

COST SHARING REGULATION

BY

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September 18, 1987

PURC WORKING PAPER NO. 115

Abstract

Cost sharing regulation requires that multiproduct firms operating in multiple markets or allocate portions of the firms "common or joint" costs to the markets using the "common or joint" inputs. Regulated firms such as electric, natural gas transmission, and telecommunications utilities are subject to this style of regulation in conjunction with traditional rate base regulation to prevent the cross subsidization between consumers. In addition, a cost allocation scheme's effect on the overall level of welfare for society depends upon the choices left to regulators and to the firm. For example, an unconstrained firm will allocate costs to maximize profits, while a regulator might choose an allocation mechanism encompassing some idea of maximizing social welfare. This paper examines the firm's incentives concerning the input choice when the firm chooses the allocator and the incentives facing the firm when the regulators change the cost allocator for policy reasons.

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COST SHARING REGULATION

1. Introduction

Cost sharing regulation requires that multiproduct firms operating in multiple markets or jurisdictions allocate portions of the firms "common or joint" costs to the markets using the "common or joint" inputs. Regulated firms such as electric, natural gas transmission, and telecommunications utilities are subjected to this style of regulation in conjunction with traditional rate base regulation to prevent the cross subsidization between consumers in different markets. Faulhaber (1976) and Fumas and Whinston (1982) examined the possibility of a firm being able to set prices so that no one market subsidized another. The ability to cross subsidize, and the amount of cross subsidization in public enterprises may be in the eye of the beholder, but the mandated use of cost allocation mechanisms in regulated industries shows that regulators believe that a cross subsidy is desired for policy reasons. In addition, a cost allocation scheme's effect on the overall level of welfare for society depends upon the choices left to regulators and to the firm. For example, an unconstrained firm will allocate costs to maximize profits, while a regulator might choose an allocation mechanism encompassing some idea of maximizing social welfare. This paper will examine the firm's incentives concerning the input choice when the firm chooses the allocator and the incentives facing the firm when the regulators change the cost allocator for policy reasons.

Wellisz (1963) pointed to this type of sharing regulation in his seminal article detailing the cost allocation rules promulgated by the Federal Power Commission which allocated costs among the markets served by natural gas companies. Many regulatory commissions adopted similar rules to prevent one group of consumers from contributing more towards shared costs than regulators thought would be fair. These allocation procedures, commonly referred to as fully distributed cost (FDC) plans, impact heavily in the natural gas transmission and the telecommunications industries even today. The study of this type of cost sharing regulation,

however, was eclipsed by the Averch-Johnson model of rate base regulation until relatively recently. In fact, the use of cost sharing regulation in conjunction with rate base regulation may hide or exaggerate input choice distortions and output mix distortions.

Braeutigam (1980) resurrected the analysis of cost sharing regulation by examining FDC pricing methodologies showing that there is no unique set of prices that satisfy the FDC pricing scheme. In addition, he showed that since the prices are based on some notion of average cost, price vectors chosen under this scheme will tend to generate a lower social welfare than Ramsey prices.

Sweeny (1982) undertook to examine the effects of FDC pricing on a firm that is partially regulated. Using the accounting formulae examined in Braeutigam, Sweeny examined the effect of two different fully distributed cost allocation formulae on the output mix of the firm. Sweeny's results showed that a firm using one of these accounting rules will choose a price vector which is dominated: i.e. there is a different set of price-output combinations that will yield greater than or equal outputs with at least the same level of overall profit. The cost equation in Sweeny's formulation treated the costs to be allocated as a fixed cost (F) that would be allocated by some formula across the different product lines. For example, cost for a two product firm was represented as

$$C(q_1, q_2) = c_1 q_1 + c_2 q_2 + F.$$

The fixed cost F was to be allocated between the jurisdictions (say) based on relative outputs, so that an allocator (α) is defined as $\alpha = q_1 / (q_1 + q_2)$; the shared cost allocated to market one and two respectively would be αF and $(1-\alpha)F$. Sweeny examined the effects of these imposed cost allocation procedures on the output mix of a partially regulated firm.

This paper shows the effects of changes in α on both the output mix and input use for a certain class of inputs. The cost allocator referred to below, however, is different from the allocator Sweeny and Braeutigam described. The first allocator examined in the following model is of no particular methodology. Rather it is an implicit allocator much like a Lindahl

price. The purpose of the model is to describe how the firm would choose an allocator if the firm were trying to maximize profits given that the firm is subject to profit regulation and that regulators require that the firm allocate costs of public inputs between markets. After the firm chooses its cost allocator, its output mix and public input, suppose the regulators decide that the allocator the firm chooses is not fair. The regulators then decide to change the allocator. The model presented in the next sections examines the problem of allocating, what have often been referred to as, joint, common or shared costs, and the effect of changing the cost allocation formula on output and input use.

2. A Two Jurisdiction Model with Shared Costs

Most regulated firms required to allocate costs between various markets share inputs which are used to produce outputs in those markets. The literature describing the differences between common and joint costs is sometimes confusing. This section defines and discusses the types of costs for the types of costs that pertain to the model in the following sections.

2.1 Definition of a Shared Input: Public Inputs vs. Joint or Common Inputs

Prior to examining the firm's incentives under various cost allocation assumptions, it is instructive to define what public, joint or common. For the purposes of this paper, a shared input is any input that enters into the production function of more than one good or service. If there is no opportunity cost of using the shared (or jointly used) input in one market in terms of lost output in another market, the input can be considered a public input with non-rival consumption by the various production functions. If the input is public, then the opportunity cost of using the input for producing one unit of (say) output A in terms of output B is zero. A good example of a public input is the local telephone network. The local telephone network (i.e. the telephone poles, wires, etc) is an input into the production of both toll and local calls. In addition, one can use the local telephone network to call locally without imposing opportunity costs in terms of lost sales in the toll market. Another example, is the electric utility's generating plant. It is used in both peak and off-peak periods. Using

the plant during peak times does not create an opportunity cost in terms of lost off-peak sales.

