EFFICIENT ESTIMATION OF TRANSLOG PRICE ELASTICITIES: CASE OF INDUSTRIAL ELECTRICITY DEMAND

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ABSTRACT

This note introduces a correction in the Hirschberg-Aigner (H-A) methodology for deriving price elasticities of various time differentiated electric inputs, using estimated cost-share elasticities in a translog formulation. The H-A method leaves out an important component of variance - due to the fact that the estimated cost share elasticities are themselves estimates and are not known quantities. This omission leads to overestimating the significance levels of price elasticities. With the correction, the price elasticity estimates (significant when derived through H-A method) now become insignificant. This points out the need for such a correction when evaluating the importance of computed price elasticity estimates, particularly when these are used in policy-making.

1. Introduction:

Since the passage of PURPA (1978), a key public policy issue has been the usefulness of TOU pricing structures. Several empirical studies, including one by Hirschberg and Aigner (H-A) [7],have tested the responsiveness of industrial customers to new time-of-use (TOU) rate structures. H-A used the translog formulation to derive the price elasticities of various time-differentiated electric inputs. This paper provides a correction to the H-A method for deriving price elasticities from estimated cost share elasticities. The correction greatly reduces the underestimation of variance involved in the H-A method. With the suggested bootstrapping procedures, the significance levels of the estimates obtained are correctly computed.

The plan of this note is as follows: first the econometric problem is explained and analysed. Next, we briefly describe the database. Then we present the results and adduce some concluding remarks.

2. Econometric Analysis:

Following H-A [7] we assume a firm has a production function of the following type:

$$Y = f(K,L,NE,E). \tag{1}$$

where: Y = Output

L = Labour

E = Energy

NE = Non-energy inputs.

Energy is then sub-divided into electric and non-electric inputs. Assuming weak separability in the electric inputs, we can write

$$Y = g(H(x), \theta) \tag{2}$$

where H = electricity input function

θ = vector of all other inputs

x = vector of time differentiated electricity inputs.

This framework implies a two-step optimisation: first, determine total electricity cost as a function of level of output and prices of all other inputs. Next, allocate "electricity consumption" (kilowatt-hours) and "demand" (kilowatts) by time-of-use as a function of total electricity cost and time-of-use pricing structure.

Using the duality of cost and production functions, they write

$$c = c(P, z, E) \tag{3}$$

where c = total electricity cost

p = vector of prices of various electric inputs

z = vector of exogenous factors such as weather, prices of other inputs

E = total energy consumed.

Using, Shephard's Lemma, one obtains the input demand functions for various electricity inputs as:

$$\frac{\partial C_i}{\partial P_i} = x_i \quad \text{and } i \varepsilon I \tag{4}$$

where x_i = cost minimizing level of electric input i

I = set of all electric inputs

The functional form that is used is the translog function.

$$\ln C = a + \sum_i \alpha_i \ln P_i + \tfrac{1}{2} \sum_i \sum_j \beta_{ij} \ln P_i \ln P_j + \sum_i \gamma_i P_i \ln E + \psi \ln E \tag{5}$$

where:

$$\sum_i \alpha_i = 1 \quad \sum_j \beta_{ij} = 0 \ \forall_i \quad \sum_i \beta_{ik} = 0 \ \forall_j \quad \sum_i \gamma_i = 0 \tag{6}$$

$$\beta_{ij} = \beta_{ji} \quad \forall \ i \neq j$$

Equation (6) assumes that the cost function (5) is positive linear homogeneous in the elements of P(the electricity input price vector). The underlying cost function (3) is assumed to be twice differentiable.

Next using Shephard's Lemma, we have

$$\frac{\partial \ln C}{\partial \ln P_i} = (1/C)P_i \frac{\partial C}{\partial P_i} = \frac{P_i X_i}{C} = M_i \tag{7}$$

where M_i = cost share of the ith electric input.

