Production Efficiency and Optimal

Pricing in Intermediate-Good Regulated Industries†

by

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Abstract

This paper reconsiders, within the context of a public sector pricing problem, the relationship that exists between the production efficiency result of Diamond and Mirrlees and the Ramsey Rule. Specifically, the paper considers a general equilibrium hierarchial structure of production in which some industries, having fixed costs, are run as regulated monopolies, while others operate competitively. One regulated industry sells at least some of its output to another regulated industry (e.g., railroads transport coal for utilities). A single regulatory agency must balance the budgets of all regulated industries in the aggregate so as to maximize the utility of a representative individual. The Ramsey Rule, implying equiproportional reductions in all regulated commodities, always holds. Pricing rules, however, are far more complex due to the vertical production relationship and the existence of unregulated prices. For example, even with zero cross price demand elasticities, price-cost margins are not inversely related to demand elasticities. The optimum may have some negative price-cost margins and may not involve production efficiency which holds only when consistent with equiproportional output reductions in all regulated commodities.

I. Introduction

It is common in industrial countries to have a range of industries subject to regulatory control by political authorities. The manner in which control is exercised varies from nationalization to arm's length regulatory supervision. Typically, the political authority sets or approves prices for these industries but is not able to do so elsewhere.

This paper reconsiders public sector optimal pricing within a hierarchical framework containing a regulated sector. An important feature is that some of the output of an upstream regulated industry is sold to a downstream regulated industry. This structure has widespread empirical significance. For example, a number of countries regulate both coal and electric utilities, with the output of the former being an input into the production process of the latter.[1]

The hierarchical structure implies that one of the regulated industries produces an intermediate good. If an intermediate good price is not set at marginal cost, not only are individuals' consumption choices distorted but the economy is placed in the interior of the production possibly set. This possibility has been addressed in the optimal tax literature. Most notably, when all commodities are potentially taxable, Diamond and Mirrlees (1971) demonstrate that production efficiency should hold which implies that, at the optimum, there is no scope for intermediate good taxation.[2]

Optimal prices depend crucially on regulatory constraints. We assume that the regulator must maintain budget balance over the regulated sector as a whole.[3] Regulators frequently are constrained to have each firm separately break even. That need not be inconsistent with the approach used here if the government can make direct revenue transfers between firms. Sector wide budget constraints are more likely with nationalized industries as in Europe than with privately owned regulated industries as in the U.S. Our model serves to emphasize that prices on transactions between regulated industries represent a means for effecting revenue transfers within the regulated sector. In addition, we assume regulators can only set prices of the firms they regulate. In so doing, they recognize their indirect effects on the quantities and prices of unregulated transactions. We demonstrate that when regulated firms are in a vertical relationship, only the equiproportional output reduction statement of the Ramsey Rule survives. Pricing rules are, however, far more complex than otherwise. For example, with zero cross price elasticities of demand, the Ramsey pricing rule is normally thought to involve price-cost-margins that are inversely related to demand elasticities. Not only does this fail in the vertical model, but the optimum may have some negative price-cost-margins and may not involve production efficiency. Production efficiency holds only when consistent with equiproportional output reductions in all regulated commodities.

II. Relation to the Literature

In a nonhierarchial context, there is a voluminous literature on "optimal pricing" and "optimal commodity taxation" (Baumol and Bradford (1970)), Ramsey (1927), and Sandmo (1976)).[4] In this literature, the government is able to tax or control all prices in the economy. Extensions include models with a production hierarchy in a general equilibrium framework; e.g. Diamond and Mirrlees (1971) where the government is still assumed able to tax all commodities.

Models in which the government can control only a subset of prices have generally been partial equilibrium models. Damus (1984) (without a production hierarchy) and Spencer and Brander (1983) (with a hierarchy) consider pricing rules for an isolated regulated industry.

The most directly relevant literature is Boiteux's seminal work on pricing rules for regulated industries (Boiteux (1971), Dreze (1964)). Boiteux specified a general equilibrium model with hierarchical production and regulated and unregulated industries at all different levels of the hierarchy. He considered cases in which the government faces a single overall budget constraint and cases when separate constraints exist for each industry. However, there has been confusion in the literature as to what assumption Boiteux made concerning whether the government controlled all or only a subset of prices. Hagen (1979) and Bos (1985, 1986) specify models with the government controlling only a subset of prices which they argue is consistent with the Boiteux framework. Their results differ in important respects from ours since they assume that the government regulator sets all regulated prices as if no other prices would be affected. This assumption, that entire sectors act as price takers, is not only myopic, it is also not the assumption made by Boiteux. The only way to interpret Boiteux in a consistent manner is to assume that the government participates in all markets and controls all prices.

Four general classes of uncontrolled prices exist: (1) prices of commodities on a separate branch of the hierarchy which are related to the regulated sector only through consumer demand; (2) prices of primary factors which are used by most or all industries in the economy; (3) prices of private firms that use regulated inputs and sell to consumers; and (4) prices for industries which use a regulated good as an input and sell (at least in part) to another regulated industry. The first two classes are independent of the production hierarchy whereas the third and fourth classes only arise in a hierarchy.

The existence of uncontrolled prices of the first class modifies Ramsey type results through cross-price elasticities. If cross-price elasticities are not zero then effects along the lines of Corlett and Hague (1953) occur. This case has no implications for production efficiency. If nonregulated prices are set by firms with market power, then a game theoretic analysis of the interaction between the regulators and these firms must be analyzed as in Ware and Winter (1986).

The effect of uncontrolled prices of the second class is considered by Stiglitz and Dasgupta (1971) and Auerbach (1985) in a nonhierarchical model. If some input price is uncontrolled, then production efficiency may fail. If the regulated sector is a small part of the economy, then effects of changes in regulated prices on these uncontrolled prices may be small and can be disregarded. Dreze (1984), also in a nonhierarchical setting, considers regulatory pricing with uncontrolled private prices belonging to the first and second classes. His main focus is on exogenously fixed prices and commodity rationing. He also treats competitive market clearing. Either case causes significant increases in complexity beyond the standard Boiteux framework.[5]

The remaining classes become important when a production hierarchy exists. No simple assumptions, such as having zero cross-price demand elasticities or having a "small" regulated sector, leave uncontrolled prices unaffected by the regulator's decisions. Feldstein (1972b) and Brown and Sibley (1986), for the third class, show that when a regulated firm sells to competitive firms instead of consumers, the results are essentially the same as when a regulated firm sells directly to consumers. Ramsey Rule type results using the derived demand curves remain valid. Production efficiency fails in the aggregate economy but is maintained within the public sector.

The fourth class of uncontrolled prices has not previously been analyzed. In this case, we show that the optimal regulated prices are Ramsey prices in the sense that they imply an equiproportional compensated reduction in all of the outputs of both regulated industries. Output levels of the final goods ultimately affected need not be reduced by this proportion and the actual prices needed to ensure this result may be <u>quite different</u> from those normally implied by the Ramsey Rule. For example, in order to induce equiproportional reductions in output, regardless of demand elasticities, it may be efficient for an upstream regulated industry to <u>subsidize</u> a downstream regulated industry. In the hierarchical context developed here, the Ramsey Rule does not reduce to a simple inverse-elasticity formula when the assumption of zero cross-price elasticities is made. Account must be taken of the fact that demand for the output of the upstream industry is reduced by the distortions in the market supplied by the downstream industry.[6]

Concerning the role of production efficiency, if there is a single primary input and if an upstream regulated industry sells some output to a downstream regulated industry, then that output should be priced at marginal cost (i.e., production efficiency within the regulated sector is desirable). In situations not satisfying these conditions, production efficiency may not hold.[7] In particular, marginal cost pricing within the regulated sector is not appropriate if some of the output of an upstream industry must be sold at a uniform price to final consumers and to the downstream industry. Thus restrictions on what prices can be controlled are crucial.

