A Note on Optimal Dynamic Incentive Schemes

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ABSTRACT

We consider a dynamic setting where both adverse selection and moral hazard concerns are present. We analyze how the power of the optimal incentive scheme varies over time according to the agent's performance history. In many circumstances, the power of later incentive schemes optimally declines as earlier performance improves. The imposition of poor performance incentives following good performance can limit initial incentives to understate one's capabilities.

1. Introduction.

The optimal design of incentive contracts in settings where the contracting parties interact more than once is an important issue, both from a theoretical and a practical standpoint. Many, if not most, relationships involve repeated interactions. For example, a regulator and a regulated firm commonly interact over many years, as do most employers and their employees.

An important question that arises in dynamic settings is how early performance should affect the terms of subsequent incentive schemes. In the regulatory arena, for example, experts are debating how AT&T's performance during the four years (1989-1993) it has operated under price cap regulation should influence future regulations governing AT&T's operations.[1] A related and important question in the marketing arena is how should the terms of future compensation for a sales force be influenced by its past sales record?

The purpose of this note is to develop a simple model that can begin to address questions of this nature. We analyze a two-period model in which, at the start of the first period, the policy designer (the principal) specifies the incentive schemes that will be implemented in the two periods to follow. The terms of the second-period incentive scheme can be based on the first-period performance of the subordinate (the agent). Incentive schemes are required to motivate the agent because neither the agent's innate ability nor the amount of effort he supplies can be observed by the principal. We focus on the question of how first-period performance optimally affects the power of the second-period incentive scheme. The power of an incentive schemes refers to the sensitivity of the agent's compensation to his performance.

A high-powered incentive scheme generally increases both social surplus and profit opportunities for the agent. Thus, one might suspect that an agent who performs well in the initial period should be rewarded with a high-powered incentive scheme in the second period.

Such a reward structure would seem to motivate the agent to work diligently in the initial period. However, our main finding is that the opposite is often true. Superior first-period performance will often call forth an incentive scheme with lower power in the second period. An agent who initially claims to have low innate ability will be punished with a low-powered second-period incentive scheme following superior first-period performance. Superior first-period performance serves as evidence of high, not low, ability. Therefore, if the agent is punished with a low-powered incentive scheme after he claims to have meager ability but his performance suggests otherwise, the agent's incentive to understate his ability can be mitigated. In essence, the adverse selection concerns outweigh the moral hazard concerns, and good performance optimally results in reduced future incentives. While this policy prescription may sound perverse when viewed from a narrow perspective, a broader view of all of the relevant incentive problems renders the conclusion quite intuitive.

Our analysis differs from most in the dynamic incentive literature because it incorporates both adverse selection and moral hazard concerns. The problems examined by Radner (1981, 1985), Lambert (1983), Rubinstein and Yaari (1983), Rogerson (1985), and Spear and Srivastava (1987) are all repeated moral hazard problems, where the agent's only private information concerns the amount of effort he has supplied. While risk aversion is the fundamental contracting friction in these models, all parties are risk neutral in our model. Risk aversion also plays a critical role in Allen's (1985) dynamic model. While Allen considers a dynamic adverse selection problem, his focus is on the advantages of long term contracts over short term contracts, not on the manner in which incentive contracts are optimally revised over time. Fudenberg, Holmstrom, and Milgrom (1990) also adopt Allen's focus, and demonstrate how the

absence of adverse selection considerations is important for short term contracts to work as well as long term contracts.

A dynamic analysis that incorporates both adverse selection and moral hazard concerns is Baron and Besanko's (1984) model in which a regulated firm has private cost information and can undertake unobservable research and development activities.[2] The authors focus on how the presence of moral hazard influences optimal pricing design, not on how observed performance affects the power of future incentive schemes. Laffont and Tirole (LT) (1993, p. 103) discuss a dynamic model that incorporates adverse selection and moral hazard. LT prove that the optimal policy has the principal commit at the outset to offer the same incentive scheme to the agent in every period, regardless of the agent's performance in preceding periods.[3] This conclusion follows from the deterministic technology in LT's model. With a nonstochastic relationship among effort, productivity, and performance, the agent can adjust his effort level to compensate perfectly for any misrepresentation of his productivity. Therefore, given the agent's equilibrium claim about his productivity, the agent's observed performance provides no new information about the agent's true productivity. In contrast, the agent's realized performance provides additional information about the agent's true productivity in our model, because the stochastic technology does not enable the agent to simply adjust his effort level to compensate perfectly for any misrepresentation of his productivity.

