The Identification of Predatory Behavior in the Presence of Uncertainty

Sanford V. Berg
Associate Professor of Economics
and Executive Director - Public
Utility Research Center,
University of Florida

&

Richard E. Romano*
Assistant Professor of Economics
University of Florida

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Abstract

Out of the ongoing debate on what constitutes predatory behavior has surfaced the idea that it must entail a sacrifice of profits. The notion is that behavior which sacrifices profits must have an ulterior motive, namely an attempt to induce exit by rivals. This note makes the point that when incumbents must make choices before the resolution of uncertain events, a policy which sacrifices expected profits could have as an ulterior motive risk reduction. The issues surrounding the identification of predatory behavior in the presence of uncertainty are discussed and a definition of predation which allows for uncertainty is presented.

Cost based tests of predatory behavior constitute an attempt to establish intent to monopolize. The basis of such tests is that a policy choice of an incumbent which sacrifices profits under the assumption of viability of rivals[1] must have an ulterior motive namely rival exit inducement. Practical cost based tests of predation delimit (hopefully) observable conditions which are sufficient to demonstrate policy choices which are inconsistent with profit maximization. For one, the Areeda-Turner (AT) test for predatory pricing is the observation of price below average variable cost, clearly a non-profitable pursuit from the standpoint of static profit maximization. Under the pure form of the AT rule, for the simplest text book case of a price taker, we would compare price to marginal cost. More generally, the ideal test would compare marginal revenue to marginal cost. The Ordover-Willig (OW) definition of economic predation extends the principle of cost based tests to more complex settings, including innovation and multiproduct price choices. We do not here wish to discuss the issue of whether cost based tests are tantamount to social efficiency tests. We do, however, maintain (as do OW and AT) that showing intent to monopolize must be a necessary condition for the identification of predatory behavior. This note demonstrates that under conditions of uncertainty profit sacrifice (under rival viability) can have as an ulterior motive risk reduction (under viability) so that cost based tests can fail to establish intent. Put differently, when uncertainty is present, the correct form of the test is utility based not cost based.

Specifically, this paper makes and examines the following points. (i) In an economic environment beset with uncertainty where ex ante policy choices are required of incumbents, one must carefully derive ex post inferences concerning those policy choices. For example, observing price below marginal cost (or average variable cost) might not signal predatory intent. (ii) A risk-neutral firm will pursue policies which maximize expected profits, but this need not imply a stationary setting that, for example, price will usually equal marginal cost. In a stationary setting the policy choices of a non-predatory, risk-neutral firm will (asymptotically) on (arithmetic) average lead to ex post observations of maximized profits (under viability of rivals). (iii) In general, the above will not be true for observations with respect to non-predatory choices of non-neutral firms. As is well known the optimal (non-predatory) policy choice of a non-neutral firm will diverge from the expected (non-predatory) profit maximizing policy choice, to the extent the firm faces a trade off between expected profits and risk levels. We deal here primarily with behavioral aspects associated with risk aversion since empirically it appears to be the norm. (iv) The nature of policies which reduce risk is fairly complex and case specific, so that where uncertainty is present, the burden on the fact finder in establishing intent is increased substantially. (v) The issues surrounding establishment of predatory intent using a utility basis are at least theoretically transparent, and useful practical prescriptions can be derived from our model.

This note proceeds as follows. Section II further discusses points (i) thru (iv) and makes use of examples to illuminate these issues. Section III proposes the correct rule to establish predatory intent under uncertainty and discusses how such a rule might be implemented. An appendix contains some details.

Section II

The general problem with which we are concerned involves a firm in an uncertain environment which must choose one or more ex ante controls in the sense of Leland.[2] Accepting the axioms of expected utility theory, the choice problem of the firm is to maximize his von Neumann-Morgenstern utility function, $U(\Pi)$, which is a function of random profits, with respect to the ex ante controls. In general, whenever the ex ante controls impact the distribution of profit, and the firm is a non-risk-neutral, the optimal control set differs from that which maximizes expected profits. We consider several examples of such a problem which highlight our concerns.

