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"Cost Allocations in a Multiproduct Natural Monopoly: Separations in Telecommunications."

CHAPTER V
REGULATION AND COST ALLOCATION BETWEEN MARKETS

A cost allocation scheme's effect on the overall level of welfare for society depends upon the choices left to regulators and to the firm. For example, an unconstrained firm will choose a cost allocation mechanism that will allow profits to be maximized while a regulator might choose an allocation mechanism encompassing some idea of maximizing social welfare. The purpose of this paper is to examine incentives to the firm and industrial performance under various institutional arrangements and policy rules.

5.1 Cost Allocations between two Regulatory Jurisdictions

Hannon (1978) studied the cost allocation problem by examining the effects of the cost allocation mechanism on the output of the firm. The approach yields a great deal of information, but does not consider the problem from the input side. Hannon implicitly treated the shared capital (K) like a common input instead of a joint or public input in his model, with the firm subject to a regulatory constraint in both markets. Although, the model is similar to the standard Averch-Johnson (1962) models, Hannon points out the necessity to use these models,

> If a minimally adequate theoretical base is to underlie the discussion of rate of return regulation, Averch-Johnson models must be expanded to encompass partial and multi-jurisdictional regulation ... the primary emphasis being on the impact of changes in the cost allocations between markets rather than analyzing or detecting A-J effects (Hannon, 1978: 31).

This point is especially valid given the well-known criticisms of Averch-Johnson models (Peles and Stein (1976), and Joskow (1974)).

Secondly, when Hannon examined the effects of a changing cost allocator (α) on the firm's behavior, he assumed that changing α had no effect on capital. As will be shown below, there could be a relationship between the cost allocation mechanism and the amount of capital used in the production process. There is no reason to assume *a priori* that Hannon's assumption is even true. In a more realistic setting (section 5.3), the effect of the cost allocation on capital becomes very important when examining the effect of a change in α on the choice of capital.

As mentioned previously, Hannon's model did not deal with joint costs, but with common costs (which can be defined loosely as those cost which are common to all production process). More precisely, however, when plant is an input in the production of two products, the cost of the input is a common cost if, in the production of one good, the amount of capacity left for production of other goods is diminished. There is, in fact, an opportunity cost of producing product one rather than product two.

Common costs could thus be allocated on the basis of cost causation since there is an opportunity cost of using the input for one particular product over another. With joint costs, however, there is no opportunity cost of producing one good over the other. In this sense too, a joint input is like a public input. In fact, while producing one output, the firm has available an input that can be used to produce another output.

The problem faced by regulators is that it is difficult to identify common costs and treat them differently than joint costs. The solution is often to lump the costs together and speak of allocating joint and common costs. In this respect Hannon's model simulates reality. Nevertheless, the firm still bases its decisions upon the marginal productivities of inputs even though it is subject to these accounting costs allocation rules.

Sweeny (1982) employed a model similar to the model described below using the accounting formulae examined in Braeutigam (1980). Sweeny examined the effect of two different fully distributed cost allocation formulas (relative output and relative revenues) on the output decisions of the firm given that it is subject to regulation in numerous jurisdictions. Sweeny's results showed that a firm using one of these accounting rules will choose a price vector which is dominated (i.e. there is a different set of price-output combinations that will yield greater than or equal outputs with at least the same level of overall profit.) The model presented in the next sections examines the problem of allocating joint costs, and the effect of changing the cost allocation formula on output and capital input use. Unlike both Sweeny and Hannon, however, this model assumes that the input choice is effected by the cost allocation formula.

5.2. A Two Jurisdiction Model with Joint Costs

This section seeks to analyze the effects of altering the cost allocation factor (α) on the firm's input choice and output mix in the short run and in the long run. The comparative statics of a stylized model of cost allocation with regulation will be analyzed to show how much more complex the firm's decisions have become and to show how the firm, if left to maximize profit, will choose a cost allocation scheme consistent with the regulatory constraints.

The production function used in these models is a general neoclassical production function. The major difference here is that one input (K) enters the production function of two different goods. The models described below have a firm producing different outputs in two markets. Thus the production functions take the following form:

$$q_i(K, z_i) \text{ for } i = 1, 2,$$

where $q_1^{*} > 0$ and $q_2^{*} > 0$. The input K is a joint input into the production of both goods 1 and two while the input z_i can be a vector of other inputs unique to the production of good i.

The analysis next proceeds in four stages. First, the short-run, and thus, the variable cost functions are derived for all levels of the public input K. Second, profit will be maximized over the choice of K, q_1, and q_2 subject to the regulatory constraint that price in each market be no greater than average cost. Third, we let the firm chose α, and fourth we will explore the implications of deviating from the firm's preferred α.

Now let $C^1(q_1) = V^1(q_1,K,w) + \alpha rK$, and

$$C^2(q_2) = V^2(q_2,K,w) + (1-\alpha)rK,$$

where $V^i(q_i,K,w) = \min_{z_i} wz_i$ s.t. $q_i^* = q_i(K,z_i)$ for i = 1,2,

Forming the Lagrangean and deriving the first order conditions,

$$L_{z_i} = w - \mu_i q_z^i = 0 \qquad i = 1,2, \qquad (5.2.1)$$

$$L_{\mu_i} = q_i^* - q_i(K,z_i) = 0 \qquad i = 1,2, \qquad (5.2.2)$$

where μ_i is the Lagrangean multiplier for market i.

By employing the implicit function theorem, the conditional demand functions for each of the variable inputs can be derived. Thus, $z_i^* = z_i^*(q_i,K,w)$ for each output and the variable cost functions V^i are obtained so that

$$V^i(q_i,K,w) = wz_i^* \qquad i = 1,2. \qquad (5.2.3)$$

This variable cost function thus has the following properties:

i) $V_i^i = (\delta V^i / \delta q_i) > 0$, for $i = 1,2$.

ii) $V_K^i = (\delta V^i / \delta K) < 0$, for $i = 1,2$.

iii) $V_K^i \equiv w*(\delta z_i / \delta K)$

The first property is that marginal (variable) costs must be positive for both outputs. The second property states that variable costs decrease as the amount of capital increases. The third property, derived from the definition of the variable cost function, states that the derivative of the variable cost function with respect to K is the marginal rate of technical substitution between z_i and K multiplied by the wage rate (w) of z_i.