Of course, our model differs from those in the literature on ratcheting and limited commitment because we assume the principal can credibly specify in advance how future incentive schemes will vary according to the agent's observed performance.[4] While more limited powers of commitment may prevail in some settings, it is important to understand how

a principal with the power to commit to future policy should optimally employ the agent's performance to update incentive schemes. In practice, most policy designers are either endowed with some ability to commit to future policy or can create this ability.[5]

Our analysis proceeds as follows. The simple model we investigate is described in section 2. A formal statement of the principal's problem is recorded in section 3. Our central finding is stated and explained in section 4. Concluding thoughts and some extensions of our analysis are discussed in section 5. The proofs of all formal results are in the Appendix.

2. The Model.

We examine a very simple model in order to demonstrate our central finding most clearly. There are only two actors in the model: the principal and the agent. The agent's effort is a critical input in the production technology owned by the principal. Production results in either a more or a less favorable outcome, with respective values V_2 and V_1 for the principal. For instance, the higher value might result from a successful innovation, while the lower value is realized if no technological breakthrough is achieved. The probability of "success" (V_2) is denoted $p(\cdot) \in [0,1]$. This probability is influenced both by the agent's innate ability, θ, and the effort, $e \geq 0$, he supplies. The higher is the agent's ability level, the greater is the total and the marginal product of his effort, i.e., $p_\theta(e \mid \theta) > 0$ and $p_{e\theta}(e \mid \theta) > 0$ for all $e > 0$ and for all θ, where subscripts on $p(\cdot)$ denote partial derivatives. Higher effort levels increase the probability of success, but at a decreasing rate, i.e., $p_e(e \mid \theta) > 0$ and $p_{ee}(e \mid \theta) < 0$ for all $e \geq 0$ and for all θ. To ensure an interior solution to the problem under consideration, standard Inada conditions are also imposed, i.e., $p_e(0 \mid \theta) = \infty$ and $p_e(\infty \mid \theta) = 0$ for all θ. In words,

initial effort supply by the agent is infinitely productive, but the marginal contribution of effort eventually falls to zero.

The functional form of the production technology, $p(\cdot)$, is common knowledge. However, the principal cannot observe the level of effort supplied by the agent. Nor can she observe the agent's ability level. The outcome of the production process (success or failure) is observed publicly.

The realized utility level of the agent is the difference between the transfer payments (T) he receives from the principal and the amount of effort he supplies. Thus, the agent is risk neutral, and his unit cost of effort is normalized to unity. The principal is also risk neutral, and seeks to maximize the difference between the expected value of the agent's output and expected payments to the agent. The principal's expectations are influenced by her beliefs about the agent's productivity. For simplicity, the agent's ability is assumed to take on one of two values, θ_1 or θ_2, where $\theta_1 < \theta_2$. $\phi_i \in (0,1)$ will denote the probability that the agent's productivity is θ_i for $i = 1,2$.

There are two periods in the model. The agent's productivity is the same in both periods. The agent learns his productivity at the start of the first period. Then the principal announces the incentive schemes available to the agent. An incentive scheme specifies the transfer payment the principal will make to the agent at the end of the second period.[6] This payment can be based on the agent's performance in both the first and the second period and on the agent's initial claim about his productivity. Formally, T^i_{jk} will denote the transfer payment delivered to the agent after he initially claims his productivity to be θ_i and then produces V_j in the first period and V_k in the second period. After observing the incentive schemes he is offered,

the agent makes a binding choice by announcing his productivity parameter.[7] Then the agent decides how much effort to supply in the first period. The first period ends with (public) observation of the agent's performance. In the second period, the agent first chooses how much effort to supply. Then the realized performance is publicly observed. Finally, the promised transfer payment is delivered by the principal to the agent.

Three elements of the model warrant emphasis. First, the principal faces both an adverse selection problem and a moral hazard problem. The adverse selection problem arises because the agent has private productivity information from the outset of his relationship with the principal. The moral hazard problem is present because the principal cannot observe the level of effort supplied by the agent. Second, risk sharing is not a concern in this model. Both the principal and the agent are risk neutral. Third, and perhaps most importantly, the principal has unrestricted commitment powers in this model. Consequently, any indexing of final payments on first period performance is designed optimally in advance. If any form of ratcheting occurs, it is not because the principal is unable to refrain from exploiting information she infers from first-period performance.