Consider the traditional dominant firm price leadership model, where for price, $p \varepsilon (0, \bar{p}]$, market demand is $X(p)$ where $X'(p) < 0$ and $2X'(p) + pX''(p) < 0$. We introduce uncertainty into this model in the aggregate fringe's supply curve, $X_f(\cdot)$. In particular, let

$$X_f(p, \varepsilon) = X_f(p) \cdot \varepsilon \qquad (1)$$

where $\varepsilon \geq 0$ is random, with $\varepsilon \sim dG(\varepsilon) = g(\varepsilon)$, $E[\varepsilon] = 1$, and $V[\varepsilon] = \sigma^2$.[3] $X_f(p)$ is assumed to have the properties $X_f(p)$, $X_f'(p)$, and $2X_f'(p) + pX_f''(p) > 0$ for $p \varepsilon (p_0, \infty)$ where $p_0 < \bar{p}$; $X_f(p) = 0$ for $p \leq p_0$; and $X_f'(p_0) = 0$. This last assumption facilitates analysis since it renders a continuously differentiable residual marginal revenue function, i.e. avoids the kink in the standard analysis, and compromises results only in that at some points strict inequalities would become nonstrict.[4]

By assumption the nature of the uncertainty requires that the dominant firm must choose a price before realization of ε, and then satisfy residual demand, $D(p, \varepsilon)$. We have

$$D(p, \varepsilon) = \begin{cases} X(p) - X_f(p)\varepsilon & \text{for } 0 \leq \varepsilon \leq \dfrac{X(p)}{X_f(p)}, \text{ and} \\ 0 & \text{for } \dfrac{X(p)}{X_f(p)} \leq \varepsilon \end{cases} \qquad (2)$$

If at the optimal price choice there is any possibility the fringe will completely supply (or over-supply) the market, residual demand is zero, so that the lower branch of $D(p, \varepsilon)$ in (2) can be relevant.

Define the deterministic analog to this problem to be the situation where $\varepsilon = 1$ with certainty. Let us assume $g(\varepsilon)$ is single peaked and symmetric, implying the median of ε equals the mean, so that the deterministic analog not only has the property that fringe output is the expected output under the uncertain case but also the output we would most frequently observe.[5] Letting $C(\cdot)$ be the total cost of the dominant firm (with the usual properties) we find the optimal price choice under the deterministic analog, p_d, in

$$\frac{D(p_d, 1)}{D_1(p_d, 1)} + p_d = C'(D(p_d, 1)) \quad , \tag{3}$$

where numerical subscripts refer to respective partials.

Consider now the problem of the risk-neutral dominant firm with constant average cost, c. The manager solves

$$\underset{p}{\text{MAX}}\; E(\Pi) = \underset{p}{\text{MAX}} \int_0^{X(p)/X_f(p)} \{(p-c)(X(p) - X_f(p)\varepsilon)\}\, g(\varepsilon)\, d\varepsilon \quad . \tag{4}$$

It is not hard to show that the associated first order condition can be written

$$\frac{D(p_N, 1)}{D_1(p_N, 1)} + p_N - c = \frac{X_f(p_N) + X_f'(p_N)}{G(X(p_N)/X_f(p_N))\,(X'(p_N) - X_f(p_N))} \cdot \int_{X(p_N)/X_f(p_N)}^{\infty} (1-\varepsilon)\, g(\varepsilon) d\varepsilon \quad , \tag{5}$$

where p_N is the maximizer. Now, if at p_d, there is no probability that $\varepsilon > \frac{X(p_d)}{X_f(p_d)}$ then the right hand side of (5) vanishes and $p_N = p_d$. This is an example of the kind of case where "uncertainty does not matter" in the sense that optimal choices are unchanged by the introduction of uncertainty. Of course, if we were to view the dominant firm's price choice ex post it would "appear" optimal only in the instances where the realization of ε happened to be 1. In any other instances, the choice would not "appear" optimal and would violate the principle of the cost based tests for predatory intent, though in this case the AT test would never be violated. This illustrates point (i).