As a bench-mark to compare with the next model let us assume we have the following total cost function:

$$C(q_1, q_2) = V^1(q_1,K) + V^2(q_2,K) + rK \qquad (5.2.4)$$

Given this function, we now desire to determine the optimal provision conditions for K. From the public goods literature (Samuelson, 1956) we know the firm will choose K to minimize costs by setting the summation of the marginal rates of technical substitution equal to the ratio of the wage rates. This can be done by taking the derivative of the above cost function with respect to K so as to yield the following first order condition:

$$V_K^1 + V_K^2 = r \qquad (5.2.5)$$

Rearrangement yields:

$$(\delta z_1/\ \delta q_1) + (\delta z_2/\ \delta q_2) = r/w \qquad (5.2.6)$$

Now that the technology can be described, the next step is to maximize profit (π) over the choice of K, q_i, and α, given the regulatory constraint that prices must be no greater than average cost as defined by the regulators. This constraint is the familiar rate of return constraint which allows the firm a specified rate of return on capital. If r is the true economic cost of capital, the regulators pick a $s_i > r$ as a cap on the firm's earnings. Therefore, price in the first jurisdiction must be:

$$p_1 \leq (V^1 + \alpha s_1 K)/q_1,$$

and price in jurisdiction two must be less than or equal to average costs as defined by:

$$p_2 \leq (V^2 + (1 - \alpha)s_2 K)/q_2.$$

The firm is allowed to choose its level of output, capital input use, and cost allocation scheme. To do so, it undertakes the following maximization:

$$\max_{K,q_i,\alpha} \pi = R^1(q_1) + R^2(q_2) - V^1(q_1,K,w) - V^2(q_2,K,w) - rK$$

$$\text{s.t. a) } R^1(q_1) - V^1(q_1,K,w) - \alpha s_1 K \leq 0$$

$$\text{b) } R^2(q_2) - V^2(q_2,K,w) - (1-\alpha)s_2 K \leq 0$$

$$\text{c) } q_i > 0,\ K > 0, \quad \text{for } i = 1,2.$$

where K represents the long run demand for K.

Before analyzing the Kuhn-Tucker first order conditions, it is necessary to make some assumptions about the revenue and cost functions. For a maximum, the Kuhn-Tucker sufficiency theorem requires the objective function to be differentiable and concave in the non-negative orthant. In addition, each constrained function must be differentiable and convex in the non-negative orthant. According to the Kuhn-Tucker sufficiency condition, if a solution satisfies the Kuhn-Tucker first order conditions it will be a global maximum (Chiang, 1974: 722). In addition, the following regularity conditions are assumed:

i) $$\frac{\delta R^i}{\delta q_i} = R^i_i > 0 \quad i = 1,2.$$

ii) $$\frac{\delta^2 R^i}{\delta q_i \delta q_i} = R^i_{ii} < 0 \quad i = 1,2.$$

iii) $$\frac{\delta V^i}{\delta q_i} = V^i_i > 0 \quad i = 1,2.$$

iv) $$\frac{\delta^2 V^i}{\delta q_i \delta q_i} = V^i_{ii} \gtrless 0, \quad i = 1,2.$$

v) $$\frac{\delta V^i}{\delta K} = V^i_K < 0, \quad i = 1,2.$$

vi) $$\frac{\delta^2 V^i}{\delta K \; \delta K} = V^i_{KK} > 0, \quad 1 = 1,2.$$

vii) $$\frac{\delta^2 V^i}{\delta K \; \delta q_j} = 0, \quad i \neq j.$$

viii) $$\frac{\delta^2 V^i}{\delta q_i \delta q_j} = 0, \quad i \neq j.$$

$M_{\mu 2}\mu_2 = 0$ (5.2.12b)

Given these first order conditions it is necessary to rule out values the choice variables may take in order to be assured a solution exists.

From the examination of the constraints, two feasible solutions exists. The values of the choice variables which are consistent with the first solution are:

$$K > 0, \mu_1 = 0, \mu_2 > 0, q_i > 0, \text{ for } i = 1,2.$$

This is the case where the firm is regulated (constrained) in only one market (market 2). From the constraint analysis the only possible value of α which can exist without a violation of any of the constraints is $\alpha = 0$. This implies that none of the joint costs are allocated to the unregulated market. If α were greater than 0, then the constraint in equation (5.2.10a) should be zero by complementary slackness. But since $\mu_1 = 0$, equation (5.2.10a) implies $-\mu_2 s_2 = 0$. However, since it was assumed μ_2 and s_2 are positive, the constraint is violated. Therefore α must be zero in this solution.

Temin and Peters (1985), in their explanation of the history of the separations process, conclude that it was in AT&T's best interests to put all of its joint plant in the local jurisdiction before the FCC took an active stance in the regulation of interstate messages. The above model partly explains why AT&T fought the introduction of the station to station method of accounting where AT&T was

The first four conditions are the typical first and second order conditions. Conditions (v) and (vi) are the same as those derived above during the discussion of the variable cost function. The last two conditions though are used to get at the public good nature of the capital input. Condition (vii) says that a change in output in one market has no effect on the cost savings attributable to K in the other market. This "independence" assumption is employed to make the analysis a bit less complicated. Condition (viii) requires that the marginal cost of one market be independent of a change in output in the other market. This will preclude the possibility of congestion in the use of the joint input.

Letting $M(q_1,q_2,K,\alpha)$ be the objective function, the Kuhn-Tucker first order conditions are:

$$M_K = -V^1_K(1-\mu_1) - V^2_K(1-\mu_2) - r + \alpha s_1\mu_1 + (1-\alpha)s_2\mu_2 \leq 0 \tag{5.2.7a}$$

$$M_K K = 0 \tag{5.2.7b}$$

$$M_1 = (R^1_1 - V^1_1)(1 - \mu_1) \leq 0 \tag{5.2.8a}$$

$$M_1 q_1 = 0 \tag{5.2.8b}$$

$$M_2 = (R^2_2 - V^2_2)(1-\mu_2) \leq 0 \tag{5.2.9a}$$

$$M_2 q_2 = 0 \tag{5.2.9b}$$

$$M_\alpha = (s_1\mu_1 - s_2\mu_2)K \leq 0 \tag{5.2.10a}$$

$$M_\alpha \alpha = 0 \tag{5.2.10b}$$

$$M_{\mu 1} = -R^1 + V^1 + \alpha s_1 K \geq 0 \tag{5.2.11a}$$

$$M_{\mu 1}\mu_1 = 0 \tag{5.2.11b}$$

$$M_{\mu 2} = -R^2 + V^2 + (1-\alpha)s_2 K \geq 0 \tag{5.2.12a}$$

required to charge a portion of its joint plant used in the production of toll and local calls to both the long distance and local jurisdictions. This Kuhn-Tucker solution is essentially the solution with the board to board method of accounting: the allocation scheme favored by AT&T prior to FCC regulation. The Kuhn-Tucker multiplier μ_1 equal to zero implies that there was no effective regulatory constraint, nor was there any reason to share costs if they could be allocated completely to the regulated jurisdiction.

