leq c_{[2]} \leq \cdots \leq c_{[N]} \). Set \( s = 0 \) and \( q_i = 0 \) \( \forall i \).
2. \( s = s + 1 \). Determine \( t_{[s]} = \max\{0, Q - \sum_{i=1}^{s-1} r_i q_i\} \) and based on this, \( q_{[s]} = \min\{y_{[s]}, t_{[s]}\} \).
3. If \( s = n \) stop, else repeat 2.

Before proceeding to determine the optimal quantity \( Q \), note that for each supplier where \( q_i = y_i > 0 \), the Lagrange multiplier \( \lambda_i \) shows the marginal profit which could be obtained by increasing the capacity of supplier \( i \) and this can be determined from Eq. (27).

To determine the optimal quantity \( Q \) which should be sourced from all the suppliers, we note that for any one supplier \( k, -c_k + s + (p - s + u)(1 - F(Q)) = 0 \) must hold. Obviously, since profits are maximized by ordering first from the lower cost suppliers, determining \( k \) iteratively as in our Theorem must hold. This concludes our proof. \( \square \)

Appendix 4. Proof of Theorem 5

**Proof.** Recall the capacitated suppliers problem with no diversification benefit is as follows:

Maximize \[ Z_{q_i} = \frac{(p - s)(b + a)}{2} - \sum_{i=1}^{N} c_i r_i q_i + s \sum_{i=1}^{N} r_i q_i - (p - s + u)ES \]

subject to

\[ q_i \leq y_i \quad \forall i, \]  
\[ q_i \geq 0 \quad \forall i, \]  
\[ q_i \geq z_i \quad \forall i. \]  

Since \( Z \) is strictly concave in \( q_i \) and the constraints are all linear, the KKT conditions for the Lagrange function \( L(L = Z + \sum_{i=1}^{N} \lambda_i (y_i - q_i) + \sum_{i=1}^{N} \theta_i (q_i - z_i)) \), are necessary and sufficient to identify the optimal solution to this problem. These conditions are
To start with assume that there is some amount $Q$ which will be sourced from all the suppliers. Then, similar to the underlying logic of Theorems 1 and 2, we know that at most one supplier can be unconstrained in the optimal solution.

References