Consider the situations where $X(p_d) < 2X_f(p_d)$ so that under our assumptions regarding the distribution of ε it is possible that at p_d the realization of residual demand will be zero. Here we find $p_N > p_d$. This follows since the right hand side of (5) must be positive and our assumptions regarding $X(p)$ and $X_f(p)$ insure the left hand side is an increasing function in p. (These are left to the reader to show.) This case illustrates point (ii). Here an observer would find that ex post the price choice would differ on average (i.e. would error on one side more often) from that price which equates realized residual marginal revenue to marginal cost. An outside observer might mistakenly infer the dominant firm is not learning from previous mistakes or is "up to something." The truth of the matter is, of course, that the policy does maximize expected profits; if the observer calculates profits ex post no policy will be (asymptotically) superior on (arithmetic) average. We can explain the seeming paradox by the lower bound of zero on the dominant firm's profits.[6]

Now consider the choice problem of the risk averse dominant firm. He solves

$$\max_{p} E[U(\Pi(p))] \quad , \tag{6}$$

and the first order condition can be written (see, for example, Blair)

$$E[\Pi'(p_A)] = -\frac{COV[U'(\Pi), \Pi'(p_A)]}{E[U'(\Pi)]} , \tag{7}$$

where p_A is the optimal price choice. We can sign the right hand side of (7) as follows. By the assumption of risk aversion $U''(\Pi) < 0$ always, and we find easily that $\frac{\partial \Pi}{\partial \varepsilon} \geq 0$, with strict inequality for some values of ε. It is straightforward to show $\frac{\partial \Pi'(p)}{\partial \varepsilon} \leq 0$, also with strict inequality for some values of ε. Combining these results we have that the covariance term in (7) is positive. Since $U'(\Pi) > 0$ always, its expectation is positive so the right hand side of (7) is negative. Finally, we find that $\Pi''(p) \leq 0$ with strict inequality for some values, so that the expectation of $\Pi'(p)$ is decreasing in p. The implication is that the price choice under risk aversion is less than under risk neutrality, i.e. $p_A < p_N$. This follows since $E[\Pi'(p_N)] = 0$ establishes p_N.

This case illustrates point (iii). Now, in a stationary setting, p_A would not have the property of being the single price choice which would (asymptotically) maximize the (arithmetic) average of the profit realizations. Put differently we could find price choices which would on average yield higher ex post profits. Indeed this price choice then violates the principle of the cost based rules, though again not in this case the AT rule.[7] Of course, the dominant firm is simply trading off some expected profits for risk reduction; a lower price reduces the variability of random profits. One can conceive of the following scenario. An equilibrium is disrupted by the entry of one or more new fringe members, which shifts out the expectation of the fringe supply, but also increases the variance. The dominant firm then cuts price substantially, not only in reaction to the new members but also in reaction to the increased variability of fringe supply. Conceivably this price cut might drive one or more of the marginal fringe members out of the industry, and a predatory pricing suit might be instigated against the dominant firm who is innocent of predatory intent.

In the context of a single product firm, what may serve to best make our point, would be to show a case where ex post we observe price less than marginal cost. If we wish to extend the model we have thus far employed to do this, we must recall that we have assumed quantity to be an ex post control of the dominant firm. The implication of this assumption is that the non-predatory firm need never choose a quantity so that marginal cost exceeds the posted price. One way to generate the result is to assume goodwill on the part of the dominant firm in that he commits himself to a policy of satisfying all residual demand at his posted price even if this requires the taking of losses on the marginal units. Alternatively one could assume simply that quantity is an ex ante control and price adjusts to clear the market, but this would entail a different conception of a dominant firm. Here we discuss the former.