Similarly, when the FCC attempted to split AT&T into two separate subsidiary companies in the Computer II (1980) decision (one regulated and the other unregulated) to allow the unregulated long distance division to compete against unregulated firms on a "fair" basis, the FCC recognized the incentive to shift its common and joint costs to the regulated local jurisdiction. In order to avoid this result the FCC instituted very detailed and complicated rules to make sure the integrated telecommunications firm would not be expanding its markets at the expense of the local telephone customer or have an unfair advantage over its competitors.

Returning to the model, we note that the second solution is the case where the firm is subjected to regulation in both markets (μ_1, μ_2, > 0), a positive amount of capital is chosen ($K > 0$), and output in both markets is positive ($q_i > 0$). The cost allocation mechanism (α) consistent with all the first order conditions, not surprisingly, can range from zero to one depending upon the system of the first order conditions.

Examining the first order condition in equation (5.2.7a) while assuming that $K > 0$, let us compare this result with the benchmark cost minimizing condition shown in equation (5.1.5). By rearranging (5.2.7a) and defining $V_K^i (1 - \mu_i) \equiv -w\hat{z}_K^i$, for $i = 1,2$, then

$$\hat{z}_K^1 + \hat{z}_K^2 = (r - \alpha s_1 \mu_1 - (1 - \alpha) s_2 \mu_2)/w \qquad (5.2.13)$$

Equation (5.2.13) says that the optimal amount of K is chosen based upon the constrained sum of the marginal rates of technical substitution (MRTS) between K and z_i being set equal to a function of the ratio of the wage rates. From the cost minimizing case in equation (5.2.2) we can see that the choice of K was based simply on the sum of the unconstrained MRTS being set equal to the ratio of the wage rates.

Now, if α is chosen by the firm and is positive, then, from equations (5.2.10a) and (5.2.10b), we know that

$$(\mu_1 s_1 - \mu_2 s_2) = 0.$$

Thus, (5.2.13) reduces to

$$\hat{z}_K^1 + \hat{z}_K^2 = (r - s_1 \mu_1)/w \qquad (5.2.13a)$$

Not surprisingly, the expression on the right hand side of (5.2.13a) is not equal to the ratio of wage rates (r/w) and in fact is greater than (r/w) since $\mu_1 s_1 > 0$. This is the standard Averch-Johnson result. Only in this case the input is a joint input into the production of both services. By allowing the firm to choose α, the firm will pick an α that will equate the effective regulatory constraints across the jurisdictions thus yielding the standard AJ result.

After examining the constraint and finding a solution, it is possible to derive the values of the choice variables that maximize profits. Using the implicit function theorem the optimal values are:

$$q_i^*(w,r,s_i) \quad i = 1,2.$$

$$K^*(w,r,s_i)$$

$$\alpha^*(w,r,s_i)$$

Thus, given a rate of return constraint requiring prices to be no greater than allowed average cost, and allowing the firm to choose its capital input and output in both markets, the firm will choose an α to allocate costs between the two divisions in such a way that is consistent with the regulatory constraints.

It is important to note that if regulators in both jurisdictions set $s_i = r$, the firm will act to set prices consistent with the new allowed rates of return. What is interesting about these prices is that they will be Ramsey prices. Equations (5.2.8a) and (5.2.9a) can be rewritten so that:

$$\frac{(p_1 - V_1^1)}{p_1} e_1 = \frac{(p_2 - V_2^2)}{p_2} e_2 \qquad (5.2.14)$$

From equations (5.2.11a) and (5.2.12a) we know that price must equal average cost which is a function of α. By inserting these prices (p_i^*) into (5.2.14) we get:

$$\frac{(p_1^* - V_1^1)}{p_1^*} e_1 = \frac{(p_2^* - V_2^2)}{p_2^*} e_2 \qquad (5.2.15)$$

These prices are equal to the 'average cost' when α is chosen to set $\mu_1 = \mu_2$ from the first order condition in equation (5.2.10a). This has the effect of constraining the firm to just one regulatory constraint which results in the standard Ramsey result. Even when $s_1 \neq r$, a "Ramsey like" result will occur because the firm attempts to set α so as to equate $\mu_1 s_1$ and $\mu_2 s_2$. The result, which is not even a second best result, will depend upon the relative demand elasticities.

Theoretically, it is not necessary for regulators to set α since regulators implicitly determine α when they set the allowed rates of return. If $s_1 > s_2$ we would expect the firm to pick a particular α, and if the sign was reversed, the firm would choose another allocation. However, the reality of the situation is that the cost allocation factor, α, is a politically determined variable. The history of the separations process detailed in Gabel

(1967), Sichter (1977), and Temin and Peters (1985) demonstrates that the telecommunications cost allocator had changed numerous times in the last four decades to rectify imbalances in interstate versus intrastate toll rates, to stave off rate increases for the local companies, and to keep AT&T from earning excess profits. The next logical question to ask is what happens to the output choices and level of capital chosen if the allocating factor is changed by the regulators to meet their policy goals.