Joint costs can be considered a special case of a cost associated with a public input. An excellent example is Marshall's mutton-wool example. The lamb to be slaughtered is a public joint input into the production of mutton and wool. The difference between a public input and a joint cost is that with a joint input one produces outputs in fixed proportions. For example, one lamb will produce upon slaughtering an amount of meat (4lbs. of mutton) and an amount of wool (20 skeins of yarn). Thus, for every unit input of lamb we will obtain a fixed amount of meat and a fixed amount of yarn. There is no opportunity cost of producing an extra unit of meat in terms of skin (assuming free disposal) or vice versa. A joint input, then, is a special case of a public input as output derived from the input is produced in fixed proportions.

Common costs, however, are distinguishable based on the presence of opportunity cost. Common costs, which are costs of inputs which enter into more than one production function are called common because the use of one input by one division involves an opportunity cost in the production of one good over another. A good example is the train station. The station itself is a common input because it serves as a station for both passenger and freight traffic. Space must be allocated to one service or the other. By allocating space to freight, for example, there is an opportunity cost in terms of serving passengers as the space can not be used for passenger services.

Since these common costs do exhibit an opportunity cost attributable to one jurisdiction over another, they can be allocated based on the opportunity cost. In the following models, however, it is assumed that the input in question is a public input.

3. A Model of Cost Allocation with a Public Input

The introduction of a public input poses few, if any, conceptual modifications to production theory. The production function is a general neoclassical production function. The major

difference here is that one input (K) enters the production function of two different goods. For simplicity, the models described below have a firm producing different outputs in two markets. Thus the production functions take the following form:

$$q_i(K, z_i) \text{ for } i = 1, 2,$$

where $q_K^i > 0$ and $q_z^i > 0$. The input K is a public input into the production of both goods one and two while the input z_i can be a vector of other inputs *unique to the production of good i*. Thus, the conditional demand function can be written as $z_i^* = z_i^*(q_i, K, w)$ ¹ for each i and the variable cost functions (V^i) are

$$V^i(q_i, K, w) = wz_i^* = \min \{wz_i \text{ s.t. } q_i(K, z_i) \geq q_i\}, \quad i = 1, 2.$$

This variable cost function thus has the following properties:

$$\text{i) } V_1^i = (\partial V^i / \partial q_i) > 0, \quad \text{ii) } V_K^i = (\partial V^i / \partial K) \leq 0,$$

First, marginal (variable) costs must be positive for both outputs, and second, variable costs decrease as the amount of capital increases.

Given the technology, a regulated firm maximizes profit (π) over the choice of K, q_i , and α , given the constraint that revenues must be no greater than costs as defined by the regulators. In this first case we are letting the firm choose its own cost allocator subject only to the two regulatory constraints for the two jurisdictions. If r is the true economic cost of capital, the regulators pick a $s_i > r$ as a cap on the firm's earnings for jurisdiction i .² Therefore, profit in jurisdiction one (π^1) must be:

¹It should be noted here that there is no such thing as "directly attributable costs." In the literature these directly attributable costs are allocated first to the market which causes the costs prior to the allocation of any common, shared, joint, or public input costs. When the variable cost function, however, includes the public input as an argument, then any change in the variable cost due to a change in wages or output will influence the derived demand for the public input. Thus, it would be incorrect to allocate variable costs caused in market 1 solely to market 1 as the variable costs in market 1 are jointly determined with the variable costs in other markets due to the public input.

²For the purpose of this discussion the choice of s_i is assumed to be exogenous. Historically, however, there may have been a trade-off between the allowed rate of return and the cost allocation mechanism. For interesting descriptions of the telecommunications industry see Gabel (1967) and Temin and Peters (1985a and 1985b).

$$\pi^1 \leq R^1(q_1) - (V^1(q_1, K) + \alpha s_1 K),$$

and profit in jurisdiction two must be:

$$\pi^2 \leq R^2(q_2) - (V^2(q_2, K) + (1 - \alpha)s_2 K).$$

The firm is allowed to choose its level of output, capital input use, and (for now) the cost allocation scheme. To do so, it undertakes the following maximization:

$$\begin{aligned} \text{Max } \pi &= R^1(q_1) + R^2(q_2) - V^1(q_1, K, w) - V^2(q_2, K, w) - rK \\ \text{K, } q_i, \alpha & \\ \text{s.t. } & \text{a) } \pi^1 - R^1(q_1) - V^1(q_1, K) - \alpha s_1 K \leq 0 \\ & \text{b) } \pi^2 - R^2(q_2) - V^2(q_2, K) - (1 - \alpha)s_2 K \leq 0 \\ & \text{c) } q_i \geq 0, K \geq 0, \text{ for } i = 1, 2. \\ & \text{d) } 0 \leq \alpha \leq 1 \end{aligned}$$

where K represents the long run demand for K . Letting $M(q_1, q_2, K, \alpha)$ be the objective function, the Kuhn-Tucker first order conditions are:

$$M_K = -\Sigma V_K^i (1 - \mu_i) - r + \alpha s_1 \mu_1 + (1 - \alpha)s_2 \mu_2 \leq 0, K \geq 0, M_K K = 0 \quad (3.1)$$