3. Statement of the Problem.

The principal's problem [P] in this dynamic setting is the following:

$$\underset{T^i_{jk}}{\text{Maximize}} \quad \sum_{i=1}^{2} \phi_i \Big\{ p(e^i \mid \theta_i) \Big\{ V_2 + p(e^i_2 \mid \theta_i)\,[V_2 - T^i_{22}] + [1 - p(e^i_2 \mid \theta_i)]\,[V_1 - T^i_{21}] \Big\}$$
$$+ [1 - p(e^i \mid \theta_i)] \Big\{ V_1 + p(e^i_1 \mid \theta_i)[V_2 - T^i_{12}] + [1 - p(e^i_1 \mid \theta_i)]\,[V_1 - T^i_{11}] \Big\} \Big\} \tag{3.1}$$

subject to:

$$\pi^i(e^i, \theta_i) \geq 0 \qquad \forall\, i = 1,2 \;; \tag{3.2}$$

$$\pi^i(e^i, \theta_i) \geq \pi^k(e^{ki}, \theta_i) \qquad \forall\, k \neq i, \;\; i,k = 1,2 \;; \tag{3.3}$$

$$e_j^{ik} \equiv \underset{e}{\operatorname{argmax}} \;\; \pi_j^i(e, \theta_k) \;\; \text{and} \;\; e_j^i \equiv e_j^{ii} \qquad \forall\, i,j,k = 1,2 \;; \text{ and} \tag{3.4}$$

$$e^{ik} \equiv \underset{e}{\operatorname{argmax}} \;\; \pi^i(e, \theta_k) \;\; \text{and} \;\; e^i \equiv e^{ii} \qquad \forall\, i,j,k = 1,2 \;; \tag{3.5}$$

where $\forall \;\; i,j,k = 1,2$:

$$\pi_j^i(e, \theta_k) \equiv p(e \mid \theta_k)\, T_{j2}^i + [1 - p(e \mid \theta_k)]\, T_{j1}^i - e \;, \text{ and} \tag{3.6}$$

$$\pi^i(e, \theta_k) \equiv p(e \mid \theta_k)\, \pi_2^i(e_2^{ik}, \theta_k) + [1 - p(e \mid \theta_k)]\, \pi_1^i(e_1^{ik}, \theta_k) - e \;. \tag{3.7}$$

The objective function, (3.1), in [P] reflects the principal's goal of maximizing the difference between the expected value (V) of the agent's performance and the expected payments (T) made to the agent. The individual rationality constraint, (3.2), states that the agent will only agree to work for the principal if he anticipates nonnegative earnings. The incentive compatibility constraint, (3.3), ensures that the agent will truthfully report his ability, thereby choosing the $\{T_{jk}^i\}$ incentive scheme when his ability is θ_i.[8] Constraints (3.4) and (3.5) define the agent's self interested choice of effort, given his true ability (θ_k), his reported ability (θ_i), and any prior performance realization (V_j). Notice that, for simplicity, there is no discounting in the model.

Before proceeding to characterize the solution to [P], it is useful to identify the policy the principal would implement if she shared the agent's knowledge of his ability. This "first-

best policy" is characterized in the following definition.

Definition. In the first-best policy: (i) $\pi^i(e^i, \theta_i) = 0$ for $i = 1, 2$; (ii) $T^i_{j2} - T^i_{j1} = V_2 - V_1$ $= V_2 - V_1$ for all $i, j = 1, 2$; and (iii) $T^i_{2j} - T^i_{1j} = V_2 - V_1$ for all $i, j = 1, 2$.

Under the first-best policy, the principal ensures total expected surplus is maximized while limiting the agent's expected net payoff to his opportunity wage. The surplus-maximizing (or first-best) effort is induced from the agent by promising him the full incremental value of his performance. In other words, the agent is paid V_2 - V_1 extra dollars for success rather than failure.

4. Findings.

Certain features of the solution to [P] are immediate. These features are recorded in Proposition 1.