5.3 Long Run Results when α is Exogenous

As mentioned above, Hannon (1978) and Sweeney (1982) either assumed or implicitly set $dK/d\alpha = 0$. In so doing, they were unable to examine the effect of exogenous changes in α on all the firm's decisions. By taking the total differential of the Kuhn-Tucker first order conditions (assuming an exogenous α set by regulators) a linear system of five equations is derived and shown in Table 5.1. Now, by dividing both sides of the system by $d\alpha$ and rearranging the system into matrix notation such that the system is now represented as $Hx = d$ where

$$H = \begin{vmatrix} 0 & 0 & M_{\mu 1K} & M_{\mu 1q1} & 0 \\ 0 & 0 & M_{\mu 2K} & 0 & M_{\mu 2q2} \\ M_{\mu 1K} & M_{\mu 2K} & M_{KK} & M_{K1} & M_{K2} \\ M_{\mu 1q1} & 0 & M_{K1} & M_{11} & 0 \\ 0 & M_{\mu 2q2} & M_{K2} & 0 & M_{22} \end{vmatrix}$$

and where $x = \begin{vmatrix} d\mu_1/d\alpha \\ d\mu_2/d\alpha \\ dK/d\alpha \\ dq_1/d\alpha \\ dq_2/d\alpha \end{vmatrix}$, and $d = \begin{vmatrix} -s_1K \\ s_2K \\ 0 \\ 0 \\ 0 \end{vmatrix}$.

Table 5.1

Total Differentiation of First Order Conditions

$$+ M_{\mu_1 K}dK + M_{\mu_1 q_1}dq_1 = -s_1 K d\alpha$$

$$+ M_{\mu_1 K}\, dK + M_{\mu_2 q_2}dq_2 = s_2 K d\alpha$$

$$M_{\mu_1 K}d\mu_1 + M_{\mu_2 K}d\mu_2 + M_{KK}dK + M_{K1}dq_1 + M_{K2}\, dq_2 = (s_2\mu_2 - s_1\mu_1)d\alpha$$

$$M_{\mu_1 q}d\mu_1 + M_{K1}dK + M_{11}dq_1 = 0$$

$$M_{\mu_2 q_2}d\mu_2 + M_{K2}dK + M_{22}dq_2 = 0$$

$M_{KK} = -\Sigma V^i_{KK}(1 - \mu_i) < 0$ $\quad$ $M_{\mu_1 K} = V^1_K + \alpha s_1 > 0$

$M_{K1} = -V^1_{K1}(1 - \mu_1) > 0$ $\quad$ $M_{\mu_1 q_1} = -(R^1_1 - V^1_1) < 0$

$M_{K2} = -V^2_{K2}(1 - \mu_2) > 0$ $\quad$ $M_{\mu\ K} = V^2_K + (1 - \alpha)s_2 > 0$

$M_{\mu_2 q_2} = -(R^2_2 - V^2_2) > 0$

$M_{11} = (R^1_{11} - V^1_{11})(1 - \mu_1) < 0$

$M_{22} = (R^2_{22} - V^2_{22})(1 - \mu_2) < 0$

Table 5.2
Comparative Statics

(1) $$\frac{dK}{d\alpha} = \frac{1}{D}\{K[-A_2B_2(\overbrace{\mu_1 s_1 - \mu_2 s_2}^{?})] + s_2A_2(\overbrace{B_1M_{22} - B_2M_{K2}}^{-}) - s_1B_2(\overbrace{A_1M_{11} - A_2M_{K1}}^{+}) \lesseqgtr 0$$

(2) $$\frac{dq_1}{d\alpha} = \frac{1}{\overset{-}{D}}\{K[A_1A_2B_2^2(\overbrace{\mu_1 s_1 - \mu_2 S_2}^{?}) + s_2A_1A_2(\overbrace{M_{K2}B_2 - M_{22}B_1}^{+}) + s_1(\overbrace{A_1B_2^2M_{K1} + 2A_2B_1B_2M_{K2} - A_2B_1^2M_{22} - A_2B_2^2M_{KK}}^{+})] \gtrless 0$$

(3) $$\frac{dq_2}{d\alpha} = \frac{1}{\overset{-}{D}}\{K[A_2^2B_1B_2(\overbrace{\mu_1 s_1 - \mu_2 s_2}^{?}) - s_1B_1B_2(\overbrace{M_{K1}A_2 - M_{11}A_1}^{-}) - s_2(\overbrace{A_2^2B_1M_{K2} + 2A_1A_2B_2M_{K1} - A_2^2B_2M_{KK} - A_2^2B_2M_{11}}^{-})] \gtrless 0$$

(4) $$\overset{-}{D} = B_1^2A_2^2M_{22} - 2B_1\overset{-}{A_2}B_2^2M_{K2} - 2M_{K1}\overset{-}{A_1}A_2B_2^2 + A_2^2\overset{-}{B_2^2}M_{KK} + M_{11}\overset{-}{A_1^2}B_1^2 < 0$$

$$B1 = \frac{\delta^2 M}{\delta K \delta \mu_2} > 0$$

$$B_2 = \frac{\delta^2 M}{\delta q_2 \delta \mu_2} > 0$$

$$A_1 = \frac{\delta^2 M}{\delta K \delta \mu_1} > 0$$

$$A_2 = \frac{\delta^2 M}{\delta q_2 \delta \mu_2} > 0$$

Using Cramer's rule to solve for the elements of **x**, the comparative statics can be derived and are shown in Table 5.2.

As can be seen in Table 5.2, the signs of the total derivatives with respect to α are not absolutely determinable, but it is still possible to obtain some information. The fact that $dK/d\alpha$ is unlikely to be zero is important in and of itself. By examining the denominator (D) we know that for a maximum, the sign of the principal minors must alternate and the sign for the determinant of the bordered Hessian must be negative. Although not shown here, this condition has been met. Equation (1) in Table 5.2 shows the comparative static for $dK/d\alpha$. The first term $(B_1M_{22} - B_2M_{K2}) < 0$ since $M_{22} < 0$, $M_{K2} > 0$ and $B_1 > 0$. Correspondingly, the second term is positive since it is preceded by a negative sign.

Before going further, the importance of differing regulatory policies should be noted in conjunction with the AJ effect. There is still the mathematical result that a firm will choose to employ more than the optimal amount of capital when allowed to earn a rate of return greater than the cost of capital. Even if some of the behavioral implications of the AJ hypothesis are subject to debate, there is the added problem of how the capital choice is affected when the firm employs capital subject to cost allocation regulation. This new distortion depends in part upon the relative size of $\mu_1 s_1$ versus $\mu_2 s_2$, or divergent regulatory treatment. This behavior may, in fact, be a more plausible reason for any capital use distortions or output distortions than the standard AJ distortion.