$$M_1 = (R_1^1 - V_1^1)(1 - \mu_1) \leq 0, q_1 \geq 0, M_1 q_1 = 0 \quad (3.2)$$

$$M_2 = (R_2^2 - V_2^2)(1 - \mu_2) \leq 0, q_2 \geq 0, M_2 q_2 = 0 \quad (3.3)$$

$$M_\alpha = (s_1 \mu_1 - s_2 \mu_2)K \leq 0, \alpha \geq 0, M_\alpha \alpha = 0 \quad (3.4)$$

$$M_{\mu_1} = -\pi^1 - R^1 + V^1 + \alpha s_1 K \geq 0, \mu_1 \geq 0, M_{\mu_1} \mu_1 = 0, \quad (3.5)$$

$$M_{\mu_2} = -\pi^2 - R^2 + V^2 + (1 - \alpha)s_2 K \geq 0, \mu_2 \geq 0, M_{\mu_2} \mu_2 = 0 \quad (3.6)$$

From the examination of the constraints, a number of feasible solutions exist. It is instructive to take a look at the implications of a few of the Kuhn-Tucker solutions. The values of the choice variables which are consistent with the first solution are:

$$K > 0, \mu_1 = 0, \mu_2 > 0, q_i > 0, \text{ for } i = 1, 2.$$

This is the case where the firm is regulated (constrained) in only one market (market 2). From the constraint analysis the only possible value of α which can exist without a violation of any of the constraints is $\alpha = 0$. This implies that none of the public input costs are allocated to the unregulated market. If α were greater than 0, then the constraint in

condition (3.4) should be zero by complementary slackness. But since $\mu_1 = 0$, condition (3.4) implies $-\mu_2 s_2 = 0$. However, since μ_2 and s_2 are assumed positive, the constraint is violated. Therefore α must be zero in this solution.³

Another interesting case surrounds the analysis of conditions (3.1) and (3.4). From (3.1) we know that the firm will try to set the sum of the constrained marginal rates of technical substitution equal to the regulation determined shadow price. For a traditional public good we would expect to see the optimal choice determined by the vertical summation of the marginal benefit set equal to the marginal cost (Samuelson: 1956) and each consumer is charged a Lindahl price determined by the intersection of the consumer's marginal benefit curve and the optimal amount of the public input chosen. Figure 1 shows the corresponding situation for the case of a public input into a multiproduct regulated firm. In this case the firm's constrained choice of the public input depends upon the vertical summation of the firm's marginal productivity of the public input (i.e. $\sum V_K^i(1-\mu_i)$). In addition, since the firm is regulated in both markets the marginal productivities are deflated by $(1-\mu_i)$. These marginal productivities show the marginal value to the particular market of increasing the public input. When the firm is allowed to choose its own allocator, then, like the traditional public good case, the firm could imply an allocator that is essentially a Lindahl price for the public input.

Two cases are worth noting. The first is, if the firm chooses its own cost allocator, the it will chose α so that condition (3.4) is true. By choosing its allocator, the firm, attempts to

³Temin and Peters (1985a), in their explanation of the history of the separations process, conclude that it was in AT&T's best interests to put all of its joint plant in the local jurisdiction before the FCC took an active stance in the regulation of interstate messages. The above model partly explains why AT&T fought the introduction of the station to station method of accounting where AT&T was required to charge a portion of its joint plant used in the production of toll and local calls to both the long distance and local jurisdictions. This Kuhn-Tucker solution is essentially the solution with the board to board method of accounting: (the allocation scheme favored by AT&T prior to FCC regulation). The Kuhn-Tucker multiplier μ_1 equal to zero implies that there was no effective regulatory constraint in the interstate market, nor was there any reason to share costs if they could be allocated completely to the regulated jurisdiction. (See also Grace: 1986).

equate the return across jurisdictions by choosing α to equate $s_1\mu_1$ and $s_2\mu_2$. In Figure 1 we see that when the firm chooses α , it sets the sum of the constrained marginal productivities of the public input ($-\Sigma V_K(1-\mu_i)$) equal to the regulation determined shadow price of the public input ($r - \mu_1s_1$). This is the standard Averch-Johnson formulation modified for the case of a public input where the public input is subject to rate base regulation.

[Insert Figure 1 Here]

Now, for the sake of comparison, let us assume that the regulators, rather than the firm, choose the cost allocator α . For this case, assume the regulation determined shadow price in market 2 is less than or equal to the regulation determined shadow price of the public input in market one (i.e. $r - \mu_1s_1 > r - \mu_2s_2$). This implies that $\mu_1s_1 \leq \mu_2s_2$. In this situation, the incentives facing the firm are to choose an amount of the public input that is not less than the Averch-Johnson level of the public input chosen (K_{AJ}) and which may be as great as (K_{CA}). The sum of the constrained marginal productivities is set equal to $r - \alpha\mu_1s_1 - (1-\alpha)\mu_2s_2$. This, of course, assumes that the regulators do not choose α to maximize the firm's profits, but based on some policy goal. If $\mu_1s_1 \leq \mu_2s_2$, then if $\alpha = 0$ (the regulators decide to allocate all the costs of the public input to market 2), the regulation determined shadow price becomes $r - \mu_2s_2$ and the firm would choose to employ K_{CA} amount of the public input. If α is chosen to be 1 (the regulators decide to charge the costs of the public input to market 1), then the firm would set its sum of constrained marginal productivities equal to $r - \mu_1s_1$ and choose to employ K_{AJ} amount of the public input. Thus, bounds are placed on the constrained choice of the public input. A similar, but opposite result can be obtained when $\mu_1s_1 \geq \mu_2s_2$. In this case, the firm's choice of the public input would be less than (K_{AJ}).