Proposition 1. At the solution to [P]:

(i) $\pi^1(e^1, \theta_1) = 0$;

(ii) $\pi^2(e^2, \theta_2) = \pi^1(e^{12}, \theta_2) > 0$;

(iii) $T^2_{22} - T^2_{21} = T^2_{12} - T^2_{11} = V_2 - V_1$;

(iv) $T^1_{12} - T^1_{11} < V_2 - V_1$; and

(v) $T^1_{22} - T^1_{21} < V_2 - V_1$.

Proposition 1 summarizes some well-known properties of optimal incentive schemes.

Because the agent has private knowledge of his ability, he can command rents when his ability is high (see property (ii) of Proposition 1), but not when his ability is low (see property (i)). The agent must be rewarded when he truthfully reports his ability to be high. Otherwise, he could always claim to have low ability, and thereby demand compensation commensurate with the high costs of achieving success implied by low ability. To limit the financial gains from understating ability, a report of low ability leads to a reduction in the agent's second-period effort supply below its first-best level. The reduced effort supply is accompanied by a reduction in expected payment to the agent equal to the avoided costs of the low-ability agent. Since the corresponding avoided costs of the high-ability (low-cost) agent are less than those of the low-ability (high-cost) agent, the reduction in effort supply and corresponding payment reduction is differentially disadvantageous to the agent with high ability. Consequently, the incentives of the high-ability agent to understate his ability are mitigated by reducing the effort supply of the low-ability agent. The reduced effort supply is ensured by restricting the agent who claims to have low ability to a second period lottery that provides an incremental payoff for success (rather than failure) below the principal's incremental valuation of success. (See properties (iv) and (v) of Proposition 1.)

Of course, when the agent's ability is truly low, he has no incentive to claim it is high. Therefore, there are no gains from distorting the second-period effort supply of the agent who truthfully reports his ability to be high. First-best effort levels are ensured by promising the agent the full incremental value of success, as reported in property (iii) of Proposition 1.[9] Thus, Proposition 1 reflects the standard conclusion that agents of greater ability optimally face incentive schemes of higher power.

It remains to determine how the power of the second-period incentive scheme is optimally influenced by the agent's first period performance. The power of an incentive scheme refers to the incremental reward for success rather than failure that the scheme provides. Notice from property (iii) of Proposition 1 that the power of the second-period incentive scheme faced by the high-ability agent in equilibrium is always V_2 - V_1, regardless of the agent's first-period performance. Proposition 2 reveals a different conclusion for the low-ability agent.

Proposition 2. Suppose $p_{eee}(e \mid \theta_i) \leq 0$ and $p_{ee\theta}(e \mid \theta) \geq 0$ for all $e \geq 0$ and for $i =$ 1, 2. Then $T^1_{22} - T^1_{21} < T^1_{12} - T^1_{11} < V_2 - V_1$ at the solution to [P].

Proposition 2 reports that under the identified regularity conditions, an agent who initially claims to have low ability faces an incentive scheme with lower power in the second period if he succeeds in the first period than if he fails in the first period. This finding may seem inconsistent with the standard intuition that agents of greater ability optimally face incentive schemes of higher power. First-period success would seem to provide evidence suggesting that the agent has high ability, and therefore that the principal should establish an incentive scheme with higher power in the second period. First-period success is indeed a signal that the agent may have high ability. However, this fact is precisely why the principal lowers the power of the second-period incentive scheme following first-period success.

Recall that the critical task of the principal is to limit the agent's incentive to understate his true ability. To best accomplish this task, the principal distorts from its first-best configuration the incentive scheme that is awarded to the agent who claims to have low ability. The distortions are designed to differentially disadvantage the agent if his ability is high rather

than low.[10] The distortion identified in Proposition 2 serves exactly this purpose. A high-ability agent is more likely to succeed than a low-ability agent under any nontrivial incentive contract. Consequently, a punishment that is imposed following an initial claim of low ability and subsequent first-period success will cause an agent who has understated his ability to suffer more than the agent who truly has low ability. The reduction in the power of the second-period incentive scheme limits the extra expected earnings of a high-ability agent, and therefore serves as a punishment. By introducing this punishment after the agent claims to have low ability but then succeeds in the first period, the principal differentially disadvantages the high-ability agent, and thereby limits his incentive to understate his ability.

The regularity conditions cited in Proposition 2 simply strengthen the notion of diminishing returns to effort and enhance the interpretation of θ as an ability parameter. The first condition ($p_{eee}(\cdot) \leq 0$) requires that returns to effort diminish with the level of effort at an increasing rate. The second condition ($p_{ee\theta}(\cdot) \geq 0$) requires that the rate of diminishing returns to effort be less pronounced the higher the agent's ability level.