Now looking at equation (2) in Table 5.2 the sign of $dq_1/d\alpha$ depends upon the relative importance of the regulatory constraints as does the sign of $dK/d\alpha$. The sign of the second and third terms are able to be determined and turn out to be positive; so, if $(\mu_1 s_1 - \mu_2 s_2)$ is positive (or relatively small) then the sign of $dq_1/d\alpha$ in equation (2) will be negative since the numerator is positive and the denominator is negative. Similarly, in equation (3) the second and third terms are negative. Coupled with the possibility that $(\mu_1 s_1 - \mu_2 s_2)$ is negative (or positive and relatively small), the sign of $dq_2/d\alpha$ in equation (2) will be positive.

For $dq_1/d\alpha > 0$ in equation (2), and $dq_2/d\alpha < 0$ in equation (3) the effect of the differences between the effective regulatory constraints $(\mu_1 s_1 - \mu_2 s_2)$ must outweigh the effects of the terms that have a definite sign. The fact that the sign of the entire differential equation can not be determined with certainty suggests that the possibility that a perverse policy result could occur. For example, say regulators increased α (thus increasing the share of the joint-costs to market one) to encourage more consumption of good two by making the price of good two lower. Then if, $dq_2/d\alpha < 0$ because of the relative effects of the effective regulatory constraint ($\mu_1 s_1 - \mu_2 s_2$) terms outweighing the terms with a definite sign, the increase in α will have the exact opposite effect from what the policy intended. Because of this possibility, however remote it might be, regulators should take care to ascertain the effects of changing the allocator.

Finally, the sign of $d\alpha/dK$ will depend, to a great extent, on the relative demand elasticities for the two markets. If demand in market 1 is relatively inelastic, the MRTS will be insensitive to changes in price, and if market 2's demand is elastic, we would expect to see an increase (shift) in the MRTS for market 2 which would offset any decrease in market 1's MRTS.

5.4 <u>Short Run Analysis with an Exogenous Change in α</u>

In the short run, when the firm cannot alter its underlying capital structure, it is interesting to note the effects of an exogenous change in the cost allocation formula on the output in the two markets. Unlike the long run results, more definitive results can be derived. As mentioned above, the cost allocation mechanism has been altered in the past for numerous policy reasons. If the regulator were to arbitrarily increase the cost allocation mechanism, the firm will shift some of the costs presently covered by one division to the other division of the firm. This would cause prices to change in order to be consistent with the regulatory constraints. By setting $dK = 0$ it is possible to see the short run affects of a change in α. Solving the system $H_1 x_1 = x_1$ for x_1 where:

$$
H_1 = \begin{vmatrix} 0 & 0 & -(R_1^1 - V_1^1) & 0 \\ 0 & 0 & 0 & -(R_2^2 - V_2^2) \\ -(R_1^1 - V_1^1) & 0 & M_{11} & 0 \\ 0 & -(R_2^2 - V_2^2) & 0 & M_{22} \end{vmatrix}
$$

$$
x_1 = \begin{vmatrix} d\mu 1/d\alpha \\ d\mu 2/d\alpha \\ dq1/d\alpha \\ dq2/d\alpha \end{vmatrix}, \text{ and } d_1 = \begin{vmatrix} -s1K \\ s2K \\ 0 \\ 0 \end{vmatrix},
$$

the following comparative statics can be derived.

$$
\frac{d\mu_1}{d\alpha} = \frac{s_1(KM_{11})}{(R_1^1 - V_1^1)^2} < 0 \tag{5.4.1}
$$

$$
\frac{d\mu_2}{d\alpha} = \frac{s_2(-KM_{22})}{(R_2^2 - V_2^2)^2} > 0 \tag{5.4.2}
$$

$$
\frac{dq_1}{d\alpha} = \frac{s_1 K}{(R_1^1 - V_1^1)} < 0 \tag{5.4.3}
$$

$$
\frac{dq_2}{d\alpha} = \frac{-s2K}{(R_2^2 - V_2^2)} > 0 \tag{5.4.4}
$$

Equation (5.4.1) is negative since $M_{11} < 0$. Similarly, for equation (5.4.2), $M_{22} < 0$, so $d\mu_2/d\alpha > 0$. These two equations are difficult to interpret, but since the Kuhn-Tucker multiplier is essentially the marginal profitability of the division, equation (5.4.1) says that if α were increased, the marginal profitability of the firm will decrease. For division two, equation (5.4.2) says that as α increases the marginal profitability of the firm will increase. This makes intuitive sense because the division's costs are being administratively altered, so describe the long run effect on the firm without an idea of the firm's capital responsiveness to a change in the allocation, it is important to know that such a response is likely. As will be shown in the next section, this responsiveness can provide some interesting results in a welfare analysis context.

5.5 An Application: Welfare and Ramsey Prices

This section shows how the cost allocation chosen by the regulator can be compared to the Ramsey optimal choice. It is possible to test the hypothesis that regulators have preferred one class of customers over another by examining how regulators chose the cost allocation (α). But, first, a short digression is in order to show how a similar technique was used to make conclusions concerning the relative social welfare weights a regulator may place on the value of consumption of different goods of a multiproduct monopolist.

Implied Social Welfare Weights

Ross (1984) has shown that it is possible to retrieve an implied social welfare weighting made by regulators between different goods. Ross asks whether it is possible to infer from existing price structures the social welfare weights that would make prices Ramsey-Optimal. To see this, assume there are two classes of customers where A customers consume only good 1 and B customers consume only good 2, and that there is an over all profit there would be an effect upon the division's marginal profitability.

For the output markets, as α increases the amount of q_1 supplied by the firm decreases reflecting an increase in costs. To satisfy the regulatory constraint, the new allocator requires an increase in prices and a decrease in quantity demanded. Equation (5.4.3) shows this result assuming that a regulated firm produces past the point where marginal revenue equals marginal costs (that is, $R_1^1 < V_1^1$). In market two, described in equation (5.4.4), as α increases costs fall to the division, and the firm is required to increase its production of q_2 to satisfy the profit constraint.

In the short run when the capital input is fixed, the regulator can influence the relative amount of output produced. The numerous changes made by the regulatory authorities from 1947 to 1982 can plausibly be explained by the simple fact that only short run considerations were taken into account by policy makers. The fact that a firm would adjust its production decisions (potentially in opposition to the regulator's policies) in the long run was never considered. If regulators did not understand the concepts embodied in a demand elasticity until the early 1960s, then it seems highly plausible that a supply response based upon these altered cost allocations would not be considered either. Although, it is not possible to describe the long run effect on the firm without an idea of the firm's capital responsiveness to a change in the allocation, it is important to know that such a response is likely. As will be shown in the next section, this responsiveness can provide some interesting results in a welfare analysis context.