Thus, when the firm is allowed to choose its cost allocator, the firm will do so to equate returns across the jurisdictions, and we see the traditional Averch-Johnson bias. This result, however, is altered if the firm is subjected to cost allocation regulation and the regulator chooses for some policy reason the cost allocator α . The firm may have incentives to over-

capitalize further or to employ less than the cost minimizing amount of capital depending on the relative regulation-determined shadow price of the public input in each market.

Another question to examine is what happens to α if $\mu_1 = \mu_2$, but $s_1 < s_2$. In this case the firm will choose α so to satisfy equation (3.4). Condition (3.4) requires that

$$\alpha = 0 \text{ if } s_1 < s_2.$$

Thus, the firm allocates all of its capital to the division with the highest rate of return by choosing an α to maximize profits. If $s_1 > s_2$, then the firm will allocate all its costs to jurisdiction 1 and if $s_2 > s_1$ the firm will allocate all its public input costs to market 2. Correspondingly, since either jurisdiction could be jurisdiction 1, we know that the firm will always allocate its public input to the jurisdiction with the highest rate of return.

Thus, if the firm is given the choice of the cost allocator, it will attempt to allocate costs to the jurisdiction with the highest rate of return. This behavior, in and of itself, will not promote efficiency or insure that the firm will act as the regulators intended. This model sheds additional light if the regulators (state and federal) have conflicting reasons for choosing a particular allowed rate of return. A gaming situation could evolve where the regulators would attempt to set the allowed rate of return in such a way as to influence the firm's cost allocator. The federal regulators would try to set the rate of return on the interstate market lower so that the multi-market firm would allocate more costs towards the jurisdiction with the higher allowed rate of return. The state regulator, trying to protect its own constituencies, would try to lower the allowed rate of return to the intrastate market.⁴ The analysis of this type of regulatory behavior is interesting, but beyond the scope of this paper.

⁴Gabel (pp. 27-43), in his history of the cost allocation process in the telecommunications industry cites evidence of the Federal Communications Commission negotiating with AT&T to lower interstate rates while ignoring cost allocation issues and formulae. In addition, while the Bell Operating Companies were asking for rate increases, the state regulators were not allowing any rate increases. Unlike the Federal Government, the state commissions were trying to change the cost allocation formulae to shift more costs to the interstate market.

The next step is to examine another type of regulatory behavior concerning the situation where a regulated multiproduct firm is either given or is allowed to choose its own cost allocator, and then for policy reasons the regulators decide to change the allocation to a particular market.

4. Effect of Exogenous Changes of Cost Allocator on Input Use

Normally, the firm is not given the choice of the cost allocator. Regulators supposedly choose the allocation based upon welfare concerns that a firm acting in its own self interest would not address. The next section examines the effects of a new regulatory policy on the input and output choices of the firm.

As Braeutigam (1980) and Sweeney (1982) point out, the methods employed by regulators to allocate costs were inefficient: the firm's prices were not based on marginal costs, thus biasing the output mix. Many such rules have been placed in the regulation of transportation, natural gas distribution, and telecommunications firms. Some of the more common include the gross revenues method (where $\alpha_i = R^i/\Sigma R^i$), the relative output method ($\alpha_i = q_i/\Sigma q_i$) or the relative costs method ($\alpha_i = V^i(q_i, K)/\Sigma V^i(q_i, K)$). Not only do these mechanisms bias the output mix, but they also affect the investment in the public input.

The next logical question to ask is what happens to the output choices and level of capital chosen if the allocating factor is changed by the regulators to meet their policy goals. This reflects what has been seen in the telecommunications industry over the past 50 years. As a means of promoting universal service more and more of the public input costs have been allocated to the toll jurisdiction so that local users will be able to subscribe.

4.1 Long Run Results under Cost Sharing Regulation

As mentioned above, Sweeney did not examine the effects of a changing cost allocator on input use. In so doing, he was unable to examine the effect of exogenous changes in α on all the firm's decisions. The changing cost allocations, however, do exert an influence on the firm's choice of the public input. By taking the total differential of the Kuhn-Tucker first

order conditions (assuming an exogenous change in α determined by regulators) a linear system of five equations is derived and shown in the appendix as Table A-1. The comparative statics can be derived using Cramer's Rule and are shown in Table A-2.

This model assumes that in the past the regulators did not set the cost allocator, leaving the choice to the firm. Then, for some policy reason, the regulator determined that the cost allocation between markets (for what ever reason) was not "fair". The regulator then decides to change the allocator by (say) increasing the amount of the public input costs allocated to market 1.

As can be seen in Table A-2, the signs of the total derivatives with respect to α are not absolutely determinable, but it is still possible to obtain some information. By examining the denominator (D) we know that for a maximum, the sign of the principal minors must alternate and the sign for the determinant of the bordered Hessian must be negative. Equation (1) in Table A-2 shows the comparative static for $dK/d\alpha$. The second term ($B_1M_{22} - B_2M_{K2}$) < 0 since $M_{22} < 0$, $M_{K2} > 0$ and $B_1 > 0$. Correspondingly, the third term is positive since it is preceded by a negative sign and $(A_1M_{11} - A_2M_{K1}) < 0$. If $\mu_1s_1 = \mu_2s_2$, then the sign of $dK/d\alpha$ will depend upon the difference between the second and third terms. The second and third terms reflect the difference in the rate of change in the marginal profitability when output is changed multiplied by the difference between the marginal productivity and the allocated return in market 2 (i.e $V_K^2 + (1-\alpha)s_2$). For market 1 (represented by the third term in the numerator of equation (1)) a similar interpretation can be made.