The details of this case where the dominant firm's cost is a quadratic are presented in Part I of the Appendix. Here we only indicate the results. The effect on the price choice of the risk neutral dominant firm of the combination of increasing marginal cost and goodwill is to reduce the optimal price choice below the deterministic analog's price.[8] The optimal price choice, p^*, can be such that

$$C'(X(p^*) - \underset{\varepsilon}{\mathrm{MIN}}\, X_f(p^*)\varepsilon) > p^* \quad . \tag{8}$$

That is, when residual demand is at its maximum, marginal cost exceeds price. Thus, the 'pure form' of the AT rule can be violated.[9]

One can extend our dominant firm pricing model quite easily to a multi-product setting to show how more complex cost based rules can go astray. Let the dominant firm now produce another product X_2 with cross elastic demand with the original product (subscripted by 1). Let the dominant firm be the only producer of X_2 and suppose market demands are certain and given by $X_1(p_1, p_2)$ and $X_2(p_1, p_2)$. As before the dominant firm's behavior in market 1 is constrained by a competitive fringe and their supply is $X_f(p_1)\varepsilon$ with the same properties. For simplicity we consider the case where the dominant firm has constant average cost of production in each market, equal to c_i $(i=1, 2)$. Then random profits of the dominant firm are

$$\Pi(p_1, p_2) = (p_1-c_1)(X_1(p_1, p_2) - X_f(p_1)\varepsilon) + (p_2-c_2)X_2(p_1, p_2) \quad , \qquad (9)$$

where we assume, for simplicity, that over the relevant domain of Π, there is no probability the fringe will completely supply the market for X_1. Assuming Π is concave (see the Appendix) the solution (p_1^*, p_2^*) to the following system then constitutes the optimal prices under the deterministic analog and under risk neutrality.

$$p_1^* + \frac{X_1(p_1^*, p_2^*) - X_f(p_1^*)}{\frac{\partial X_1}{\partial p_1}(p_1^*, p_2^*) - X_f'(p_1^*)} = c_1 - \frac{(p_2^*-c_2)\frac{\partial X_2}{\partial p_1}(p_1^*, p_2^*)}{\frac{\partial X_1}{\partial p_1}(p_1^*, p_2^*) - X_f'(p_1^*)} \quad . \qquad (10.1)$$

$$p_2^* + \frac{X_2(p_1^*, p_2^*)}{\frac{\partial X_2}{\partial p_2}(p_1^*, p_2^*)} = c_2 - \frac{(p_1^*-c_1^*)\frac{\partial X_1}{\partial p_2}(p_1^*, p_2^*)}{\frac{\partial X_2}{\partial p_2}(p_1^*, p_2^*)} \quad . \qquad (10.2)$$

Of course, the second terms on the right hand side of (10.1) and (10.2) account for the cross elasticity effects. For example, if X_1 and X_2 are substitutes, these terms represent the marginal cost of increasing output associated with the lost sales of the other product.

Now consider the problem if the dominant firm is risk averse. The optimal price vector then satisfies the system

$$E\left[\frac{\partial\Pi(p_{1A}, p_{2A})}{\partial p_1}\right] = -\frac{COV[U'(\Pi(p_{1A}, p_{2A})), \frac{\partial\Pi}{\partial p_1}(p_{1A}, p_{2A})]}{E[U'(\Pi(p_{1A}, p_{2A}))]} \quad , \qquad (11.1)$$

and

$$\frac{\partial\Pi(p_{1A}, p_{2A})}{\partial p_2} = 0 \quad . \qquad (11.2)$$

In Part II of the Appendix it is shown that $p_{1A} < p_1^*$. The optimal price vector choice $[p_{1A}, p_{2A}]$ does not have the property that (asymptotically) the arithmetic average of expost profits will be maximized. Then this price vector violates the spirit of the OW approach to defining predatory behavior for multiproduct firms.[10] Moreover, since the price chosen by the risk averter is lower than the expected profit maximizing choice, this example is of the class where predatory suits are likely, since marginal competitors might be driven from the market. A scenario to consider is where the dominant firm develops X_2, a substitute for X_1, and in introducing it lowers the price of the latter.

We close this section with a discussion of two points. The examples we have presented have in common two characteristics. For one, the ex ante control of the firm has been price or a price vector. Like the concept of predation (and should be rules to identify it), our points concerning uncertainty apply to other policy instruments. OW describe how R&D investments should enter into cost based tests of predation. We have developed models where risk aversion leads to "excessive" R&D outlays, i.e. outlays which exceed the expected profit maximizing outlay. Other instruments which should be considered include capital expenditures, marketing expenditures, and advertising outlays. The second common strain of our examples is that risk aversion has led to behavior which can stimulate predatory suits, in particular, in all our examples risk aversion leads to a lowering or price. One cannot, however, infer that risk aversion generally leads to choices which damage competitors. The effect of risk aversion is, unfortunately, highly case specific. For example, we have developed a model with capacity choice as ex ante control under uncertain demand, where risk aversion induces less investment in capacity than under risk neutrality, implying a lower probability of the observation of excess capacity in the former case. We say more regarding this general issue below.