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$$\max_{p_i} W = \Gamma^A Z^A(p) + \Gamma^B Z^B(p) + \Theta(\pi - \pi^*) \quad (5.5.1)$$

and gives the following first order conditions:

$$W_1 = -q_1^A \Gamma^A + \Theta(q_1 + (p_1 - c_1)\frac{dq_1}{dp_1}) = 0 \quad (5.5.2)$$

$$W_2 = -q_2^B \Gamma^B + \Theta(q_2 + (p_2 - c_2)\frac{dq_2}{dp_2}) = 0 \quad (5.5.3)$$

where $Z^A(p)$ and $Z^B(p)$ are the consumers' surplus for consumers A and B, Γ^A and Γ^B are the regulator's weights for each class of consumers, and Θ is the value the firm's profits takes in the regulator's social welfare function. From Shephard's Lemma we also know that $\delta Z/\delta p_i = -q_i$.

Dividing the FOC's by q_i, and using a slight manipulation we get:

$$\frac{(p_1 - c_1)}{p_1} e_1 = \frac{\Theta - \Gamma^A}{\Theta} \quad (5.5.4)$$

$$\frac{(p_2 - c_2)}{p_2} e_2 = \frac{\Theta - \Gamma^B}{\Theta} \quad (5.5.5)$$

These are the Ramsey results when there are differing social welfare weights. If the weight for A equaled the weight for B, then we would have the traditional formulation where the percentage price mark-up over marginal costs multiplied by the price elasticity is a constant across all markets.

Now by rewriting (5.5.4) and (5.5.5) as

$$\Gamma^A/\theta = 1 - \frac{(p_1 - c_1)}{p_1} e_1 = 1 - m_1 e_1, \quad (5.5.4a)$$

$$\Gamma^B/\theta = 1 - \frac{(p_2 - c_2)}{p_2} e_2 = 1 - m_2 e_2, \quad (5.5.5a)$$

it is possible to derive a simple expression relating the welfare weights by dividing the first equation by the second equation. This yields:

$$\frac{\Gamma^A}{\Gamma^B} = \frac{1 - m_1 e_1}{1 - m_2 e_2} . \quad (5.5.6)$$

To find the relative weights we need only the mark-up over price and the price elasticity of demand for each good. Then it is possible to econometrically test to see if Γ^A/Γ^B is significantly different that one to determine whether the class of A customers are favored over the class of B customers.

Determining the Optimal α

Now let us introduce a slightly different formulation which is consistent with the model described in section 5.4. If we assume that there are two regulatory jurisdictions and that there is a required cost allocation between the two jurisdictions, then the maximization problem becomes:

$$\max_{p,\alpha} W = Z^A(p_1) + Z^B(p_2) + \mu_1[\pi_1^* - R^1 - V^1 - \alpha rK] +$$
$$+ \mu_2[\pi_2^* - R^2 - V^2 - (1-\alpha)rK] \quad (5.5.7)$$

This problem yields the following first order conditions.

$$W_1 = \delta Z^A/\delta p_1 - \mu_1(q_1 + p_1\delta q_1/\delta p_1 - V_1^1 \delta q_1/p_1] = 0 \quad (5.5.8)$$

$$W_2 = \delta Z^B/\delta p_2 - \mu_2(q_2 + p_2\delta q_2/\delta p_2 - V_2^2 \delta q_2/p_2] = 0 \quad (5.5.9)$$

$$W_\alpha = \Sigma\delta p_i/\delta\alpha[\delta Z^j/\delta p_i - \mu_i(q_i + p_i\delta q_i/\delta p_i - V_i^i \delta q_i/p_i]$$
$$+ rK(\mu_1 - \mu_2) = 0. \quad (5.5.10)$$

where $j = A$, B, and $i = 1,2$. Since equations (5.5.8) and (5.5.9) are equal to 0, equation (5.5.10) reduces to $\mu_1 = \mu_2$. Thus by choosing the optimal level of α the regulators equate the Lagrangean multipliers across jurisdictions. This is the same result we would get if the firm chose the cost allocator. Thus, by choosing α optimally, the regulator effectively sets Ramsey prices. To the extent that α is not chosen to satisfy (5.5.10) one could econometrically test to see how far regulatory policies diverge from the Ramsey efficient result. By rearranging equations (5.5.8) and (5.5.9) using the steps outlined in equations (5.5.4) - (5.5.6) we can see that

$$\frac{\mu_1 - 1}{\mu_1} = (p_1 - V_1)e_1/p_1 = (p_2 - V_2)e_2/p_2 = \frac{\mu_1 - 1}{\mu_1}.$$

This in turn implies that

$$1 = \frac{1 - m_1 e_1}{1 - m_2 e_2} . \qquad (5.5.11)$$

It is thus possible to test whether equation (5.5.11) is greater than, less than, or equal to one. Notice that this is the same test that Ross proposed in equation (5.5.6) to determine whether regulators were employing social welfare weights when pricing decisions. Thus from the same evidence Ross would use to extract social welfare weights it is possible to make conclusions, not about welfare weights, but about the optimal cost allocation. To the extent that (5.5.11) is (say) greater than 1 it would be difficult to say with confidence that regulators chose to weight the welfare of one group of consumers over another since an alternative conclusion would be that the regulators chose a sub-optimal cost allocator. Given the fact that regulators do not always attempt to set Ramsey prices either due to lack of knowledge or a lack of technical ability, then the second conclusion that an incorrect cost allocation has been chosen is more applicable.