Now, looking at equation (2) in Table A-2, the sign of $dq_1/d\alpha$ depends less upon the relative importance of the regulatory constraints than does the sign of $dK/d\alpha$. The sign of the second and third terms are able to be determined and both turn out to be positive. The second term is similar to the second term in equation (1); it shows the effect of increasing K on market 2. The third term in equation (2) shows the interaction in the changes in cost function as K is changed. Since, the second and third term are positive, if $(\mu_1s_1 - \mu_2s_2)$ is

positive (or relatively small) then the sign of $dq_1/d\alpha$ in equation (2) will be negative since the numerator is positive and the denominator is negative. An increase in the public input costs allocated to market 1 yields a decrease in the output of that market. Similarly, in equation (3) the second and third terms, which are similar to the second and third terms of equation (2), are negative. Coupled with the possibility that $(\mu_1s_1 - \mu_2s_2)$ is negative (or positive and relatively small), the sign of $dq_2/d\alpha$ in equation (2) will be positive.

If regulators increase the cost allocation to one market for a particular policy reason (i.e. attempt to make telephone service affordable for everyone). By decreasing the cost allocation to one market, however, the above model shows that a perverse result could occur. For example, if regulators decreased α to say the local telephone company and the firm was no longer able to set $\mu_1s_1 = \mu_2s_2$, then it is possible that $dK/d\alpha < 0$. Since $V_K < 0$, the costs of marginal costs providing service in both markets increases due to a decrease in K and output in both markets decreases. Thus, the new cost sharing policy actually worked to the detriment of the consumers who were the intended beneficiaries.

4.2 Special Cases

For the sake of exposition and to obtain more feeling for the results, let us make some simplifying assumptions. For example, in case 1, assume the firm has already chosen α so that $\mu_1s_1 = \mu_2s_2$. The government for a policy reason decides that this is not a fair cost allocation and mandates the firm use a new allocator determined by the government. Suppose further that the government desires to increase the allocation to market 1. Finally, let us assume that $A_1 = \partial^2 M / \partial K \partial \mu_1$ and $B_1 = \partial^2 M / \partial K \partial \mu_1$ are positive⁵ and that $A_2 = \partial^2 M / \partial q_1 \partial \mu_1$, and $B_2 = \partial^2 M / \partial q_2 \partial \mu_2$ are also both positive as it is assumed that a regulated firm will always produce beyond the level of output selected by the profit maximizing firm. With these assumptions in mind, it is possible to see results from Table A-2 more clearly. The choice of

⁵This merely assumes that $r > \alpha s_1$ and that $r > (1-\alpha)s_2$. The author will supply a simple proof if desired.

capital is affected by the change in the cost allocator, but once again its sign is indeterminate. The sign depends upon whether the additional benefit of public input to a particular jurisdiction outweighs the additional cost in the other jurisdiction. In addition, output moves in the expected manner. In market 1, output falls with an increase in the cost allocator to market 1, while in market 2 we see an increase in output as the cost allocated to that jurisdiction decreases.

For the second case, assume once again, that $\mu_1 s_1 = \mu_2 s_2$, and A_1 and $B_1 > 0$. The difference here is it is assumed that $A_2 = \partial^2 M / \partial q_1 \partial \mu_1 = 0$ and $B_2 > 0$. The point of this assumption is to see what happens if the firm is allowed set price at marginal cost in one market (i.e. the regulatory constraint is not binding). So, if the firm can set marginal revenue equal to marginal cost in market 1, an increase in the cost allocator to market 1 will cause the firm to decrease the amount of the public input it will employ. In addition, output falls in market 1 since marginal costs increase (as capital decreases) and in market 2, output increases.

Finally, the third case examines just the opposite situation from case 2. Assume that the firm is able to set marginal revenue equal to marginal costs in market 2 and not market 1. Thus, $B_2 = 0$ and $A_2 > 0$. Under these assumptions if the regulators increase α so the cost share to market 1 increases; if marginal revenue in market 2 equals marginal cost, the amount of the public input chosen increases, output in market 1 decreases and output in market 2 increases. Cases 2 and 3 show that if the allocator is increased to the market with monopoly power, the amount of the public input chosen by the firm will decrease but, if the allocator is increased to the regulated market the amount of the public input increases.

An application of the model to the telecommunications industry sheds some light on potential distortions induced by cost sharing regulation. The FCC, in conjunction with state regulators, regularly increased the cost allocator to AT&T's Long Lines division during the last 40 years. If AT&T's Long Lines division was the division that was able to set $R_1^i = V_1^i$ and the

local operating companies were the divisions forced to set price such that $R_1^i \leq V_1^i$, then inferring from the above stylized model, this regulatory policy resulted in a lower investment in the public input than would have been contemplated if regulators did not increase the cost allocation. Given that the firm's response depends on many interactions, with minor assumptions, the effect of a changing policy on cost sharing scheme impacts the behavior of the firm in its investment choice, and the cost sharing scheme can impact adversely the customer groups that were the intended beneficiaries. An additional wrinkle to this same problem can be seen in the next section where the regulator's incentives are analyzed.