These latter results suggest an important reinterpretation of the relationship between production efficiency and the Ramsey Rule. Production efficiency is not always desirable whereas, as already stated, the Ramsey Rule (in quantities) is always valid. When production efficiency is desirable, it is precisely to ensure that all regulated goods, both intermediate and final, are equiproportionately reduced. This implies that production efficiency should not be considered as an independent principle. Reinterpreting the Diamond and Mirrlees (1971) contribution, production efficiency can now be shown to follow from the fact that Ramsey equiproportional reductions hold for all taxable commodities, be they final or intermediate. That is, production efficiency in their context is necessary to ensure that the Ramsey Rule holds for all taxable commodities.

Our results therefore stand in contradistinction to those of Boiteux who showed that production efficiency should be maintained within the regulated sector when there is one overall budget constraint. That something as fundamental as production efficiency fails highlights the importance of including in analases of public sector pricing regulated firms that participate only in a subset of markets, especially when there are intermediate goods.

III. General Equilibrium Framework

The hierarchical industrial structure consists of five industries, indexed 0 through 4. Each industry uses two primary inputs supplied by consumers denoted L^{1i} and L^{2i} where i refers to the industry using the input.[8] These inputs are combined with produced inputs purchased from other industries. Three of these industries, 1, 2, and 3, have constant returns to scale while two, 0 and 4, have constant variable returns to scale plus fixed costs.[9] These two industries have single firms which would have deficits at marginal cost pricing but which are regulated so as to just cover costs in the aggregate. All five industries, produce homogeneous outputs. The outputs of regulated industry j sold to industry i or to final consumers f are denoted by Z^{ji} and Z^{jf} respectively. Outputs of competitive firm j sold to industry i or to final consumer f are denoted by X^{ji} and X^{jf} respectively. As indicated in Figure 1, not all firms or consumers buy every output. Upstream regulated industry 0 sells all its output to competitive industries 1 and 2. Downstream regulated industry 4 sells some output to competitive industry 3 and some directly to consumers. Competitive industry 2 sells output directly to consumers and to regulated industry 4. Thus, industry 0 sells part of its output indirectly to industry 4 through industry 2. The unregulated industries are constrained by competition to charge a single price for all of their output, while regulated industries are permitted to price discriminate.[10] Consumer input L^{2i} is the numeraire while w, the price of L^{1i}, and p_1, p_2, and p_3, the prices for the outputs of competitive industries 1, 2, and 3, are all market determined. Let q_{ji} denote the price of the output of regulated industry j sold to customer i. These q_{ji} are the optimal breakeven prices to be determined in this paper.

This structure highlights the most interesting cases with intermediate goods, and yet remains tractable. Three of the four types of uncontrolled prices are present; w belongs to the second class, p_1 and p_3 fall into the third class, and p_2 belongs to the crucial fourth class.

Since this paper is concerned with economic efficiency rather than equity, the general equilibrium structure is set in the context of a representative individual, who is assumed to have a twice continuously

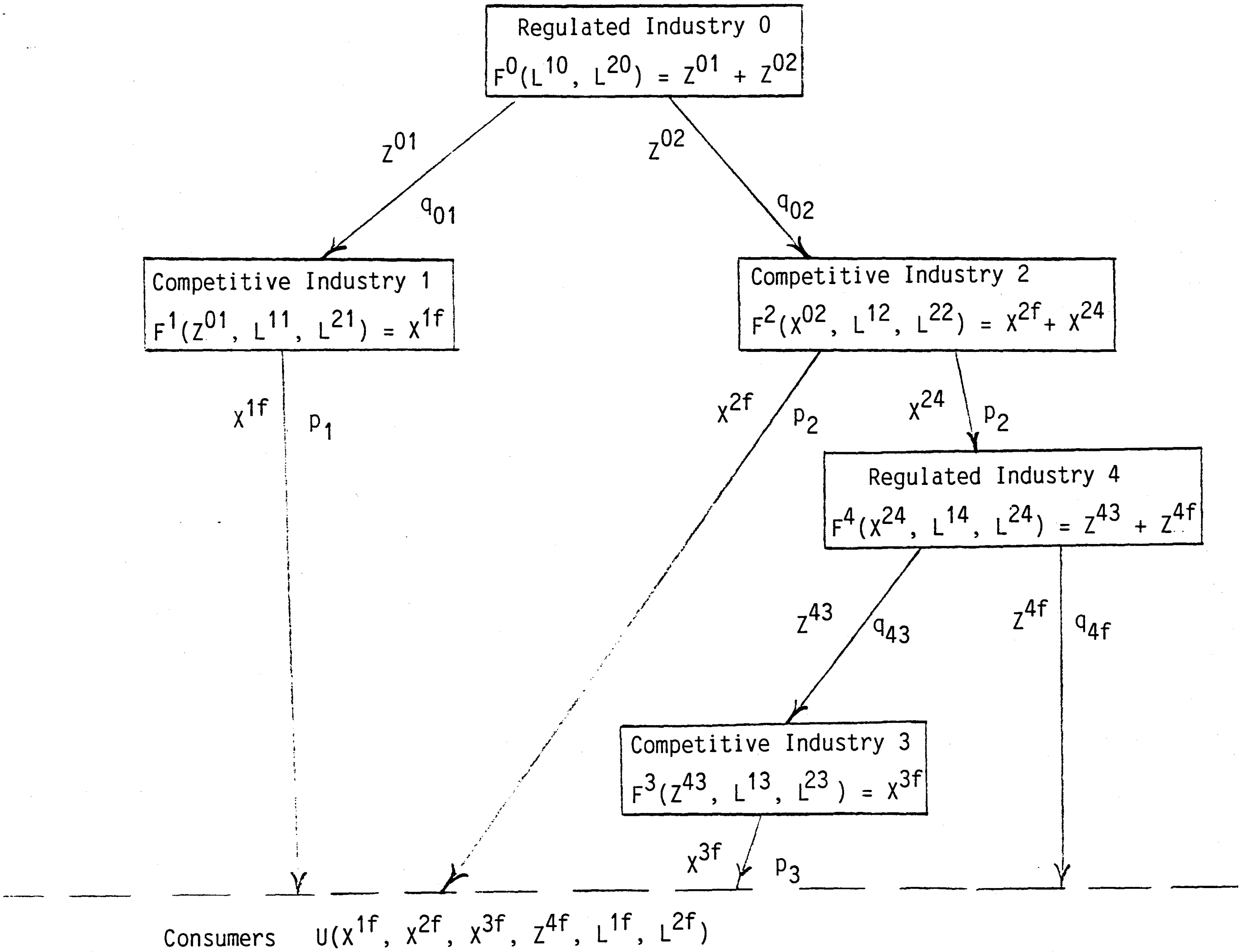
Regulated Industry 0
F^0(L^10, L^20) = Z^01 + Z^02
Z^01
q_01
Z^02
q_02
Competitive Industry 1
F^1(Z^01, L^11, L^21) = X^1f
Competitive Industry 2
F^2(X^02, L^12, L^22) = X^2f + X^24
X^1f
p_1
X^2f
p_2
X^24
p_2
Regulated Industry 4
F^4(X^24, L^14, L^24) = Z^43 + Z^4f
Z^43
q_43
Z^4f
q_4f
Competitive Industry 3
F^3(Z^43, L^13, L^23) = X^3f
X^3f
p_3
Consumers U(X^1f, X^2f, X^3f, Z^4f, L^1f, L^2f)

FIGURE 1

differentiable, monotonic, strictly quasi-concave, utility function $U(X^{1f},X^{2f},X^{3f},Z^{4f},L^{1f},L^{2f})$. This is maximized subject to the budget constraint $p_1X^{1f} + p_2X^{2f} + p_3X^{3f} + q_{4f}Z^{4f} + wL^{1f} + L^{2f} = w\bar{L}^{1f} + \bar{L}^{2f}$, where $\bar{L}^{1f}$ and $\bar{L}^{2f}$ are the initial endowments of the inputs. The shadow price associated with this budget constraint is denoted as β. Solving this yields demands for the X^{if} and Z^{4f} and supplies of L^{if} as functions of the p_i, q_{4f}, and w. An interior solution is assumed.