5.6 Welfare and Passive Regulation

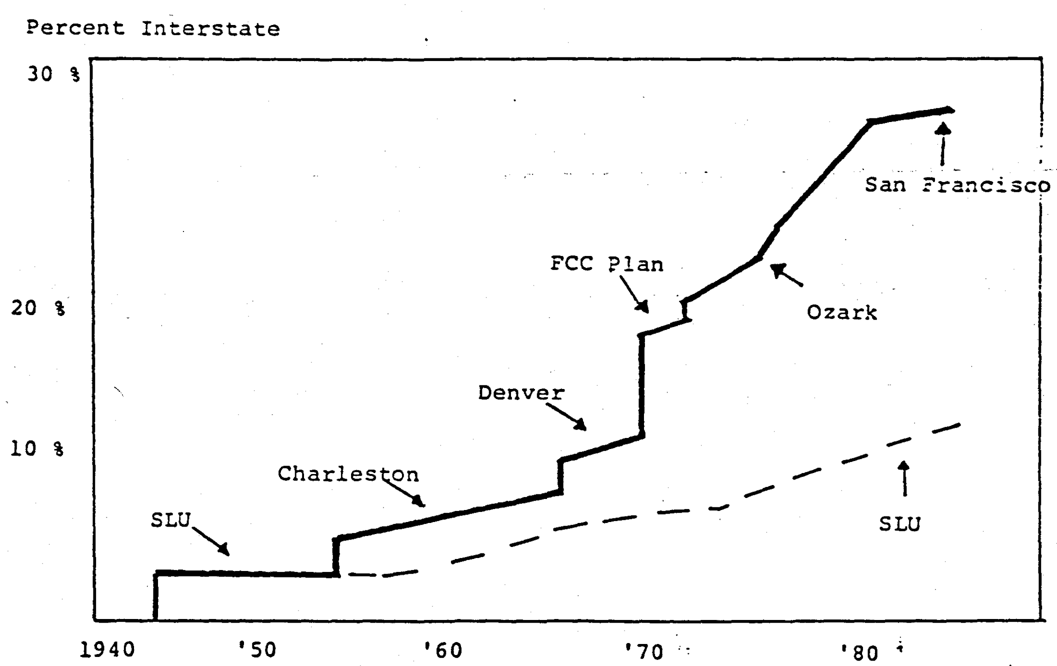
As mentioned above, the FCC and state public service commissions have used the cost allocation mechanisms to transfer rents between local and toll jurisdictions. This regulatory behavior is not the traditional rate base regulation found in the Averch-Johnson line of models. In fact, the behavior is more like that modeled by Joskow (1974). Joskow hypothesized that regulatory agencies seek to maximize their utility by minimizing conflict and criticism appearing in the regulatory environment. Consumer groups, legislators, investors, and the regulated industries themselves generate signals to the regulators to which the regulators respond. Regulation, under Joskow's approach, is characterized by an "equilibrium relationship" where regulatory procedures and instruments are well defined and are used repetitively and predictably.

Joskow hypothesized that during the 1950's and 1960's the regulation of the electric utility industry was in this equilibrium mode as regulators were most concerned with keeping nominal prices from increasing. Electricity prices declined as costs declined. Firms which could increase their profitability without increasing prices or allowing overall profits to go above a "reasonable" level were not subjected to strict rate base regulation. Rate decreases were regularly allowed without investigation, and it was only when a firm asked for a rate increase did the regulators inquire into the firm's costs and revenues. Thus, the regulation was passive and firms were permitted to earn almost any rate of return as long as it did not go above an "unreasonable" amount (Hope Natural Gas, 194). So, there was no "allowed" rate of return which regulators forced the companies to earn.

Although Joskow only used examples of the electric industry to illustrate his theory, the telecommunications industry also faced similar passive regulation. Local companies were not subject to strict scrutiny unless asking for rate increases. Regulators never forced the local companies to give back excess earnings until very recently. Rate reviews were only called for when the local company asked to raise its rates. It was only when enough states had companies asking for rate increases was the cost allocation between local and toll calls altered so that the local company rates no longer needed an increase. Costs were shifted from the intrastate jurisdiction to the interstate jurisdiction through the allocation change to avoid increases in nominal prices.

This model is consistent with the joint state-federal regulation of the telephone industry. It should be noted that this joint regulation was not really an amicable arrangement. The states and federal government each had different constituencies. Although not as passive as the states, the FCC negotiated with AT&T to reduce rates from time to time. The state governments resented these rate reductions because it made the rates for interstate toll calls cheaper than those placed within-state. This had the effect of increasing the criticism the state regulators received. The state utility regulators banded together to fight the FCC and wrestle away some of the FCC's jurisdiction. In order to avoid losing the fight, the FCC (to some extent) allowed the states (in the form of the National Association of Regulated Utility Commissioners) some say over how costs are allocated (Temin and Peters 1985). Table 5.3 shows the cost allocation to the interstate jurisdiction over time. Since 1947, the cost allocation of the joint plant to the interstate jurisdiction has increased from approximately 5 percent to 25 percent in 1984. Gabel (1967) cites numerous examples throughout his monograph of changes in the cost allocation supposedly to stave off local rate increases.

If the Joskow approach is valid and the cost allocation was altered to keep local rates from increasing, the explanation as to why regulators were able to shift these costs with apparent impunity were twofold. First, costs were falling in the long distance market due to a combination of economies of scale and technological change. Regulators could "tap" the rents from the long distance market (without raising nominal prices in the long distance market) and apply them to the local jurisdiction to keep rates from increasing. Second, there were no long distance telecommunications substitutes, so long distance consumers were not able to change telecommunications suppliers.



Source: Leggette (1985).
AT+T

Table 5.3
Amount of NTS Allocated to Interstate over Time

If we assume the firm maximizes profit subject to some type of profit constraint like average cost plus a mark-up. It could even be the traditional AJ type of regulation described in the models in section 5.2. The only necessary assumption is that the regulator only acts to change the cost allocation formula when one of the firm's divisions meets a binding regulatory constraint and thus comes before the commission to ask for a change in the allowed rate of return. If the constraint becomes binding the firm must get approval from the regulatory commission to increase its rates. But, instead of granting a increase in the division's mark-up, the regulators shift some of the burden of the joint cost to the other jurisdiction.

The regulators problem can be examined under two sets of regulatory behavior. The first is when regulators concern themselves only with the effects of their policies on the consumer. The second way of looking at this problem is when the regulators, in addition to being concerned about the consumers surplus, are also concerned about the firms profits.