5. Conclusions and Extensions.

We have examined a very simple model to show most clearly why better initial performance by an agent may optimally result in his facing worse long run incentives in a dynamic setting with adverse selection and moral hazard concerns. Despite the many special features of our model, the basic insight seems to be robust. Better performance can signal that an initial claim of low ability may have been false. By imposing a low-powered incentive scheme following an initial claim of low ability and subsequent good performance, incentives

to understate one's ability can be mitigated.

This basic insight might usefully be extended in a number of directions. Alternative characterizations of the information asymmetry between principal and agent might be considered, for example. One alternative would likely give rise to a finding that is complementary to the finding presented here. Suppose that the agent had private information about his opportunity wage, and that the agent's opportunity wage is positively correlated with his ability working for the principal. To limit the agent's incentive to exaggerate his opportunity wage in this setting, the principal will optimally distort the incentive schemes she presents to the agent who claims to have a particularly high opportunity wage.[11] Following the logic developed above, the distortion will likely include an increase in the power of later incentive schemes following poor performance by an agent who implicitly claimed to have high ability by reporting a large opportunity wage. In this setting, then, poor performance may be "rewarded" with enhanced incentives (which are differentially disadvantageous to an agent with low ability).

Another extension of the simple model analyzed here might allow for second sourcing.[12] The threat of terminating an agent and replacing him with another agent can be a useful motivational device. The question that arises in the dynamic setting analyzed here is when and how the second source is optimally employed. The logic developed above suggests that the agent who initially claims to have little ability and subsequently performs well may face the greatest threat of replacement by the second source (since termination provides future incentives that are particularly low powered). This finding would give rise to a policy prescription that, on the surface, seems counterintuitive: good performance optimally increases one's chance of termination. The conclusion is more intuitive, however, if one recognizes how the threat of

termination limits initial incentives to misrepresent one's capabilities.

A third extension of our model would endow the principal with additional policy instruments. For example, the principal might be able to supply inputs to the production process. In static models of adverse selection, undersupply of a productive resource optimally accompanies a report of low ability in order to limit incentives to understate ability.[13] A different conclusion seems possible in dynamic models where adverse selection and moral hazard are present. If an input makes realized performance a more reliable signal of the agent's ability, expanded supply of the input may optimally accompany reports of low ability, to limit incentives for understatement of ability.

In closing, we briefly mention an empirical implication of our simple model. Our analysis suggests a possible reason for limited stock market response to observed (research and development) success by a firm. Although success may represent "good news" on the surface, it can call forth diminished incentives for future success under an optimal incentive policy. Consequently, early success can signal bad news about potential future profitability, leading to a limited stock market response. This and other empirical implications of our model warrant additional investigation.

FOOTNOTES

1. See Mitchell and Vogelsang (1991) for a useful description of the price cap regulation under which AT&T operates.

2. Baron and Besanko (1984) analyze this model in the penultimate section of their paper. The majority of their paper focuses on a dynamic adverse selection problem, where moral hazard is absent. (See the next footnote.) Besanko (1985b) adopts this same focus.

3. Structurally, Laffont and Tirole's (1993) dynamic model is very similar to the dynamic model of adverse selection on which Baron and Besanko (1984) focus their analysis. Baron and Besanko reach the same conclusion drawn by LT for the setting in which the agent's productivity does not change over time.

4. Among the many articles in these literatures are Weitzman (1980), Holmstrom (1982), Baron and Besanko (1987), Laffont and Tirole (1987, 1988a), and Lewis and Sappington (1991).

5. See Sappington (1986).

6. There is no loss of generality in assuming payments are made to the agent only at the end of the second period because the agent does not have the option of leaving the principal's employ after the first period, because both principal and agent are risk neutral, and because there is no discounting in our model.

7. The agent also has the option of rejecting all incentive schemes that are offered. By refusing all schemes, the agent terminates his relationship with the principal, and works

elsewhere, earning his (known) opportunity wage of zero. We assume that production by the agent is sufficiently valuable that the principal always induces the agent to work for her.

8. The revelation principle (e.g., Myerson (1979)) ensures there is no loss of generality by focusing on the equilibrium in which the agent is induced to truthfully reveal his private information.