The unregulated industries are assumed to have well behaved production functions F^i which exhibit constant returns to scale. The cost minimization problem of industry 1 may be represented as:

$$C^1(q_{01}, w, 1)X^{1f} \equiv \text{Min} \quad q_{01}Z^{01} + wL^{11} + L^{21}$$
$$\text{s.t. } F^1(Z^{01},L^{11},L^{21}) = X^{1f}$$

where the cost function $\hat{C}^1$ has the multiplicative form C^1X^1 due to the assumption of constant returns to scale. All output of this industry is sold at the same price which must equal average cost. Assuming interior first-order conditions and using Shephard's Lemma, the following are immediate:

$$L^{11}/X^{1f} = \partial L^{11}/\partial X^{1f} = C^1_2(q_{01},w,1) \tag{1A}$$

$$Z^{01}/X^{1f} = \partial Z^{01}/\partial X^{1f} = C^1_1 1(q_{01},w,1) \tag{1B}$$

$$p_1 = C^1(q_{01},w,1) \tag{1C}$$

Analogously, the production decisions of industry 2, are determined by:

$$C^2(q_{02}, w, 1)(X^{2f} + X^{24}) \equiv \text{Min } q_{02}Z^{02} + wL^{12} + L^{22}$$
$$\text{s.t. } F^2(Z^{02},L^{12},L^{22}) = X^{2f} + X^{24}$$

where, as before:

$$L^{12}/(X^{2f} + X^{24}) = \partial L^{12}/\partial X^{2f} = \partial L^{12}/\partial X^{24} = C^2_2(q_{02},w,1) \tag{2A}$$

$$Z^{02}/(X^{2f} + X^{24}) = \partial Z^{02}/\partial X^{2f} = \partial Z^{02}/\partial X^{24} = C^2_1(q_{02},w,1) \tag{2B}$$

$$p_2 = C^2(q_{02},w,1) \tag{2C}$$

Note again, that the output of inbetween competitive industry 2 is sold at the same price both to final consumers (X^{2f}) and to downstream regulated industry 4 (X^{24}).

Industry 3's problem is defined by:

$$C^3(q_{43},w,1)X^{3f} \equiv \text{Min } q_{43}Z^{43} + wL^{13} + L^{23}$$
$$\text{s.t. } F^3(Z^{43},L^{13},L^{23},) = X^{3f}$$

where

$$L^{13}/X^{3f} = \partial L^{13}/\partial X^{3f} = C^3_2(q_{43},w,1) \tag{3A}$$

$$Z^{43}/X^{3f} = \partial Z^{43}/\partial X^{3f} = C^3_1(q_{43},w,1) \tag{3B}$$

$$p_3 = C^3(q_{43},w,1) \tag{3C}$$

Upstream regulated industry 0 has a fixed cost $D^0>0$, measured in terms of the numeraire, and produces subject to a homogeneous of degree one production function. The regulated industry can charge different prices for Z^{01} and Z^{02}. The regulated industry must sell whatever output is demanded by its customers at the regulated prices. Its behavior may be described as follows:

$$C^0(w,1)(Z^{01} + Z^{02}) + D^0 \equiv D^0 + \text{Min } wL^{10} + L^{20}$$
$$\text{s.t. } F^0(L^{10},L^{20}) = Z^{01} + Z^{02}$$

where:

$$L^{10}/(Z^{01}+Z^{02}) = \partial L^{10}/\partial Z^{01} = \partial L^{10}/\partial Z^{02} = C^0_1(w,1) \tag{4A}$$

Similarly, the possibilities facing downstream regulated industry 4 which has a fixed cost $D^4 > 0$, may be represented as:

$$C^4(p_2,w,1)(Z^{43} + Z^{4f}) + D^4 \equiv D^4 + \text{Min } p_2X^{24} + wL^{14} + L^{24}$$

$$\text{s.t. } F^4(X^{24},L^{14},L^{24}) = Z^{43} + Z^{4f}$$

$$L^{14}/(Z^{43}+Z^{4f}) = \partial L^{14}/\partial Z^{43} = \partial L^{14}/\partial Z^{4f} = C^4_2 2(p_2,w,1) \qquad (5A)$$

$$X^{24}/(Z^{43}+Z^{4f}) = \partial X^{24}/\partial Z^{43} = \partial X^{24}/\partial Z^{4f} = C^4_1(p_2,w,1) \qquad (5B)$$

Given constant returns to scale, unregulated markets clear as long as price equals average costs in these markets. The only commodity requiring an explicit market clearing equation is $\overline{L}^{1f}$ where this equation is:

$$L^1(p_1,p_2,p_3,w,q_{01},q_{02},q_{43},q_{4f}) \equiv \sum_{i=0}^{4} L^{1i} + L^{1f} = \overline{L}^{1f} \qquad (6)$$

The equations for demands and supplies together with equations (1) through (6) determine equilibrium prices p_i and w and equilibrium quantities as functions of q_{01}, q_{02}, q_{43}, and q_{4f} where the latter are chosen to maximize the utility of a representative individual subject to market clearing and balanced budget requirements. The regulatory agency's maximization problem may be represented as follows:

$$\underset{q_{01},q_{02},q_{43},q_{4f}}{\text{Max}} \quad U(X^{1f},X^{2f},X^{3f},Z^{4f},L^{1f},L^{2f})$$

$$\text{s.t. } q_{01}Z^{01}+q_{02}Z^{02}+q_{43}Z^{43}+q_{4f}Z^{4f} = w(L^{01}+L^{41})+L^{02}+L^{42}+p_2X^{24}+D^0+D^4$$

One regulator chooses all regulated prices to attain budget balance for the regulated sector. The multiplier on this constraint is denoted by λ.

It is notationally convenient for what follows to reinterpret the pricing conditions in their equivalent price-cost excess form (see Baumol and Bradford (1970)), where these are defined by:

$$t_i \equiv q_{0i} - C^0,\ i=1,2,\ t_3 \equiv q_{43} - C^4, \text{ and } t_4 = q_{4f} - C^4 \qquad (7)$$

From the definitions of C^0 and C^4, the constraint in the maximization above becomes $t_1Z^{01} + t_2Z^{02} + t_3Z^{43} + t_4Z^{4f} = D^0 + D^4$. The regulatory agency chooses t_1 through t_4 to maximize the same utility function as above.

IV. The Ramsey Rule and Conditions for Production Efficiency

The relationship between the Ramsey Rule and production efficiency is shown in the following propositions, the proofs of which are presented in the appendix. Define $dZ^{jic} \equiv \sum_{k=1}^{4} t_k\,(\partial Z^{jic}/\partial t_k) + \sum_{k=1}^{4} t_k(\partial Z^{jic}/\partial w)(\partial w^c/\partial t_k)$ as the reduction along the tangent plane approximation to the compensated demand curve for Z^{ji} due to the divergences from marginal cost pricing in both regulated industries.[11] $\partial Z^{jic}/\partial t_k$ is the derivative of compensated demand due to a change in t_k with w held fixed, $\partial Z^{jic}/\partial w$ is the derivative of compensated demand for Z^{ji} due to a change in w with all t_j held constant, and $\partial w^c/\partial t_k$ is the change in w needed to maintain market clearing for L^1 as given in (6) due to a compensated change in t_k.

Proposition 1. The Ramsey Rule, that regulated prices should be chosen so as to induce equiproportional compensated reductions in all regulated commodities, always holds:

$$dZ^{01c}/Z^{01} = dZ^{02c}/Z^{02} = dZ^{43c}/Z^{43} = dZ^{4fc}/Z^{4f} = (\beta-\lambda)/\lambda + T$$

The term T involves income effects arising from compensations. $(\beta-\lambda)/\lambda + T$, the standard proportionality factor in the optimal pricing and taxation literature, indicates the benefits that could be gained if distortionary taxation were replaced by lump-sum taxation. It therefore can reasonably be assumed to be negative yielding reductions in outputs.[12] Note that final demands are not proportionately reduced. Instead, the reductions occur in the "taxable" commodities. As discussed in Section V below, the implementation of Ramsey prices in the present context is not the same as in the nonhierarchical context.