Section III

We consider this note an extension of the work of AT and OW, not a criticism. We agree with the principle of OW that predatory intent must be associated with non-optimal choice under rivals' viability. Here we propose a necessary condition for the identification of predatory intent which allows for uncertainty.

The general problem of a firm in its choice of an exante control I is to

$$\max_{I} E[U(w + \tilde{\Pi}(I))] \quad , \tag{12}$$

where w is the firm's initial equity and $\tilde{\Pi}$ random profits. (At this point it is useful to identify random variables with a '~'.) Following Pratt we implicitly define the risk premium, θ in

$$E[U(w + \tilde{\Pi}(I))] \equiv U(w + E(\tilde{\Pi}) - \theta) \quad , \tag{13}$$

where θ is a function of w, and depends on the distribution of $\tilde{\Pi}$ and thus I. θ has the interpretation as the amount the decision maker would pay to receive the expected value of random profits with certainty as opposed to taking the risk.[11] Following Pratt, using a Taylor expansion we find

$$\theta = \tfrac{1}{2}\, \sigma_{\Pi}^{2}\, r(w + E(\tilde{\Pi})) + O(\sigma_{\Pi}^{2}) \quad , \tag{14}$$

where $r(Z) \equiv -\frac{\mu''(Z)}{\mu'(Z)}$ is Pratt's measure of risk averseness, σ_{Π}^2 is the variance of profits, and $0(\sigma_{\Pi}^2)$ is a sum of terms of smaller order than σ_{Π}^2, in particular a linear combination of central moments greater than two.

If we are willing to make the assumptions that r is constant, i.e. of constant absolute risk aversion, and that central moments greater than two are small, then the first order condition associated with (12) can be written

$$E\left[\frac{\partial\tilde{\Pi}}{\partial I}\right] = \tfrac{1}{2}r\frac{\partial\sigma_{\Pi}^2}{\partial I} \quad . \tag{15}$$

In words (15) indicates that for optimality, I ought to be expanded until the expected incremental profit increase (decrease) equals the marginal valuation of the associated increase (decrease) in risk. If, for example, I is R&D investment, and increased R&D investment leads to a decreased variance in profits, then the risk averter optimally expands its R&D outlays beyond where expected profits are maximized, namely to the point where the marginal loss in expected profits is just compensated for by the valuation of the risk reduction.

A violation of (15) constitutes a necessary condition for predatory behavior where $\tilde{\Pi}$ is taken to be random profits under the assumed viability of rivals. We advocate an amending of the OW approach, by requiring of predation than <u>a firm pursues a policy choice which sacrifices utility calculated under the presumed viability of its rivals</u>.

As any reader is well aware, implementation of the modified rule requires substantial information. To become fully informed, a fact finder would need to (i) ascertain the ex ante perceptions of uncertainty of the firm, (ii) understand how the policy instruments in question are perceived to affect uncertainty, and (iii) ascertain the firm's valuation of risk reduction. Of course, complete knowledge is not always necessary for correct evaluations. For example, if a defendant claims a price reduction was motivated out of an attempt to mitigate risk, but there is no logical connection between price and risk, the claim is probably false. The logical order for the fact finder to proceed in gathering information is as the information types are listed. If there is no reason to believe uncertainty is important in the policy choice in question, the fact finder need go no further. One important set of cases would be when the controls in question need not be chosen before resolution of the uncertainty. (See the caveat in supra note 2 on this.)

Understanding the relationship between a policy instrument and risk requires a case by case approach. We have no general prescriptions here but view this step as the least problematic.