After some manipulation one obtains:

$$M_i = \alpha_i + \sum \beta_{ij} \ln P_j + \gamma_i \ln E \ \forall_i \tag{8}$$

Then, the own-price and cross-price elasticities are estimated from the parameters of the cost share equations in the following way:

own-price elasticity:

$$\eta_{ii} = \frac{\beta_{ij}}{M_i} + M_i - 1 \tag{9}$$

Cross-price elasticity of ith input with respect to the jth input price is:

$$\eta_{ij} = \frac{\beta_{ij}}{M_j} + M_j \qquad \begin{matrix} i \neq j \\ i,j \in I \end{matrix} \tag{10}$$

Since the price elasticities are functions of cost shares, they are variable over observations. So, in estimating the standard error of η_{ij} (price elasticities) we have to take into account the fact that the β_{ij} (cost share elasticity) is itself an estimate rather than a known quantity. In ignoring this aspect, H-A obtain standard errors that are not correct. Following Anderson and Thursby [1], H-A hold that computation at mean cost share is more likely to result in normally distributed price elasticities than at any other observation. However, it should be noted that given the access to computers it is better to check out the distribution by repeated sampling techniques, such as bootstrapping, and thereby derive correct standard errors. This adjustment is particularly important if computed elasticities are borderline cases on significance level tests.

We do bootstrapping (see Efron [3,4]) for η_{ii} estimates and generate a distribution for η_{ii} and the "right" standard errors. The basic problem is, when we get η_i from β_i values, we have to correct for β_{ij}'s characteristic as an "estimate", in the sense that we are getting values of η_{ii} and η_{ij} based on one value of β_{ij}'s. So to control for that variance, we run our system regression several times.

In particular, we had a sample size of 90 observations. From that we estimated the system $\underline{y} = \underline{f}(x,\beta)$ and $\underline{\beta}$. Next, we construct the residuals

$$\hat{\underline{u}} = \underline{y} - \underline{f}(x,\hat{\beta}).$$

Next, we take 50 drawings of 90 values of $\hat{u}$ in each selection with replacement and get sets of $\hat{\hat{u}}$. Next, construct $\hat{\hat{\underline{y}}} = \underline{f}(x,\hat{\beta}) + \hat{\underline{u}}$. Then estimate the model system using the artificial data generated:

$$\hat{\hat{\underline{y}}} = \underline{f}(x,\beta) + \underline{u}.$$

Get $\hat{\hat{\beta}}$'s from the above series of system estimation and we will get $\hat{\hat{\eta}}$ from these values. So then, we will have a set of 50 $\hat{\hat{\eta}}$. Next, we compute the mean η as usual and the right standard error as:

$$s.d_{\eta} = \sum_{i=1}^{50} \frac{(\eta_i - \eta)^2}{49} \tag{11}$$

This procedure controls for the unknown distribution of η and the "estimate" characteristic of estimated cost-share elasticities.

3. Data:

The data used were provided by Load Research Group at Florida Power and Light, Miami, Florida. The overall database included the electricity consumption of 22 companies belonging to GSLDT-3 and CST-3 groups (companies with over 4,000 KW demands). Due to data problems with some companies, we included only 6 companies and restricted ourselves only to GSLDT-3 class for this study. This still provided 96 observations of which we left out about 6 observations due to some missing components. Also for some (eg., KWH) the data were from rate and revenue systems while others were from load research systems. To the extent that there are discrepancies of this sort, our results will be affected. Also, exact start dates and closing dates of billing periods for each company were not known; to that extent, a further errors-in-variables problem creeps into our results. While there were several price variations in energy charges, we had only one price variation in the demand charges. This point also has to be noted when we consider the results. The time series used was from September 1982 to December 1983. As this is a pooled cross section time series dataset, like H-A, we have a error-components problem; but following them, we also abstract from it; the essential point of this paper is unaffected.

4. Results:

The findings are presented in two sets. In both sets, the cost elasticities are computed using Zellner's SURE method [10] and the price elasticities reported are derived using the formula described in section 2. Table 1 presents computed price elasticities and t-statistics following H-A. The results are generally similar to results obtained by H-A, although the specific coefficients are quite different. The various elasticities are significantly positive in sign, a finding that corroborates the H-A study. As H-A point out, this situation might be the result of several factors such as misspecification of cost function; the equation may omit some pertinent explanatory variables such as the prices of other inputs. A key difference between the H-A study and this is that the values of various elasticities are relatively high.