Although the Ramsey Rule in outputs always holds, such is not the case with production efficiency. The following Propositions elaborate on the circumstances which determine the desirability of production efficiency.

> <u>Proposition 2</u>: If the inbetween competitive industry 2 sells its entire output, directly or indirectly, to the downstream regulated industry and if the price of the first primary factor is independent of the price of Z^{02} ($dw/dq_{02} = 0$), then the upstream regulated industry should price the output it sells to the inbetween industry at marginal cost ($t_2=0$).

The assumptions given in this proposition are satisfied if X^{2f} is not desired by consumers and if the first primary factor is not used in any industry. Under these circumstances, the upstream regulated industry is selling some output (Z^{02}) which reaches the final consumer only by being embodied in X^{3f} and Z^{4f}. Therefore, all Z^{02} that reaches the consumer is first processed by the downstream regulated industry. There is then no gain to be had by using non-marginal cost pricing on that output since the only effect of such a policy would be to transfer revenue between the two regulated industries at the cost of an extra distortion. Although full production efficiency is not maintained (t_1 and t_3 need not equal zero), transactions between the regulated industries are production efficient. At first glance, it might appear that the desirability of production efficiency implied by Proposition 2 (i.e.,$t_2=0$) is inconsistent with Ramsey's equiproportional reductions which by Proposition 1, are always optimal. That the two results are consistent can be seen by recognizing that, given the terms of Proposition 2, equiproportional reductions in Z^{43} and Z^{4f} imply, by constant returns to scale, a similar proportional reduction in input usage on the part of the inbetween competitive industry and hence an identical proportional reduction in the demand for Z^{02} as required. Were t_2 to be nonzero, there would then be additional nonproportional changes in Z^{02}, Z^{43}, and Z^{4f} thereby violating the Ramsey Rule.

When either condition in Proposition 2, that $X^{2f}=0$ and that $dw/dq_{02}=0$, does not hold, then production efficiency will generally fail. This is seen in the next two propositions.

Proposition 3: If the inbetween competitive industry sells some of its output directly to the consumer ($X^{2f}>0$), then, in general, production efficiency is violated ($t_2 \neq 0$). In particular, when the second primary factor is not useful in production ($dw/dq_{02}=0$) and when the cross-derivatives of either compensated or ordinary demands are zero, then $t_2>0$ holds and is larger the larger is the share of the inbetween competitive industry's output sold directly to the consumer.

The significance of X^{2f} in Propositions 2 and 3 arises from the specific way in which it enters the production hierarchy. When X^{2f} is positive, there is a constraint on the regulatory body's ability to affect independently the price of each final good which directly or indirectly uses Z^{02}. Thus, if the inbetween competitive industry were irrelevant in that it used no scarce resources, the structure has imposed the extra constraint that the output going directly to the consumer via X^{2f} and the output going to the downstream regulated industry via X^{24} must have the same price.

Such an interpretation of the role of X^{2f} indicates that production efficiency may not be desirable for similar reasons in more general circumstances than those described by this framework. Any restrictions on the ability of a government agency to affect independently the prices of different final commodities can yield this outcome. For example, an analogous result in a nonhierarchical optimal tax context is given by Stiglitz and Dasgupta (1971) who show that placing certain constraints on the tax instruments available to the government also typically leads to violations of production efficiency.

Even when X^{2f} is zero, as the following Proposition shows, the existence of more than one primary factor can also result in production inefficiency.

Proposition 4: When one regulated industry, directly or indirectly, sells all of its output to other regulated industries, if several primary factors exist which can be used in variable proportions in each industry, then production efficiency in general no longer holds.

The failure of production efficiency occurs even when one regulated industry sells all its output to another. This may seem surprising since the direct effect of nonmarginal cost pricing is only to transfer revenue between the regulated industries. However, when there is more than one primary factor, the regulator can act analogously to a monopsonist, using its control of the upstream industry's output to alter w. Of course it does this to raise the consumer's welfare.

The magnitude and even direction of the nonmarginal cost pricing is difficult to determine since there are many effects occuring simultaneously. Consider a rise in w. This raises costs and hence the revenues needed by the regulator. It also directly raises consumer income and hence welfare. On net these effects have the sign of $\beta-\lambda$. Although theoretically ambiguous, this term is usually presumed to be negative. Due to these factors, a rise in w would tend to lower welfare. In addition, a change in w has ambiguous cross price effects on the regulator's demands and hence on the size of distortions needed to meet the revenue requirements. In sum, in some circumstances the regulator may seek a rise in w and in others a fall in w in order to maximize consumer welfare. The change in q_{02} needed to create the desired change in w is also ambiguous. The sign of dw/dq_{02} depends upon the derivatives of demand for L^1 with respect to w and q_{02} where each of these derivatives can take either sign. Except for an unlikely balancing of different effects, t_2 will be nonzero but its sign cannot be determined from general principles. However, t_2 is likely to be small when X^{2f} is zero. If the regulated sector is a small part of the economy and if demand for L^1 is at least as responsive to changes in w as to changes in q_{02}, then dw/q_{02} will be small. In these circumstances, for all practical purposes, the regulatory agency would be justified in making a "partial equilibrium" assumption to disregard this effect.[13]

One point to note is that this result indicates that when one regulated industry sells directly to the other, their vertical integration would reduce social welfare. Once integrated, the combined industry would presumably operate to minimize costs and thus would internally use marginal cost pricing, setting the now implicit value of t_2 equal to zero. Essentially the regulatory agency would have lost an instrument which it desired to use for its indirect effects. If the regulatory body could directly control the input mix of the combined industry, it would not need to use t_2 and the integration would not be harmful.

These Propositions yield important insights into the relationship between production efficiency and the Ramsey Rule. The former should be viewed as a corollary of the latter. The production inefficiency in Propositions 3 and 4 is needed to get the equiproportionate output reductions of Proposition 1. In Proposition 3, if $t_2=0$ held, then Z^{02} would be reduced only because reductions in Z^{43} and Z^{4f} would reduce its demand. Since consumer demand X^{2f} is only changed by cross price effects, it will generally not be reduced by the same proportion as Z^{43} and Z^{4f}. The fraction of Z^{02} used to produce X^{2f} will also be reduced by a different proportion and overall Z^{02} will be reduced differently from the outputs of industry 4. A nonzero t_2 is then needed to achieve equiproportionate reductions. Similarly, in Proposition 4, a primary factor price change does not affect all industries identically so a nonzero t_2 is generally required to ensure equiproportional reductions.

This relation of production efficiency to the Ramsey Rule is valid more generally. The argument applies to the analysis of optimal taxation in Diamond and Mirrlees (1971). Distorting all final good prices to induce equiproportionate reductions in final good demands causes the same proportionate reductions in all intermediate goods without any intermediate good price distortions. Any production distortions are ruled out since they would lead to violations of the Ramsey Rule. Diamond and Mirrlees (1971) also allow nonconstant variable returns to scale, which does not effect the essential relation between production efficiency and the Ramsey Rule.[14]

V. Properties of the Optimal Pricing Rule

To this point, the hierarchical structure has been used to throw light on the relationship between production efficiency and the Ramsey Rule. However, the optimal pricing behavior it implies is important, especially given real world examples such as railroads (an upstream regulated industry) transporting coal for utilities (a downstream regulated industry). While the propositions above show that the pricing rules are sensitive to the assumptions made concerning the hierarchical structure, some general observations are possible.