If the problem goes beyond the latter stage, an estimate of the firm's valuation of risk reduction is needed. Issues regarding the nature of ownership and firm portfolio diversification arise here. Such considerations are beyond the scope of this note. Implicitly our analysis has focused on the case of a privately owned firm engaged in a single risky activity. In this case, insurance markets can be useful in estimating a firm's valuation of risk reduction.[12] In sum, the information requirements are weighty and more research is required to develop standards which can be readily implemented by the courts.

Appendix

Part I

In the first part of this appendix we analyze the behavior of a risk-neutral dominant firm with quadratic costs. It is shown that if the dominant firm must satisfy all residual demand at its posted price, it is possible that marginal cost will exceed price. This behavior can be rationalized by the existence of goodwill.

Let

$$C(X_1) = aX_1 + \frac{b}{2} X_1^2 \quad , \tag{A.1}$$

where X_1 is the dominant firm's output and $a,b > 0$. Then the choice problem of the dominant firm under the deterministic analog to the uncertain problem is

$$\underset{p}{\text{MAX}}\ p(X(p) - X_f(p)) - C(X(p) - X_f(p)) \tag{A.2}$$

with first order condition:

$$p_d^* + \frac{D(p_d^*, 1)}{\dfrac{\partial D(p_d^*, 1)}{\partial p_d}} = a + b\, D(p_d^*, 1) \quad . \tag{A.3}$$

Under uncertainty the choice problem of the risk-neutral dominant firm is

$$\underset{p}{\text{MAX}} \int_0^{X(p)/X_f(p)} \{(p-a)(X(p) - X_f(p)\varepsilon) - \frac{b}{2}(X(p) - X_f(p)\varepsilon)^2\}\ g(\varepsilon)\ d\varepsilon \quad . \tag{A.4}$$

If at the optimal solution there is probability zero that the fringe supplies the entire market then the first order condition may be written

$$p_d^{**} + \frac{D(p_d^{**}, 1)}{\frac{\partial D(p_d^{**}, 1)}{\partial p_d}} = a + bD(p_d^{**}, 1) + \frac{bX_f(p_d^{**})X_f'(p_d^{**})\sigma^2}{\frac{\partial D(p_d^{**}, 1)}{\partial p_d}} \quad . \tag{A.5}$$

Noting that the last term on the right hand side of (A.5) is negative by comparison of (A.3) and (A.5) we find that $p_d^{**} < p_d^*$. This result is due to the interaction of the multiplicative error term and the existence of increasing marginal cost. At a fixed price, under increasing marginal cost the dominant firm's profits are concave in ε. For a single peaked symmetric distribution of ε (as we have assumed) the expectation of random profits is decreasing in the variance (in this case). Lowering price lowers the variance in residual demand due to the multiplicative random component of residual demand. Thus an expected profit maximizer chooses a price below p_d^*. (A "median profit maximizer" would choose p_d^*.)

The easiest way to show there is a probability of observing marginal cost in excess of price is with an example. Let the dominant firm's marginal cost be given by $MC_1 = X_1$, market demand by $X = 100 - .1p$, and fringe supply by $X_f = p$. Let ε take on the three values .5, 1, and 1.5 with respective probabilities .3, .4, and .3. We find $p_d^{**} \simeq 60.87$ and $X(p_d^{**}) \simeq 93.91$. In the situations where $\varepsilon = .5$, residual demand is approximately 63.48 which equals the marginal cost of the dominant firm. Thus 30 percent of the time we would expect to observe dominant firm production where marginal cost exceeds price.

Part II

In this part of this appendix we show how (11.1) is evaluated and demonstrate that $p_{1A} < p_1^*$. We assume the sufficiency conditions for strict concavity of $\Pi(p_1, p_2)$ as defined in (9) with $\varepsilon = 1$ are met, namely that

Π_{11}, $\Pi_{22} < 0$ and $\Pi_{11}\Pi_{22} - \Pi_{12}^2 > 0$. Should the price vector and distribution of ε be such that it is possible for the fringe to supply the entire X_1 market, then in these cases (referred to as fringe dominance) the profits of the dominant firm are given by the second term on the right hand side of (9). Consider the right hand side of (11.1). We have $\frac{\partial U'}{\partial \varepsilon} = U'' \frac{\partial \Pi}{\partial \varepsilon} \leq 0$, with strict equality only in the case of fringe dominance. Also $\frac{\partial^2 \Pi}{\partial p_1 \partial \varepsilon} \leq 0$ again with equality only in the case of fringe dominance. These results imply the covariance term in (11.1) is negative, and since $U' > 0$ always, the right hand side of (11.1) is positive.