5. Optimal Cost Sharing Regulation: Ramsey Revisited

The above models made no assumptions concerning the regulators behavior in determining the mandated cost allocation procedure. This is because, in reality, the cost allocation procedure is often determined in a political arena where the actors have imperfect information. If it were possible for a group of regulators to decide upon the allocator, and this decision was based upon efficiency, an interesting, yet, quite reasonable result can be shown. The regulators choosing an optimal α will choose an allocation that will be consistent with Ramsey pricing rules. Suppose that there is one regulator allowed to choose prices, the amount of the public input, and the cost allocator. In addition, the regulator has some preconceived weighting scheme to value consumer surplus. For example, the regulator desires to help consumers of good 1, so the regulators weight consumer surplus of good 1 by β and weight consumer surplus of good 2 by $(1-\beta)$. By using a formulation adapted from Baumol and Bradford (1970) and Ross (1984) we can undertake the following maximization.

$$\max W(p,K,\alpha) = \beta Z_1(p_1) + (1-\beta)Z_2(p_2) + \lambda_1(\pi^1 - (R^1 - V^1 - \alpha rK)) + \lambda_2(\pi^2 - (R^2 - V^2 - (1-\alpha)rK))$$

$$W_1 = \beta \partial Z_1 / \partial q_1 - \lambda_1(\partial q_1 / \partial p_1 p_1 + q_1 - V_1^1 \cdot \partial q_1 / \partial p_1) \leq 0, W_1 p_1 = 0 \quad (5.1)$$

$$W_2 = (1-\beta) \partial Z_2 / \partial p_2 - \lambda_2(\partial q_2 / \partial p_2 p_2 + q_2 - V_2^2 \cdot \partial q_2 / \partial p_2) \leq 0, W_2 p_2 = 0 \quad (5.2)$$

$$W_K = \lambda_1(V_K^1 + \alpha r) + \lambda_2(V_K^2 + (1-\alpha)r) \leq 0, W_K K = 0 \quad (5.3)$$

$$W_\alpha = rK(\lambda_1 - \lambda_2) \leq 0, W_\alpha \alpha = 0 \quad (5.4)$$

$$W_{\lambda_1} = -R^1 + V^1 + \alpha rK \leq 0, W_{\lambda_1} \lambda_1 = 0, \quad (5.5)$$

$$W_{\lambda_2} = -R^2 + V^2 + (1-\alpha)rK \leq 0, W_{\lambda_2} \lambda_2 = 0, \quad (5.6)$$

Using the rule that $\partial Z_i / \partial p_i = -q_i$, equation (5.1) can be rewritten so that

$$\beta / \lambda_1 = m_1 e_1 - 1, \quad (5.1a)$$

and (5.2) can be rewritten so that

$$(1-\beta) / \lambda_2 = m_2 e_2 - 1, \quad (5.2a)$$

where $m_i e_i = [(p_i - V_i^i) / p_i] e_i$ where e_i is the elasticity of demand for market i . Ross showed that it is possible to discern the regulator's implied welfare weights and econometrically test for whether the regulators favored one group of consumers over another. By taking the ratio of (5.1a) and (5.2a) we see that

$$\frac{\beta / \lambda_1}{(1-\beta) / \lambda_2} = \frac{m_1 e_1 - 1}{m_2 e_2 - 1} \quad (5.7)$$

Thus, the regulator will choose prices to satisfy the perceived weights on the consumer surplus of various consumer groups. Ross showed that for a regulated firm the left hand side of the ratio would be set equal to the ratio of the welfare weights. In the above model the possibility exists that there ^{could} be divergent weights on the profits in each jurisdiction. For example, the Kuhn-Tucker multipliers (λ_i) are the marginal value of producer's surplus to overall welfare. Since there is a trade off in each market between producer and consumer surplus, the ratio of weights must reflect this trade off. Thus, the regulator chooses prices that determined by the ratio of the weights between consumer in producer surplus. In equation (5.7) the left hand side of the expression reflects this result.

Now, looking at condition (5.4) the welfare maximizing regulator will choose α so that $\lambda_1 = \lambda_2$. This means that the regulators equate (by the choice of α) the marginal profitability between jurisdictions. To the regulators a dollar of producer surplus in market 1 should be equal to a dollar of producer surplus in market 2. When this is true it is self evident that the welfare maximizing regulator will set prices merely based upon the welfare weights

between customers. In addition, if the welfare maximizing regulator does not prefer one group of consumers over another, then the prices chosen will be ramsey prices. (See Ross).

The next question to address is: What does this mean for policy purposes? To the extent regulators have implied welfare weights for various customer classes, the above model suggests that the regulators set prices that give the desired weighted ramsey result *and* their choice of a cost allocator should be independent of the welfare weight. The allocator's purpose is to equate the marginal profitability of the jurisdictions. Suppose the regulator employed both a welfare weighting scheme and a cost allocator not based on equating the λ_i ; what would be the result? What if the regulators ex ante chose a β and then decided to choose an α so that $\lambda_1 \neq \lambda_2$? This would result in an ex post policy quite different than the ex ante policy as

$$\beta/(1-\beta) \gtrless \beta'/(1-\beta') = \beta/\lambda_1/(1-\beta)/\lambda_2 ,$$

where β' is the ex post welfare weight and β is the ex ante welfare weight. The relationship depends on whether the ratio of the Kuhn-Tucker multipliers are less than or equal to one. Thus, an attempt to use a cost allocator after setting ex ante welfare weights is potentially self defeating. Telecommunications regulators have traditionally attempted to shift costs (increase the allocator) from the local to the toll markets. At the same time, the regulators, arguably have preconceived welfare weights on the surplus of various customer classes. Because of the political nature of the policy to subsidize local service and the use of the cost allocation mechanism, the above model shows how regulators may, in fact, reverse their ex ante welfare weights by changing the cost sharing regulations.