As mentioned earlier, the implementation of the Ramsey Rule in the hierarchical context is more complicated than usual since the reduction in any given Z^{ji} arises from a number of sources. This can be seen from the definition of $dZ^{jic} \equiv \sum_{k=1}^{4} t_k(\partial Z^{jic}/\partial t_k) + \sum_{k=1}^{4} t_k(\partial Z^{jic}/\partial w)(\partial w^c/\partial t_k)$, where, for example, from (1B) $\partial Z^{01c}/\partial t_1 = X^{1f}C^1_{11} + (C^1_1)^2 X^{1fc}_1$. First, for any, Z^{ji}, there is the reduction induced by the distortion in its price. If the distorted market is for an intermediate good, this reduction is absorbed via two channels. The price increase induces a rise in the price of at least one final good. This reduction percolates back through the hierarchy via reductions in input demands, eventually reducing Z^{ji}. In addition, the distortion causes substitution effects in input choices of downstream firms which cause output effect reductions in demand for Z^{ji}. For example, for $\partial Z^{01c}/\partial t_1$ given above, $(C^1_1)^2 X^{1fc}_1$ measures the reduction in Z^{01} due to the change in final good demand while $X^{1f}C^1_{11}$ is the production substitution effect. Note that C^1_{11} can be shown to be equal to - $Z^{01}\sigma/q_{01}$ where $\sigma>0$ is the elasticity of substitution. As σ increases, indicating greater substitutability, $\partial Z^{01}/\partial t_1$ will tend to be larger in magnitude.

Second, distortions in other markets at the same level of the hierarchy as the given Z^{ji} also ultimately affect final good prices leading to compensated cross substitution effects on those final goods which directly or indirectly use Z^{ji} as an input. These cross-price effects move upstream, as described above, and cause further output effect changes in Z^{ji}. Third, reductions in Z^{ji} can occur due to changes in relative primary factor prices which induce similar output and substitution effects to those described above. These own-, cross-, and factor-price effects are qualitatively similar, although more complicated, to those described in the standard nonhierarchical optimal taxation and pricing models. However, there is a final source of the reduction in Z^{ji} arising from the nature of the hierarchy itself. There are distortions in the prices of goods above or below Z^{ji} in the hierarchy.[15] These lead to price increases in all goods which either directly or indirectly use the output produced in those distorted markets. These price increases ultimately cause output effect reductions in Z^{ji} where these effects operate in either direction in the hierarchy. For example, an increase in q_{43} will lead to reduction in Z^{02} and an increase in t_2 will lead to reductions in Z^{43}.

The following example illustrates how these various effects can operate. It shows that there can exist assumptions on production functions and demands which even result in t_2 being positive while t_3 and t_4 are zero. Assume industries 2 and 4 do not use either primary factor so that their cost functions become linear in q_{02} and p_2 and that industries 1 and 3 have Cobb-Douglas technologies in the primary factor and the intermediate input they purchase. Also assume that the individual has a Cobb-Douglas utility function over the consumption goods. Substituting these restrictions into the budget constraint and first order conditions allows one to solve explicitly for the t_i's yielding $t_1=t_2=((\lambda-\beta)/\beta)C^0$ with $t_3=t_4=0$. The reason for this solution is that the distortion in t_2 raises q_{43} and q_{4f} which then cause reductions in Z^{43} and Z^{4f}. In fact, the special assumptions give t_2 an equivalent ability to affect Z^{43} and Z^{4f} as have t_3 and t_4. However, t_2 is a better instrument in this case because it also spreads the effect onto Z^{02}. Cobb-Douglas preferences imply that X^{2f}, Z^{43}, and Z^{4f} are all reduced in the same proportion yielding the same reduction in Z^{02}. Having t_2 nonzero does not cause a production distortion since industry 2 uses only one factor of production. If industry 2 also used a primary factor then a distortion would arise from t_2 not being zero. The presence of this distortion, would induce a smaller value of t_2 and would make t_3 and t_4 positive.

In the standard, nonhierachical optimal pricing framework, the pricing rules can sometimes be reduced to inverse elasticity formulae, though it is well known that the Ramsey Rule is more general (Baumol and Bradford (1970)). Specifically, the inverse elasticity rule is consistent with equiproportional reductions only under the assumption of zero compensated cross-price elasticities. In the hierarchical context, even that assumption is insufficient for this outcome. The linkages within the chain of production would remain so that in setting the distortions at one level, the regulator must take account of the implications of nonzero distortions at the other level.

The one exception to this last point occurs when the conditions specified in Proposition 2 hold so that production efficiency is desirable with t_2 equal to zero. There are then no distortions on Z^{02} to affect the downstream industry. A type of inverse elasticity rule for Z^{01}, Z^{43}, and Z^{4f} then results. Since the change in Z^{02} is induced by the downstream distortions, the inverse elasticity rule does not apply to it.

$$(t_1/q_{01})E(Z^{1c}, q_{01}) = K \tag{10A}$$

$$(t_3/q_{43})E(Z^{3c}, q_{43}) = K \tag{10B}$$

$$(t_4/q_{4f})E(Z^{4c}, q_{4f}) = K \tag{10C}$$

Where $E(x, y)$ is the elasticity of x with respect to y. Note that these elasticities include factor market as well as final good effects.

VI. Conclusions

This paper considers a hierarchical production structure in which two of the industries are regulated. One of them, directly or indirectly, sells output to the other. A single regulator controls only the prices of the outputs of the regulated industries and must satisfy a combined breakeven constraint. With a single regulator but separate breakeven constraints, a form of the Ramsey Rule continues to hold: within each industry, percentage reductions in the compensated quantity demanded, due to distortions measured at their shadow values, are equal. Across industries, nonproportionate reductions arise since one industry cannot directly subsidize the other. Which industry's outputs are reduced more depends on whether the upstream industry should tax or subsidize the downstream industry. When each regulated industry has a separate regulator, assumptions made by one regulator about the other's pricing behavior are critical. The actual reductions in regulated outputs are markedly different from the optimal ones.[16] Thus, the results in this paper, even when not directly applicable in rate setting, do serve as a useful benchmark that gives some idea of the social losses from the inefficiencies of separate constraints and regulators.

The model illustrates that making only modest modifications to the standard model (e.g., having uncontrolled prices in a hierarchical setting) yields important insights. The results modify and extend the classic results of Boiteux by including a primary factor whose price is uncontrolled and by having an uncontrolled competitive industry between the two regulated industries which prevents the regulator from independently affecting several final good prices.

The unregulated primary factor keeps production efficiency from being optimal. Further, if some of a regulated intermediate good is sold to a regulated industry via an unregulated intermediary, with the balance going directly to consumers, then marginal cost pricing is not optimal. In such cases the divergence from marginal cost pricing <u>is not</u> primarily affected by the elasticity of the derived demand. The impact of distortions elsewhere is the crucial factor. These results are in sharp contrast to those of Boiteux who made the special assumption that all prices are controlled.

It is worth noting that, when all prices are controllable, a very general structure can be assumed. It really does not matter what the production structure is or how inputs flow in the hierarchy. However, once only a subset of prices are controllable, this generality no longer applies. The specific structure matters if the change in uncontrolled prices with respect to controlled prices is to be determined. Consider the case in which coal companies buy rail services to ship coal. Essentially, there is a system of delivered pricing. Assume exactly the same production structure but have the sales system differ with cosumers directly purchasing all coal at the mine.

Consumers then also purchase shipping services. Even though the technologies are identical, the results are quite different. In particular, in the latter model, the railroad regulator can set different rates for coal shipping to different customers. In the former, it cannot. Production efficiency will hold in the latter but not in the former as shown in Propositions 2 and 3.

Thus, one way to view the Boiteux model is as the public sector pricing analogue to the Arrow-Debreu model of uncertainty which is based upon an assumption of complete markets. In that very general model, specific assumptions on the nature of utility functions, information, and production, are irrelevant to existence and optimality. However, once this benchmark assumption of complete markets is not made, then results crucially depend upon specific structural assumptions. In public sector pricing, Boiteux provided a major benchmark paper based upon the assumption that the government controls all prices. Under this assumption, very general production relations can be assumed. If the government does not control all prices, then general results are not as available. The specific structure is crucial.