Now we show $p_1^* > p_{1A}$ by contradiction. Assume the converse. From $\Pi_2(p_1, p_2) = 0$, find the function $p_2 = p_2(p_1)$. Then using (10.1) and (11.1) and the above results we have $\Pi_1(p_1^*, p_2(p_1^*)) = 0$ and $\Pi_1(p_{1A}, p_2(p_{1A})) > 0$. Then it must be that somewhere in the interval $[p_{1A}, p_1^*]$ that $\frac{d\Pi_1}{dp_1} > 0$. We have $\frac{d\Pi_1}{dp_1} = \Pi_{11} + \Pi_{12}p_2'(p_1) = \Pi_{11} + \Pi_{12}(- \frac{\Pi_{12}}{\Pi_{22}} = \frac{\Pi_{11}\Pi_{22} - \Pi_{12}^2}{\Pi_{22}}$. The positivity of this contradicts the concavity conditions, so that $p_1^* > p_{1A}$.

Footnotes

1. That profit sacrifice under the assumption of rivals' viability is necessary for predatory intent is a point made by Ordover and Willig. It is clear that this condition is necessary for the policy choice to be noninnocent. Scheffman critisized Ordover and Willig (we think incorrectly) for not being explicit regarding what it meant by rivals' viability. Rivals' viability is meant to be synonymous with costless reentry. Thus an optimizing incumbent can not engage in predatory behavior unless there are reentry barriers. Ordover and Willig point out that the existence of reentry barriers is another necessary condition for predatory behavior.
2. If a firm commits itself to a policy before the resolution of an uncertain event, and it need not have constrained itself at that early date, this behavior may in itself constitute predatory intent. If it can be shown there is no other potential gain (like providing consumers information) from premature precommitment, predation is indicated.
3. For a fuller exposition of the dominant firm price leadership model with uncertainty in fringe supply see Blair and Romano.
4. See Blair and Romano for exposition of this point.
5. We are being somewhat informal for a continuous ε, but the point is clear.
6. See Blair and Romano for a fuller exposition of this point.
7. Though OW do not establish a specific test for a single product firm, their ideal would compare marginal revenue to marginal cost; here, expected marginal revenue exceeds marginal cost.
8. We have shown this only for the case where the dominant firm's cost is quadratic, ε enters multiplicatively, and the distrition of ε is such that there is no probability of 0 residual demand.

9. Of course, OW would label this behavior as predatory too. No doubt an example could be constructed where the practical form of the AT rule fails due to uncertainty. If the loss on the units produced beyond where price equals marginal cost outweighs the profits (inclusive of all non-sunk costs) on the other units then there is such a violation. This could occur at times when expected profits are positive.

10. Whether OW's actual rule for identifying predatory behavior would be violated or not depends on the parameters of the problem. This is so because their rule is a sufficient condition for failure to maximize profits, so constructed with real information constraints in mind.

11. A risk averter can be defined as a decision maker for whom $\theta > 0$ and a risk seeker as one for whom $\theta < 0$.

12. Suppose there is a probability of $(1-p)$ of an accident which results in damages of an amount Δ. Insurance may be purchased at a rate below the actuarially fair rate, in particular let the net return on a premium of y dollars be $\gamma\frac{p}{1-p}\,y$, where $0 < \gamma < 1$. The decision maker with initial wealth w_0 solves $\underset{y}{\text{MAX}}\ p\,U(w_0 - y) + (1-p)U(w_0 - \Delta + \gamma\frac{p}{1-p}\,y)$. Letting y^* be the maximizer for a decision maker with constant absolute risk aversion as defined by Pratt we find $r = \dfrac{\log \gamma}{y^*\left(\frac{1 - p(1-\gamma)}{1-p}\right) - \Delta}$.

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