9. The first-best effort level will be induced from the high-ability agent in the first-period also.

10. Recall the explanation of Proposition 1.

11. See Lewis and Sappington (1989) for a static model with this feature.

12. See Anton and Yao (1987), Demski, Sappington, and Spiller (1987), and Laffont and Tirole (1988b), for example.

13. See Sappington (1983) and Besanko (1985a).

APPENDIX

Proof of Proposition 1.

The proof proceeds by solving a modified version of [P], called [P]′, in which constraint (3.2) is not imposed for $i = 2$ and constraint (3.3) is not imposed for $i = 1$. It is straightforward to verify that the omitted constraints are satisfied at the identified solution, so the solution to [P]′ is the solution to [P].

To characterize the solution to [P]′, let λ_i, λ_{ik}, γ_j^{ik}, and γ^{ik} denote the Lagrange multipliers associated with constraints (3.2), (3.3), (3.4), and (3.5), respectively, and let $\gamma_j^i \equiv \gamma_j^{ii}$ and $\gamma^{ii} \equiv \gamma^i$. Also replace the argmax formulation in (3.4) and (3.5) with the obvious first order condition. Differentiating the Lagrangian function with respect to T_{22}^1, T_{21}^1, T_{12}^1, and T_{11}^1 provides (A1.1), (A1.2), (A1.3) and (A1.4), respectively.

$$p(e^1 \mid \theta_1)p(e_2^1 \mid \theta_1)[-\phi_1 + \lambda_1] - \lambda^{21}p(e_{12} \mid \theta_2)p(\theta_2 \mid e_2^{12}) + \gamma_2^1 p_e(e_2^1 \mid \theta_1)$$
$$+ \gamma_2^{12} p_e(e_2^{12} \mid \theta_2) + \gamma^1 p_e(e^1 \mid \theta_1)p(e_2^1 \mid \theta_1) + \gamma^{12} p_e(e^{12} \mid \theta_2)p(e_2^{12} \mid \theta_2) = 0; \qquad \text{(A1.1)}$$

$$p(e^1 \mid \theta_1)[1 - p(e_2^1 \mid \theta_1)][-\phi_1 + \lambda_1] - \lambda_{21}p(e^{12} \mid \theta_2)[1 - p(e_2^{12} \mid \theta_2)] - \gamma_2^1 p_e(e_2^1 \mid \theta_1)$$
$$- \gamma_2^{12} p_e(e_2^{12} \mid \theta_2) + \gamma^1 p_e(e^1 \mid \theta_1)[1 - p(e_2^1 \mid \theta_1)]$$
$$+ \gamma^{12} p_e(e^{12} \mid \theta_2)[1 - p(e_2^{12} \mid \theta_2)] = 0 ; \qquad \text{(A1.2)}$$

$$[1 - p(e^1 \mid \theta_1)]p(e_1^1 \mid \theta_1)[-\phi_1 + \lambda_1] - \lambda_{21}[1 - p(e^{12} \mid \theta_2)]p(e_1^{12} \mid \theta_2) + \gamma_1^1 p_e(e_1^1 \mid \theta_1)$$
$$+ \gamma_1^{12} p_e(e_1^{12} \mid \theta_2) - \gamma^1 p_e(e^1 \mid \theta_1)p(e_1^1 \mid \theta_1) - \gamma^{12} p_e(e^{12} \mid \theta_2)p(e_1^{12} \mid \theta_2) = 0 ; \qquad \text{(A1.3)}$$

$$[1-p(e^1 \mid \theta_1)][1-p(e_1^1 \mid \theta_1)][-\phi_1+\lambda_1]-\lambda_{21}[1-p(e^{12} \mid \theta_2)][1-p(e_1^{12} \mid \theta_2)]$$

$$-\gamma_1^1 p_e(e_1^1 \mid \theta_1) - \gamma_1^{12} p_e(e_1^{12} \mid \theta_2) - \gamma^1 p_e(e^1 \mid \theta_1)[1-p(e_1^1 \mid \theta_1)]$$

$$-\gamma^{12} p_e(e^{12} \mid \theta_2)[1-p(e_1^{12} \mid \theta_2)] = 0 \ . \qquad \text{(A1.4)}$$

Differentiating the Lagrangian with respect to e_2^1 and e_1^1 provides (A1.5) and (A1.6), respectively.