FOOTNOTES

1. Other examples could be cited. In many countries, railroads and airlines are regulated and provide transportation services to other regulated industries. In this context, there is the specific U.S. example of coal transportation by railroads. The shipping rates charged by the railroads were regulated by the federally instituted ICC and various state agencies. Approximately 80 percent of all coal shipped by railroads goes to locally regulated utility companies, where the income from this activity accounts for a significant fraction of railroad revenues. For a discussion of rate setting in this context see Damus (1982) and Zimmerman (1979).

2. Intermediate goods taxation could exist. Its only role would be to replicate final goods taxation. For example, a proportional tax on all factor inputs used by a single product firm is equivalent to an excise tax on that firm's output. There are many such tax equivalencies (Atkinson and Stiglitz (1980)).

3. The alternative regimes of having a single regulatory agency with separate budget constraints for each regulated industry and of having separate regulatory agencies with separate budget constraints are considered in companion papers (Ebrill and Slutsky (1987, 1988)).

4. For extensions and elaborations on the literature, see, for example, Feldstein (1972a), Faulhaber (1975), Braeutigam (1979, 1980), Vogelsang and Finsinger (1979), Guesnerie (1980), and Damus (1981).

5. The first two classes of uncontrolled prices provide some justification for the Hagen-Bos approach provided that there are zero cross price elasticities and that a partial equilibrium assumption is a reasonable approximation. If the latter does not hold, then, even if uncontrolled prices are fixed, rationing constraints are significant as in Dreze (1984).

6. Brown and Sibley (1986) consider when simple inverse elasticity type rules are appropriate in a model with a single regulated firm selling to consumers and to unregulated businesses. They are thus concerned with the third class of unregulated prices. They allow imperfect competition in the setting of such prices.

7. Within the context of our model, overall production efficiency occurs either when marginal cost pricing is employed everywhere or when all distortions occur in final good markets. Production efficiency within the regulated sector occurs when the upstream industry uses marginal cost pricing for the output it sells either directly or indirectly to the downstream industry. In the text, both cases are referred to as production efficiency with the context making clear which case is relevant.

8. These could have a variety of interpretations such as labor and variable capital or skilled and unskilled labor.

9. These fixed costs, D^0 and D^4, are assumed to be amounts of L^{1i} that must be employed before production can begin. D^0 and D^4 appear as demands in the redundant market clearing equation for L^2. If the fixed costs arose from a need to use other commodities in addition to L^2, the essential nature of the results would be unchanged but would become more complicated. Changes in regulatory decisions could alter the prices of these commodities and thus change the amount of the fixed cost in terms of the numeraire. In setting optimal prices, induced changes in the cost of such fixed inputs would have to be considered. Complications would arise similar to those in Proposition 4 due to changes in the price of L^1. Production efficiency could fail for this reason.

10. Alternatively, the regulated outputs could be different commodities produced at different marginal costs and sold at different prices. For example rail transport of individuals and of coal are different commodities. To model formally this interpretation would complicate the notation but not change the results at all.

11. Ramsey Rule results are conventionally based on such approximations. The approximate reductions will equal actual reductions if the derivatives of the compensated demand curves are constant or if the approximation is taken for small distortions in the neighborhood of zero deficits.

12. Consider the matrix A where a_{ij} is the term multiplying t_j in dZ^{jic}. A quadratic form is found by pre and post multiplying A by the vector of distortions t_j. This equals the proportionality factor times the deficit from the first order conditions determining dZ^{jic}. In the standard optimal tax literature, A is just the Slutsky matrix whose negative definiteness is sufficient to show that the proportionality constant is negative. In this case, A is significantly more complicated. It can be shown to be the sum of two matrices one of which is negative definite and the other has only one nonzero term, arising in the A_{22} term, which contains cross derivatives and hence is ambiguous. A will be negative definite if this last term is negative or is small enough to be dominated by the other matrix. Even if A is not negative definite, the proportionality constant can still be negative since it is really only at the optimum t_j^* for which $(t_1^*,\ldots t_4^*)[A](t_1^*,\ldots t_4^*)^t$ must be negative. For example, in the case discussed below for which $t_2=0$ is optimal,

reductions must occur since this ambiguous term is multiplied by zero.

13. For further comments on other circumstances in which production efficiency is not desirable, see Stiglitz and Dasgupta (1971) and Munk (1978).

14. For example, with decreasing returns to scale, if there is a 100 percent profits tax as well as commodity taxation, then production efficiency is desirable (Diamond and Mirrlees (1971) and Munk (1978)). Production efficiency can be shown to follow from a modified Ramsey Rule. Equiproportinate reductions still arise. The numerator of these ratios continues to be the approximation to the reduction in compensated demand due to distortions. For final goods, as in the standard results, the denominator is the actual demand. However, for intermediate goods, the denominator is the linear approximation of the demand not the demand itself. Given this formulation, production efficiency arises if and only if equiproportionate reductions of intermediate goods occur when there are distortions on final goods only. Stiglitz and Dasgupta (1971) define modified Ramsey Rules in a nonhierarchiacal setting. Although actual reductions are nonproportionate, a type of equiproportional reductions occurs.

15. These distortions could be due to second best optimal choices as considered in this paper or they could be due to inefficient distortions arising from noncompetitive industries as considered in Spencer and Brander (1983). For whatever reason they exist, the presence of distortions at one level of the hierarchy must be considered when choosing optimal distortions at another level.

16. These results are shown in Ebrill and Slutsky (1987, 1988).

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APPENDIX

The propositions follow from the first order conditions of the maximization problem. In order to derive these conditions and demonstrate the propositions, some preliminary results are needed. The endogenous variables in the model, p_i, $i=1,2,3$, w, C^0, C^4, L^{1i} and L^{2i}, $i=1,\ldots, 5$ and f, Z^{01}, Z^{02}, Z^{43}, Z^{4f}, X^{24}, and X^{if}, $i=1,2,3$, are specified by the consumer demand functions and equations (1) to (6) and as functions of the regulator's choice variables, t_1 to t_4 or q_{01}, q_{02}, q_{42}, q_{4f} depending upon which formulation of the maximization problem is being analyzed. When the t_i are the choice variables, the q_{0i} and q_{4i} are endogenously determined by (7) and when the q_{0i} and q_{4i} are the choice variables, the t_i are determined by (7). For convenience, the endogenous variables will be viewed as functions of the regulator's decision variables and w, with w then found endogenously from (6). Consumer preferences are more conveniently specified by the indirect utility function $V(p_1,p_2,p_3,q_{4f},w,1)$ which satisfies standard properties such as Roy's formulas, $V_i=-\beta X^{if}$, $i=1,2,3$, $V_4=-\beta Z^{4f}$, $V_5=-\beta(L^{1f}-\bar{L}^{1f})$, and $V_6=-\beta(L^{2f}-\bar{L}^{2f})$.

Assume the choice variables of the regulators are the t_i. Let dV/dw be the derivative of indirect utility found by holding the t_i constant but including all changes in the p_i, q_{0i}, and q_{4i} induced by changes in w. Then, using Roy's formulas,

$$dV/dw = -\beta\left[\sum_{i=1}^{3} X^{if}(\partial p_i/\partial w) + Z^{4f}(\partial q^{04}/\partial w) + L^{1f} - \bar{L}^{1f}\right]$$

From equations (1) to (5),

$$\begin{aligned} dV/dw &= \beta[(C_1^1C_1^0+C_2^1)X^{1f} + (C_1^2C_1^0+C_2^2)X^{2f} + (C_1^3(C_1^4(C_1^2C_1^0+C_2^2)+C_2^4)+C_2^3)X^{3f} \\ &\quad + (C_1^4(C_1^2C_1^0+C_2^2)+C_2^4)Z^{4f} + L^{1f} - \bar{L}^{1f}] \\ &= \beta\left[\sum_{i=0}^{4} L^{1i} + L^{1f}-\bar{L}^{1f}\right] \end{aligned}$$

Then from equation (6),

$$dV/dw = 0 \tag{A1}$$

Note that (A1) is valid only when t_1 to t_4 are constant and would not hold if q_{0i} and q_{4i} were held constant. It does not say that an increase in efficiency which changes w would have no effect on welfare since in this case the t_i would change.