For the first approach, assume that there are two classes of customers toll (T) and local (L). Local customers only purchase good 2 which is local service and toll customers only purchase good 1 which is toll service. The regulator is thus concerned soley with the effect of changing the cost allocation scheme on the sum of the consumer's surplus of these two classes of customers (i.e. the regulator perceives the welfare weight on the firms profits to be zero). Let B^L and B^T be the benefit functions for customers of local and toll services respectively. Given that the regulators are going to change α, the maximization takes the following form:

$$\max_{\alpha} W(q_1, q_2, \alpha) = B^T(q_1(\alpha)) + B^L(q_2(\alpha)) \qquad (5.6.1)$$

where W is the social welfare function regulators are trying to maximize. The first order condition is:

$$W_\alpha = \frac{\delta B^T}{\delta q_1}\frac{\delta q_1}{\delta \alpha} + \frac{\delta B^L}{\delta q_2}\frac{\delta q_2}{\delta \alpha} = 0 \qquad (5.6.2)$$

This can be rearranged so that:

$$\frac{\delta B^T}{\delta q_1}\frac{\delta q_1}{d\alpha} = - \frac{\delta B^L}{\delta q_2}\frac{\delta q_2}{\delta \alpha} = 0 \qquad (5.6.3)$$

Equation (5.6.3) says that regulators alter α so that the marginal benefits from consumption of these two goods is equated. As α increases, consumer surplus falls in the toll market and increases in the local market. What this type of regulation does is take some of the surplus from toll consumers and transfer it to local consumers so as to equate the social marginal benefit across markets.

A more realistic view of the regulatory process could be described as follows. Through some objective determination regulators attempt to set the allowed rate of return so that is comparable to similarly situated assets. The firm is thus allowed to set "reasonable" prices for each output and chose its level of joint input. The regulators do not affirmatively act to set rates, and only investigates the firm's revenue and costs when the firm asks for a rate increase. The regulator's response in this model when confronted with a request for a rate increase is to alter the cost allocation between markets rather than allow prices to increase in the local jurisdiction. The regulator's choice variable is thus the cost allocator.

Now by adding the firm's maximized profit function to the social welfare function, and maximizing over the choice of α, the regulator faces the following problem:

$$\max_{\alpha} W(q_1, q_2, \alpha) = B^T(q_1(\alpha)) + B^L(q_2(\alpha)) + \pi^*(q_1(\alpha),\ q_2(\alpha),\ K(\alpha))$$

where $\pi^*(q_1(\alpha), q_2(\alpha), K(\alpha))$ is the firm's maximized profit function (i.e the firm has already chosen outputs and the joint input). For this problem the first order conditions are:

$$W_\alpha = \frac{\delta B^T}{\delta q_1}\frac{dq_1}{d\alpha} + \frac{\delta B^L}{\delta q_2}\frac{dq_2}{d\alpha} + \frac{\delta \pi^*}{\delta q_1}\frac{dq_1}{d\alpha} + \frac{\delta \pi^*}{\delta q_2}\frac{dq_2}{d\alpha} + \frac{\delta \pi^*}{\delta K}\frac{dK}{d\alpha} = 0$$

Rearranging once again this yields:

$$W_\alpha = \left[\frac{\delta B^T}{\delta q_1} + \frac{\delta \pi^*}{\delta q_1}\right]\frac{dq_1}{d\alpha} + \left[\frac{\delta B^L}{\delta q_2} + \frac{\delta \pi^*}{\delta q_2}\right]\frac{dq_2}{d\alpha} + \frac{\delta \pi^*}{\delta K}\frac{dK}{d\alpha} = 0 \qquad (5.6.4)$$

Equation (5.6.4) shows that if a regulator knew that there would be additional effects of changing α on the firm's choice variables, then the regulator would chose α so that equation (5.6.4) is satisfied. Secondly, if regulators did not know about the effect of α on output and capital choice and chose α to maximize the "perceived" welfare using the maximization in equation (5.6.3), then the only way the outcome would be truly welfare maximizing (as defined by equation 5.6.4) would be if the following assumptions were true:

i) $dK/d\alpha = 0$, and

ii) $(\delta\pi/\delta q_1)(dq_1/d\alpha) = -(\delta\pi/\delta q_2)(dq_2/d\alpha)$

In this case regulators assume that capital is insensitive to changes in α and the effect of changing α on profits nets out. These are not implausible assumptions concerning what a regulator would know given the state of regulatory knowledge during the 1940's to the 1970's. Equation (5.6.4) yields a much different solution to the welfare agent's problem than the simple maximization of consumer surplus.

As mentioned above, the reasons why regulators could allocate costs from one jurisdiction to another during the past 40 years was due to the lack of substitutes for certain services and the availability of increasing returns to scale upon which the regulators could use to transfer rents between jurisdictions.

In addition, the structural benefits available to regulators when allocating costs have disappeared with the introduction of toll substitutes and the present existence of constant returns to scale in the toll market. Since 1969 when MCI entered the long distance market the number of substitutes from long distance service provided by AT&T has increased. There is also some evidence that (at least on the over all level) AT&T Long-Lines were operating at a point of constant returns to scale (Meyer: 1980). So, the factors which allowed regulators to shift costs from one jurisdiction to another based soley upon the firm's derived demand considerations are no longer present. Continuation of this type of cost allocation regulation poses additional dangers for regulators and the multiproduct firm as it could tend to undermine the competitiveness of the toll division of the multiproduct firm. This is the state of affairs from which the access charge arrangements must be developed.

5.7 Conclusion

From the first model in section 5.2 we found that a firm could select its own cost allocation (α) to maximize its profits given an average cost constraint. Further, if the the regulators set the allowed rate of return equal to the cost of capital ($s = r$) in both jurisdictions and allowed the firm to chose output in both markets, capital stock, and the cost allocator, the firm would pick a Ramsey price vector.

It was also shown that the firm's choice of the allocator could be influenced by the relative rates of return in each jurisdiction. If a firm was given an exogenous cost allocation mechanism and allowed to maximize profits given the choice of output and capital stock, a solution would exist consistent with the rate of return constraints. Comparative statics were then derived to show how the long run and short run decisions were affected by exogenous changes in the cost allocation mechanism. It was shown that in the long run that a change in the cost allocator could have a positive or negative effect upon the relative output mix and the choice of capital. In the short run the results were more definite as outputs changed in the expected manner when costs were shifted between jurisdictions.

Using Rosses' approach we demonstrated that it was possible to determine the optimal allocator based on just Ramsey price information, and it would be possible to test just how far the actual allocation differed from Ramsey optimality. Finally, using Joskow's approach we examined the welfare aspects of administratively allocating costs. We showed that by making simplifying assumptions concerning the effect of changing the cost allocation on the choice of input and output mix that regulators were ignoring important firm responses. By including the firm's responses into the analysis we show the proper cost allocation decision.