$$\phi_1 p(e^1 \mid \theta_1) p_e(e_2^1 \mid \theta_1)[V_2 - V_1 + T_{21}^1 - T_{22}^1] + \gamma_2^1 p_{ee}(e_2^1 \mid \theta_1)[T_{22}^1 - T_{21}^1] = 0 \ ; \text{ and} \qquad \text{(A1.5)}$$

$$\phi_1[1-p(e^1 \mid \theta_1)] p_e(e_1^1 \mid \theta_1)[V_2 - V_1 + T_{11}^1 - T_{12}^1] + \gamma_1^1 p_{ee}(e_1^1 \mid \theta_1)[T_{12}^1 - T_{11}^1] = 0 \ . \qquad \text{(A1.6)}$$

The corresponding necessary conditions for a maximum with respect to e_2^{12}, e_1^{12}, and e^{12} reveal immediately that $\gamma_2^{12} = \gamma_1^{12} = \gamma^{12} = 0$. Using this finding, the necessary conditions for a maximum with respect to T_{ij}^2 $(i, j = 1,2)$ provide $\gamma_2^2 = \gamma_1^2 = \gamma^2 = 0$ and $\lambda_{21} = \phi_2$. Property (ii) of the Proposition then follows from complementary slackness. Using the fact that $\gamma_2^2 = \gamma_1^2 = 0$, the necessary conditions for a maximum with respect to e_2^2 and e_1^2 ensure property (iii) holds.

Adding equations (A1.1) and (A1.2) provides:

$$p(e^1 \mid \theta_1)[-\phi_1 + \lambda_1] - \phi_2 p(e^{12} \mid \theta_2) + \gamma^1 p_e(e^1 \mid \theta_1) = 0 \ . \qquad \text{(A1.7)}$$

Similarly, adding (A1.3) and (A1.4) provides:

$$[1-p(e^1 \mid \theta_1)][-\phi_1 + \lambda_1] - \phi_2[1-p(e^{12} \mid \theta_2)] - \gamma^1 p_e(e^1 \mid \theta_1) = 0 \ . \qquad \text{(A1.8)}$$

Adding (A1.7) and (A1.8) reveals $\lambda_1 = 1$, so property (i) holds. Furthermore, from (A1.7):

$$\gamma^1 = \phi_2[p(e^{12} \mid \theta_2) - p(e^1 \mid \theta_1)]/p_e(e^1 \mid \theta_1) > 0 \ . \tag{A1.9}$$

The inequality holds because $p_e(\cdot) < 0$, $p_{ee}(\cdot) > 0$ and $p_e(e \mid \theta_2) > p_e(e \mid \theta_1) \quad \forall\, e > 0$.

Using (A1.9), (A1.1) and (A1.3) provide:

$$\gamma_2^1 = \phi_2 p(e^{12} \mid \theta_2)\,[p(e_2^{12} \mid \theta_2) - p(e_2^1 \mid \theta_1)]/p_e(e_2^1 \mid \theta_1) > 0 \ ; \text{ and} \tag{A1.10}$$

$$\gamma_1^1 = \phi_2[1 - p(e^{12} \mid \theta_2)]\,[p(e_1^{12} \mid \theta_2) - p(e_1^1 \mid \theta_1)]/p_e(e_1^1 \mid \theta_1) > 0 \ . \tag{A1.11}$$

Properties (iv) and (v) of the Proposition then follow immediately from (A1.5) and (A1.6), since $p_{ee}(\cdot) < 0$. ■

<u>Proof of Proposition 2.</u>

It follows from (A1.5), (A1.6), (A1.10), and (A1.11) that:

$$T_{22}^1 - T_{21}^1 = \frac{\phi_1 p(e^1 \mid \theta_1) p_e(e_2^1 \mid \theta_1)\,[V_2 - V_1]}{D_2^1} \ , \tag{A2.1}$$

where

$$D_2^1 = \phi_1 p(e^1 \mid \theta_1) p_e(e_2^1 \mid \theta_1)$$

$$+ \phi_2 \,|p_{ee}(e_2^1 \mid \theta_1)\,|\, p(e^{12} \mid \theta_2)[p(e_2^{12} \mid \theta_2) - p(e_2^1 \mid \theta_1)]/p_e(e_2^1 \mid \theta_1) \ ; \text{ and}$$