Next consider dV/dt_i, the total change in welfare from a change in t_i:

$$dV/dt_i = \sum_{j=1}^{3} V_j(\partial p_j/\partial t_i) + V_4(\partial q_{4f}/\partial t_i) + (dV/dw)(dw/dt_i), \quad i=1,\ldots,4$$

where $\partial p_j/\partial t_i$ involves only the direct effects of changes in p_j caused by the changes in t_i and excludes the effects through induced changes in w since these are included in the dV/dw term. Using Roy's formulas and (A1):

$$dV/dt_i = -\beta[\sum_{j=1}^{3}(\partial p_j/\partial t_i)X^{jf} + (\partial q_{4f}/\partial t_i)Z^{4f}]$$

Substituting for prices and demands from (2)-(5) and (7) yields:

$$dV/dt_i = -\beta Z^{0i},\ i=1,2,\ dV/dt_3 = -\beta Z^{43},\ \text{and}\ dV/dt_4 = -\beta Z^{4f} \tag{A2}$$

These equations extend to underlying parameters Roy's formulas which are generally derived for final consumer good prices. Again this result only occurs if the other t_k are held fixed. For convenience of the mathematical exposition, we will respecify the variables Z^{0i} and Z^{4i} by Y^j where $Y^1 = Z^{01}$, $Y^2 = Z^{02}$, $Y^3 = Z^{43}$, $Y^4 = Z^{4f}$.

Let M be a lump sum income of the consumer. Then $\partial X^{if}/\partial M$, $\partial Y^i/\partial M$, and $\partial L^1/\partial M$ are derivatives of the various demands with respect to lump sum income found under the assumption that all prices are held constant. Consider derivatives of X^{if}, Y^i, and L^1 with respect to t_j and w. Such derivatives can be of either uncompensated or compensated demands where compensated derivatives are denoted with a superscript "c". The compensated derivatives can be found by changing w or t_j and giving the individual a change in lump sum income sufficient to hold V constant. The amount of the income which must be given is determined from (A1) and (A2) as 0 in the case of w and Y^j in the case of t_j. This yields the "Slutsky" equations for derived demands:

$$\partial X^{if}/\partial w = \partial X^{ifc}/\partial w \quad i=1,\ldots,3 \tag{A3a}$$

$$\partial Y^{i}/\partial w = \partial Y^{ic}/\partial w \quad i=1,\ldots,4 \tag{A3b}$$

$$\partial L^{1}/\partial w = \partial L^{1c}/\partial w \tag{A3c}$$

$$\partial X^{if}/\partial t_j = \partial X^{ifc}/\partial t_j - Y^j(\partial X^{if}/\partial M) \quad i=1,\ldots,3,\ j=1,\ldots,4 \tag{A4a}$$

$$\partial Y^{i}/\partial t_j = \partial Y^{ic}/\partial t_j - Y^j(\partial Y^{i}/\partial M) \quad i=1,\ldots,4,\ j=1,\ldots,4 \tag{A4b}$$

$$\partial L^{1}/\partial t_j = \partial L^{1c}/\partial t_j - Y^j(\partial L^{1}/\partial M) \quad j=1,\ldots,4 \tag{A4c}$$

The derivatives in (A3) hold all t_j constant whereas those in (A4) assume that w and t_k, $k \neq j$, are held constant, Given (A1), equations of the same form as (A4) would hold if w were allowed to vary but such terms as $\partial X^{if}/\partial t_j$ would have different values in this case. Viewing w as dependent upon the t_j, equation (6) can be used to find derivatives of w with respect to the t_j.

$$dw/dt_j = -(\partial L^1/\partial t_j)/(\partial L^1/\partial w) \tag{A5a}$$

$$dw^c/dt_j = -(\partial L^{1c}/\partial t_j)/(\partial L^{1c}/\partial w) \tag{A5b}$$

It therefore follows using (A3c) and (A4c) that:

$$\begin{aligned} dw/dt_j = \partial w^c/\partial t_j - Y^j(\partial w/\partial M) &= -(\partial L^1/\partial t_j)/(\partial L^1/\partial w) \\ &\quad + Y^j(\partial L^1/\partial M)/(\partial L^1/\partial w) \end{aligned} \tag{A6}$$

As a final preliminary, note that the compensated effects can be found directly by specifying the changes in derived demands in terms of the changes in compensated final demands. Doing this, and comparing terms using symmetry of the cross effects of compensated final demands shows that:

$$\partial Y^{ic}/\partial t_j = \partial Y^{jc}/\partial t_i, \quad i=1,\ldots,4,\ j=1,\ldots,4 \tag{A7a}$$

$$\partial Y^{ic}/\partial w = \partial L^1/\partial t_i, \quad i=1,\ldots,4 \tag{A7b}$$

Thus, symmetry of the substitution matrix extends to compensated derivatives of derived demands with respect to distortions.

The Lagrangian for the maximization problem where the distortions are chosen is

$$L = V(p_1,p_2,p_3,q_{4f}, w, 1) + \lambda[\sum_{i=1}^{4} t_i Y^i - D^o - D^4] \qquad (A8)$$

Using (A2), the first order conditions are

$$\partial L/\partial t_i = -\beta Y^i + \lambda Y^i + \lambda[\sum_{j=1}^{4} t_j(\partial Y^j/\partial t_i + (\partial Y^j/\partial w)(\partial w/\partial t_i)] \qquad (A9)$$

$$\equiv (\lambda-\beta)Y^i + A^i + \lambda dY^{ic} - \lambda dY^{ic} = 0$$

Then

$$(\beta-\lambda)Y^i = \lambda dY^{ic} + [A^i-\lambda dY^{ic}]$$

Substituting $dY^{ic} = \sum_{k=1}^{4} t_k(\partial Y^{ic}/\partial t_k) + \sum_{k=1}^{4} t_k(\partial Y^{ic}/\partial w)(\partial w^c/\partial t_k)$ and using (A7a) yields:

$$(\beta-\lambda)Y^i = \lambda Y^{ic} + \lambda[\sum_{k=1}^{4} t_k[(\partial Y^k/\partial t_i) - (\partial Y^{ic}/\partial t_k)]$$

$$+ \sum_{k=1}^{4} t_k[(\partial Y^k/\partial w)(\partial w/\partial t_i) - (\partial Y^i/\partial w)(\partial w^c/\partial t_k)]$$

Substituting (A4), (A6), and (A7b) yields:

$$(\beta-\lambda)Y^i = \lambda dY^{ic} - Y^i\lambda[\sum_{k=1}^{4} t_k(\partial Y^k/\partial M + (\partial Y^k/\partial M)(\partial L^1/\partial M)/(\partial L^1/\partial w))] \qquad (A10)$$

<u>Proof of Proposition 1</u>:

Reexpressing (A10) gives for $i=1,\ldots,4$:

$$dY^{ic}/Y^i = (\beta-\lambda)/\lambda + \sum_{k=1}^{4} t_k(\partial Y^k/\partial M) + (\sum_{k=1}^{4} t_k(\partial Y^k/\partial M))(\partial L^1/\partial M)/(\partial L^1/\partial w),$$

$$\equiv (\beta-\lambda)/\lambda + T$$

The right hand side is independent of i showing the Ramsey rule.

Q.E.D.