6. Conclusions

When regulators allocate costs, or actually, when regulators change their allocation rules based upon dictates of public policy (i.e. "subsidizing" local telephone service) there will be an effect on the multiproduct firm's output mix and an effect upon the firm's choice of a public input. If the firm was given the choice of α , the firm equalized marginal public input returns

across jurisdictions and was merely subject to the traditional AJ style capital bias. However, if the firm was subject to a regulatory mandated cost allocator through cost sharing regulation, the amount of input use distortion could be greater than the traditional AJ story or less than the what AJ would predict depending on the relative differences in returns to the public input in different jurisdictions. In addition, there are distinct effects of changing the mandated allocator on the firm's output mix and the choice of public input. It is important to note that empirical investigations focusing solely on the the traditional AJ model could over or under estimate the impacts of rate base regulation since they do not account for the role of cost sharing regulation. Finally, the incentives facing the welfare maximizing regulator are to set ramsey prices and choose an allocator consistnat with those prices. If, however, the regulator is given both the ability to allocate costs, and the ability to use a preconceived notion of a welfare weight for the surplus of various customer classes, the regualtors could defeat their own goals by employing a cost allocation scheme that does not value the producers' surplus equally among all the markets the in which the firm produces.

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Table A-1
Total Differentiation of First Order Conditions

$$\begin{aligned}
 &+ M_{\mu_1 K} dK + M_{\mu_1 q_1} dq_1 = -s_1 K d\alpha \\
 &+ M_{\mu_2 K} dK + M_{\mu_2 q_2} dq_2 = s_2 K d\alpha \\
 M_{\mu_1 K} d\mu_1 + M_{\mu_2 K} d\mu_2 + M_{KK} dK + M_{K_1} dq_1 + M_{K_2} dq_2 &= (s_2 \mu_2 - s_1 \mu_1) d\alpha \\
 M_{\mu_1 q_1} d\mu_1 + M_{K_1} dK + M_{11} dq_1 &= 0 \\
 M_{\mu_2 q_2} d\mu_2 + M_{K_2} dK + M_{22} dq_2 &= 0
 \end{aligned}$$

$$\begin{aligned}
 M_{KK} &= -\Sigma V_{KK}^i (1 - \mu_i) < 0 & M_{\mu_1 K} &= V_K^1 + \alpha s_1 > 0 \\
 M_{K_1} &= -V_{K_1}^1 (1 - \mu_1) > 0 & M_{\mu_1 q_1} &= -(R_1^1 - V_1^1) > 0 \\
 M_{K_2} &= -V_{K_2}^2 (1 - \mu_2) > 0 & M_{\mu_2 K} &= V_K^2 + (1 - \alpha) s_2 > 0 \\
 M_{\mu_2 q_2} &= -(R_2^2 - V_2^2) > 0 & M_{ii} &= (R_{ii}^i - V_{ii}^i) (1 - \mu_i) < 0
 \end{aligned}$$

Now, by dividing both sides of the system by $d\alpha$ and rearranging the system into matrix notation such that the system is now represented as $Hx = d$ where

$$H = \begin{bmatrix} 0 & 0 & M_{\mu_1 K} & M_{\mu_1 q_1} & 0 \\ 0 & 0 & M_{\mu_2 K} & 0 & M_{\mu_2 q_2} \\ M_{\mu_1 K} & M_{\mu_2 K} & M_{KK} & M_{K_1} & M_{K_2} \\ M_{\mu_1 q_1} & 0 & M_{K_1} & M_{11} & 0 \\ 0 & M_{\mu_2 q_2} & M_{K_2} & 0 & M_{22} \end{bmatrix}$$

and where $x = \begin{bmatrix} d\mu_1/d\alpha \\ d\mu_2/d\alpha \\ dK/d\alpha \\ dq_1/d\alpha \\ dq_2/d\alpha \end{bmatrix}$ and $d = \begin{bmatrix} -s_1 K \\ s_2 K \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Case 1: Initial Case

$$dK/d\alpha = 1/D[K(s_2A_2^2B_1M_{22} - B_2M_{K2}) - s_1B_2^2(A_1M_{11} - A_2M_{K1})] > 0$$

$$dq_1/d\alpha = K/D[s_2A_1A_2(M_{K2}B_2 - M_{22}B_1) + s_1(A_1B_2^2M_{K1} + 2A_2B_1B_2M_{K2} - A_2B_1^2M_{22} - A_2B_2^2M_{KK})] < 0$$

$$dq_2/d\alpha = -K/D[s_1B_1B_2(M_{K1}A_2 - M_{11}A_1) + s_2(A_2^2B_1M_{K2} + 2A_1A_2B_2M_{K1} - A_2^2B_2M_{KK} - A_1^2B_2M_{11})] > 0$$