The relevant elasticities are presented in the following format. The entries along the diagonal are own price elasticities while those off the diagonal are cross-price elasticities. It can be seen that the elasticities have the wrong sign, and more notably they are quite significant at 1% and 5% levels of significance. The t-statistics are given below each entry in parenthesis. As an example of how the table should be read, the own-price elasticity of peak energy is 0.60, while cross-price elasticity of peak energy with respect to off-peak energy price is -0.50.

In Table 2 we present the standard errors computed by using the bootstrapping technique alluded to earlier. It can be seen that the values for various standard errors are different. Notably, we can see that the value for own-price elasticity of peak energy which was significant in Table 1 obtained using the H-A methodology is now not significant. In such cases, Efron [3,4] suggests a "bias correction" be attempted in the following way. Let us call the point estimates noted in Table 1 as $\hat{\eta}$. Further more call the estimate obtained by bootstrap replications as $\bar{\eta}$. Then the bias is defined as:

$$\text{bias} = (\bar{\eta}-\hat{\eta}).$$

Then the "corrected" estimate Efron says is $\hat{\eta}-(\bar{\eta}-\hat{\eta}) = 2\hat{\eta}-\bar{\eta}$. The bias corrected estimates are noted on top of each entry in Table 2.

We observe as a general point that the elasticity values are all less than one suggesting that the response to TOU rates is quite inelastic. Specifically, the values are relatively higher than obtained in other studies and the characteristics are changed when "bootstrapping" is attempted. While the estimate for own-price elasticity of peak energy becomes insignificant with bootstrapping, the own-price elasticity of off peak demand becomes significant. Also when the estimates are "bias corrected" a lá Efron, we see that while peak energy and peak demand register an increase in their values from .60 to .79 and .32 to .53 respectively; off-peak energy and off peak register a decrease in their values. The results for the various cross-price elasticities are also mixed, as can be readily checked by reading Table 2.

So we observe that significance of estimates obtained is sensitive to the way the standard errors are derived. To the extent that the "bootstrap" standard errors are closer to the "true" errors (see Finke and Theil [5]), we should use this method to arrive at conclusions regarding the significance of various estimates.

Table 1

Zellner's SURE method without bootstrapping

	Peak Energy	Off-Peak Energy	Peak Demand	Off-Peak Demand
Peak Energy	0.60 (2.40)	-0.44 (-2.58)	-0.27 (-2.25)	-0.36 (-4.50)
Off-Peak Energy	-0.20 (4.00)	0.36 (6.00	-0.02 (-0.66)	-0.06 (-1.50)
Peak Demand	-0.23 (-2.87)	-0.04 (-0.80)	0.32 (2.66)	0.01 (-0.33)
Off-Peak Demand	0.24 (6.00)	0.21 (2.62)	0.01 (0.25)	0.29 (1.70)

Table 2

Zellner's SURE method with bootstrapping

	Peak Energy	Off-peak Energy	Peak Demand	Off-peak Demand
Peak Energy	[0.79] 0.41 (1.57)	[-0.16] 0.72 (-4.00)	[-0.48] -0.06 (-0.28)	[-1.15] 0.43 (3.90)
Off-peak Energy	(-0.06) -0.34 (-4.25)	(0.24) 0.48 (4.80)	(0.01) -0.05 (-0.55)	(-0.11) -0.01 (-2.00)
Peak Demand	(-0.36) -0.05 (-0.27)	(0.02) -0.10 (-0.58)	(0.53) 0.11 (0.61)	(-0.08) 0.10 (1.00)
Off-peak Demand	(-0.06) 0.58 (3.86)	(0.47) -0.05 (-0.35)	(0.14) 0.16 (1.06)	(0.26) 0.32 (2.90)

Note: [bias corrected estimates]
(t-statistics)

Concluding Remarks:

This note set out to introduce a correction in the H-A methodology deriving price elasticities of various time differentiated electric inputs from estimated cost-share elasticities in a translog formulation. It was seen how H-A method leaves out an important component of variance due to the fact that the estimated cost share elasticities are themselves estimates and are not known quantities, thereby yielding overestimates of the significance levels of price elasticities. After the introduction of the correction, it was found that the price elasticity estimates (significant when derived through H-A method) now become insignificant. This points out the need for such a correction when evaluating the importance of price elasticity estimates computed price, particularly when these are used in policy-making.

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