To show the propositions relating to production efficiency, consider the equivalent maximization problem in which the q_{0i} and q_{4i} are the decision variables. Differentiating the indirect utility function, using Roy's formulas and equations (1)-(6), yields:

$$dV/dw = \beta[L^{1f} + L^{14} + (X^{24}/(X^{2f} + X^{24}))L^{12}] \quad \text{(A11)}$$

$$dV/dq_{01} = -\beta Y^1 + (dV/dw)(\partial w/\partial q_{01}) \quad \text{(A12a)}$$

$$dV/dq_{4i} = -\beta Y^i + (dV/dw)(dw/dq_{4i}) \ \ i=3,\ f \quad \text{(A12b)}$$

$$dV/dq_{02} = -\beta(X^{2f}/(X^{2f} + X^{24})) + (dV/dw)(\partial w/\partial q_{02}) \quad \text{(A13)}$$

Note that with w allowed to vary, these do not have the simple form of the ordinary Roy's formulas. Let $R^0 = t_1Y^1 + t_2Y^2$ and $R^4 = t_3 + t_4Y^4$ be the net revenues earned by industries 0 and 4. Then differentiating and using (1)-(5) yields:

$$dR^0/dw = -L^{1f} + t_1(dY^1/dw) + t_2(dY^2/dw) \quad \text{(A14)}$$

$$dR^4/dw = -L^{14} - (X^{24}/(X^{2f}+X^{24}))L^{12}+t_2(dY^3/dw) + dY^4/dw \quad \text{(A15)}$$

Comparing (A11) to (A14) and (A15) indicates why $dV/dw\neq 0$ now holds as contasted with the result in (A1). The increase in w here is not passed on to the consumer when the q_{0i} and q_{4i} are taken as given. Therefore, without allowing for any factor substitution, the regulated industries suffer a loss of revenue due to higher costs that are not passed on. This loss equals the amount of the second primary factor used directly or indirectly. This revenue loss to firms is an implicit welfare gain to consumers. When deriving (A1) all costs increases were passed through so there was no implicit welfare change.

Differentiating the Lagrangian, and using these equations, yields the first-order conditions:

$$\partial L/\partial q_{4i} = (\lambda-\beta)Y^i + [dV/dw + \lambda((dR^0/dw) + (dR^4/dw))](\partial w/\partial q_{4i})$$
$$+ \sum_{k=1}^{4} t_k\lambda(\partial Y^k/\partial q_{4i}) = 0, \quad i=3,f$$
$$= (\lambda-\beta)Y^i + [(\beta-\lambda)(L^{1f} + L^{14} + (X^{24}/(X^{2f}+X^{24}))L^{12})]$$
$$+ \sum_{k=1}^{4} t_k\lambda(dY^k/dw)(\partial w/\partial q_{4i}) + \sum_{k=1}^{4} t_k\lambda(\partial Y^k/\partial q_{41}) = 0 \qquad \text{(A16)}$$

where $\partial Y^k/\partial q_{41}$ include only direct effects of changes in q_{4i} and not those through induced changes in w. There is an analogous equation for q_{01}.

$$\partial L/\partial q_{02} = (\lambda-\beta)[X^{2f}/(X^{2f}+X^{24})]Y^2$$
$$+ [dV/dw + \lambda((dR^0/dw) + dR^4/dw)](\partial w/\partial q_{02})$$
$$+ \sum_{k=1}^{4} t_k\lambda(\partial Y^k/\partial q_{02}) = 0 \qquad \text{(A17)}$$

<u>Proof of Proposition 2</u>:

The relevant first order conditions for this proposition can be found from (A16) and (A17). The conditions of the proposition, that the first primary factor is not utilized in production and that consumers do not desire to consume X^{2f}, drop out terms involving w, X^{2f}, and derivatives of consumer demands with respect to p_2. Since q_{02} only affects consumer demands in general by affecting p_2, then $Y^1 = C_1^1(q_{01})X^{1f}$, $Y^3 = C_1^3(q_{43})X^{3f}$, and Y^4 are not affected by q_{02}. Thus, (A17) reduces to $\lambda t_2(\partial Y^2/\partial q_{02}) = 0$ where $\partial Y^2/\partial q_{02} =$ $C_{11}^2C_1{}^4Y^4 + C_1^2C_{11}^4Y^4C_1^2 + C_{11}^2C_1^4C_1^3X^{3f} + C_1^2C_{11}^4C_1^2C_1^3X^{3f}$ which is unambigously negative from the negative definiteness of the cost functions. Since $\lambda>0$ whenever the regulator's constraint is binding, $t_2=0$ is implied.

Q.E.D.

Proof of Proposition 3:

By definition, $Y^2 \equiv ((X^{2f} + (Y^3+Y^4)C_1^4)C_1^2$ so that

$$\frac{dY^{2c}}{Y^2} - \frac{d(Y^{3c}+Y^{4c})}{Y^3+Y^4} = \alpha\left[\frac{dX^{2fc}}{X^{2f}} - \frac{d(Y^{3c}+Y^{4c})}{Y^3+Y^4}\right] + (1-\alpha)\left[\frac{dC_1^4}{C_1^4} + \frac{dC_1^2}{C_1^2}\right] \tag{A18}$$

where $\alpha \equiv X^{2f}/(X^{2f}+X^{24})$. With only one primary factor

$$\frac{dY^{2c}}{Y^2} - \frac{d(Y^{3c}+Y^{4c})}{Y^3+Y^4} = \alpha\left[\frac{dX^{2fc}}{X^{2f}} - \frac{d(Y^{3c}+Y^{4c})}{Y^3+Y^4}\right] + t_2\left[(1-\alpha)\frac{C_1^2C_{11}^4}{C_1^4} + \frac{C_{11}^2}{C_1^2}\right] \tag{A19}$$

The term dX^{2fc} can be divided into two parts, the own effects $t_2X_2^{2fc}C_1^2$ and the cross price effects $d\hat{X}^{2fc}$. Hence

$$\frac{dY^{2c}}{Y^2} - \frac{d(Y^{3c}+Y^{4c})}{Y^3+Y^4} = \alpha\left[\frac{d\hat{X}^{2fc}}{X^{2f}} - \frac{d(Y^{3c}+Y^{4c})}{Y^3+Y^4}\right] + t_2\left[\frac{\alpha X_2^{2fc}}{X^{2f}} + (1-\alpha)\frac{C_{11}^4}{C_1^4}C_1^2 + \frac{C_{11}^2}{C_1^2}\right]$$

Since proportionate reductions hold from Proposition 2,

$$t_2 = \alpha\left[\frac{d(Y^{3c}+Y^{4c})}{Y^3+Y^4} - \frac{d\hat{X}^{2fc}}{X^{2f}}\right] \Bigg/ \left[\left(\alpha\frac{X_2^{2fc}}{X^{2f}} + (1-\alpha)\frac{C_{11}^4}{C1^4}\right)C_1^2 + \frac{C_{11}^2}{C_1^2}\right]$$

The denominator is negative. Hence, t_2 will equal zero if the numerator is zero. When $X^{2f} \neq 0$, this occurs only if the cross price effects on X^{2f} cause it to have the same percentage reductions as $Y^3 + Y^4$, an unlikely prospect in general. If there are zero compensated cross-price effects then $t_2 > 0$ holds under the presumption that the distorted quantities are reduced. If the other derivatives remain constant, then an increase in X^{2f} will lead to an increase in t_2.

Similar results hold if uncompensated cross price effects are zero since it follows from (A17) that:

$$t_2/q_{02} = -(\alpha(\lambda-\beta)/\lambda)/E(Y^2, q_{02})$$

When $(\lambda-\beta)$ is positive as is normally presumed, then t_2 is positive and increasing in α if the other terms stay constant as X^{2f} increases.

Q.E.D.

Proof of Proposition 4:

From Equation (A17),

$$t_2=\left[\left(\frac{\lambda-\beta}{\lambda}\right)\quad(L^{1f}+L^{14}+L^{12})-\sum_{i\neq 2} t_i\frac{dY^i}{dw^i}\right]\left[\frac{\partial w}{\partial q_{02}}\Big/\left(\frac{\partial Y^2}{\partial q_{02}}+\frac{dY^2}{dw}\quad\frac{\partial w}{\partial q_{02}}\right)\right]$$

Given the multiple ambiguities, it is difficult to sign t_2 but it will equal 0 only under unlikely balancing of different effects when $\partial w/\partial q_{02}\neq 0$.

Q.E.D.