Case 2: Monopoly Pricing in Market 1

$$dK/d\alpha = -K(s_1B_2^2A_1M_{11} - A_2M_{K1})/D < 0$$

$$dq_1/d\alpha = K(s_1A_1B_2^2M_{K1})/D < 0$$

$$dq_2/d\alpha = KM_{11}A_1B_2(s_1B_1 + s_2A_1)/D > 0$$

Case 3: Monopoly Pricing in market 2

$$dK/d\alpha = K(s_2A_2^2B_1M_{22})/D > 0$$

$$dq_1/d\alpha = -KA_2M_{22}B_1(s_2A_1 + s_1B_1)/D < 0$$

$$dq_2/d\alpha = -K(s_2A_2^2B_1M_{K2})/D > 0$$

Table A-2
Comparative Statics

$$(1) \quad dK/d\alpha = K/D \left[\underbrace{-A_2^2 B_2^2 (\mu_1 s_1 - \mu_2 s_2)}_{?} + \underbrace{s_2 A_2^2 (B_1 M_{22} - B_2 M_{K2})}_{-} - \underbrace{s_1 B_2^2 (A_1 M_{11} - A_2 M_{K1})}_{+} \right] \lesssim 0$$

$$(2) \quad dq_1/d\alpha = K/D \left[\underbrace{A_1 A_2 B_2^2 (\mu_1 s_1 - \mu_2 s_2)}_{?} + \underbrace{s_2 A_1 A_2 (M_{K2} B_2 - M_{22} B_1)}_{+} + \underbrace{s_1 (A_1 B_1^2 M_{K1} + 2A_2 B_1 B_2 M_{K2} - A_2 B_1^2 M_{22} - A_2 B_2^2 M_{KK})}_{+} \right] \gtrsim 0$$

$$(3) \quad dq_2/d\alpha = K/D \left[\underbrace{A_2^2 B_1 B_2 (\mu_1 s_1 - \mu_2 s_2)}_{?} - \underbrace{s_1 B_1 B_2 (M_{K1} A_2 - M_{11} A_1)}_{-} - \underbrace{s_2 (A_2^2 B_1 M_{K2} + 2A_1 A_2 B_2 M_{K1} - A_2^2 B_2 M_{KK} - A_1^2 B_2 M_{11})}_{-} \right] \gtrsim 0$$

$$(4) \quad D = B_1^2 A_2^2 M_{22} - 2B_1 A_2^2 B_2 M_{K2} - 2M_{K1} A_1 A_2 B_2^2 + A_2^2 B_2^2 M_{KK} + M_{11} A_1^2 B_2^2 < 0$$

$$A_1 = \partial^2 M / \partial K \partial \mu_1$$

$$A_2 = \partial^2 M / \partial q_1 \partial \mu_1$$

$$B_1 = \partial^2 M / \partial K \partial \mu_2$$

$$B_2 = \partial^2 M / \partial q_2 \partial \mu_2$$

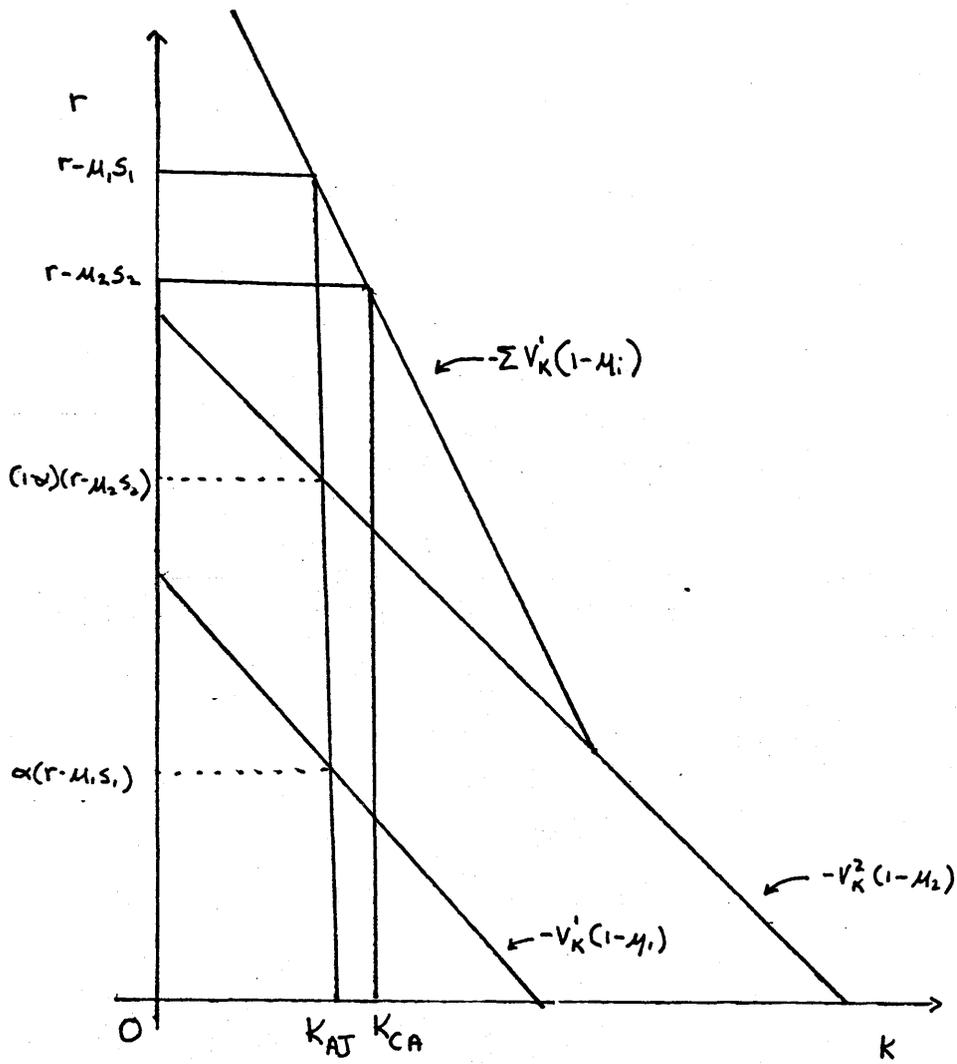


Figure 1 -- choice of Public Input