$$T_{12}^1 - T_{11}^1 = \frac{\phi_1[1 - p(e^1 \mid \theta_1)]\,p_e(e_1^1 \mid \theta_1)\,[V_2 - V_1]}{D_1^1} \ , \tag{A2.2}$$

where

$$D_1^1 = \phi_1[1 - p(e^1 \mid \theta_1)]\,p_e(e_1^1 \mid \theta_1)$$

$$+ \phi_2 \,|p_{ee}(e_1^1 \mid \theta_1)\,|\,[1 - p(e^{12} \mid \theta_2)]\,[p(e_1^{12}, \theta_2) - p(e_1^1 \mid \theta_1)]/p_e(e_1^1 \mid \theta_1).$$

From (3.4), second-period effort levels are determined by the following equality:

$$p_e(e \mid \theta)\ \Delta = 1\ , \tag{A2.3}$$

where Δ denotes the relevant incremental payoff for success rather than failure. Totally differentiating (A2.3) provides

$$\frac{de}{d\Delta} = -\frac{p_e(e \mid \theta)}{\Delta\, p_{ee}(e \mid \theta)} > 0\ . \tag{A2.4}$$

Differentiating (A2.4) with respect to θ provides

$$\frac{\partial}{\partial\theta}\left(\frac{de}{d\Delta}\right) \overset{s}{=} p_e(\cdot)p_{ee\theta}(\cdot) - p_{ee}(\cdot)p_{e\theta}(\cdot) > 0 \quad \text{if } p_{ee\theta}(\cdot) \geq 0\ . \tag{A2.5}$$

Suppose $T^1_{22} - T^1_{21} \geq T^1_{12} - T^1_{11}$. Then $e^1_2 \geq e^1_1$ from (A2.4). Hence, $p(e^1_2 \mid \theta_1) \geq p(e^1_1 \mid \theta_1)$. Furthermore, $p_e(e^1_2 \mid \theta_1) \leq p_e(e^1_1 \mid \theta_1)$ since $p_{ee}(\cdot) < 0$. In addition, if $p_{eee}(\cdot) \leq 0$, then $|p_{ee}(e^1_2 \mid \theta_1)| \geq |p_{ee}(e^1_1 \mid \theta_1)|$. Furthermore, if $p_{ee\theta}(\cdot) \geq 0$, then from (A2.5), $e^{12}_2 - e^{12}_1 > e^1_2 - e^1_1$. Hence $p(e^{12}_2 \mid \theta_2) - p(e^{12}_1 \mid \theta_2) > p(e^1_2 \mid \theta_1) - p(e^1_1 \mid \theta_1)$ since $p_{e\theta}(\cdot) > 0$. Therefore,

$$p(e^{12}_2 \mid \theta_2) - p(e^1_2 \mid \theta_1) > p(e^{12}_1 \mid \theta_2) - p(e^1_1 \mid \theta_1)\ . \tag{A2.6}$$

With $T^1_{22} - T^1_{21} \geq T^1_{12} - T^1_{11}$, (A2.1) and (A2.2) imply:

$$\frac{p(e^{12} \mid \theta_2)}{p(e^1 \mid \theta_1)} \cdot \frac{[p(e^{12}_2 \mid \theta_2) - p(e^1_2 \mid \theta_1)]\,|p_{ee}(e^1_2 \mid \theta_1)|}{[p_e(e^1_2 \mid \theta_1)]^2}$$

$$\leq \frac{1 - p(e^{12} \mid \theta_2)}{1 - p(e^1 \mid \theta_1)} \cdot \frac{[p(e^{12}_1 \mid \theta_2) - p(e^1_1 \mid \theta_1)]\,|p_{ee}(e^1_1 \mid \theta_1)|}{[p_e(e^1_1 \mid \theta_1)]^2}\ . \tag{A2.7}$$

But given (A2.6) and the conclusions recorded immediately above (A2.6), the inequality in (A2.7) can only hold if

$$\frac{p(e^{12} \mid \theta_2)}{1 - p(e^{12} \mid \theta_2)} < \frac{p(e^{1} \mid \theta_1)}{1 - p(e^{1} \mid \theta_1)} . \tag{A2.8}$$

But (A2.8) can only hold if $p(e^{12} \mid \theta_2) < p(e^{1} \mid \theta_1)$, which cannot be true. Hence, the proof is complete by contradiction. ■

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