MODIFICATIONS TO THE DCF STOCK VALUATION MODEL

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November 1984

PURC Working Paper Series

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In 1938, J. B. Williams [5] developed the concept that a stock's value is determined as the present value of its expected future dividend stream. Following up on Williams' work, Gordon and Shapiro [2] solved the discounted cash flow (DCF) stock valuation model for k, the required rate of return, and then discussed the use of this DCF k as the firm's cost of equity. Gordon and others went on to use the DCF model for many purposes, and versions were developed to deal with situations where the expected future growth rate is variable as well as constant. However, in almost all of the publications we have seen, including the leading textbooks, applications of the DCF model are based on two incorrect assumptions: (1) that dividends are paid annually, whereas they are actually paid quarterly, and (2) that the analysis takes place on a dividend payment date, so that the next dividend will be received exactly one year after the current date.[1]

[1]The dividend payment date problem involves both the ex-dividend date and the actual payment date. In the typical DCF analysis, people generally assume implicitly (1) that we are at the ex-dividend date and (2) that the payment date and the ex-dividend date are the same. This is clearly not true. For example, at the bottom of the Value Line report on BellSouth dated July 27, 1984, it is noted that the stock will go ex dividend on September 24, with payment to be made on November 1. Thus, an investor who buys the stock on September 23 will receive the November 1 dividend, so he or she will have to wait only 38 days before receiving a payment. On the other hand, an investor who buys the stock on September 24 will not receive the November dividend, and he or she will have to wait more than one full quarter before receiving the first dividend payment (37 + 90 = 127 days). The price of a share of stock normally falls by the tax-adjusted present value of the next dividend to be paid when the stock goes ex-dividend. Other things held constant, stock prices rise between ex-dividend dates, decline on the ex-dividend date, and continue to cycle in this manner over time, but with an upward drift to reflect long-term growth.

In a forthcoming paper, Charles M. Linke and J. Kenton Zumwalt [3] took a major step forward by deriving a quarterly DCF model. However, their paper was focused on utilities rather than on firms in general, and they restricted their discussion to the case of a constant growth analysis performed on a dividend payment date. In this paper, we extend the general quarterly DCF model to include both the constant and nonconstant growth cases, and we show the adjustments needed if the analysis is not performed on a payment date. In addition, the DCF analyses performed in this paper are completely general, in that they are applicable to all firms.

The use of a quarterly DCF model has at least two important implications. First, when quarterly dividend payments are taken into account, required rates of return on stocks are significantly higher than those estimated by an otherwise equivalent annual dividend payment model. Second, given the first implication, it is clear that whenever returns on stocks, bonds, T-bills, or any other securities are being compared it is important to convert all returns to a common basis--the <u>effective annual rate</u> or <u>APR</u>.

To illustrate the importance of this point, assume that an analyst is comparing the expected returns on a semiannual payment bond and a share of preferred stock. Both securities have a 13 percent coupon rate and sell at par, and both securities will have "reported yields" of 13 percent. However, the bond's APR is 13.42 percent:

$$APR = (1 + \frac{0.13}{2})^2 - 1.0 = 13.42\%.$$

The preferred stock ($100 par) pays interest of $3.25 each quarter. Using an annual DCF model, the simple annual yield on the preferred would also be 13 percent:

$$\text{Annual yield} = \frac{(\$3.25)(4)}{\$100} = 13.00\%.$$

However, the APR yield on the preferred will be 13.65 percent:

$$\text{APR} = (1 + \frac{0.13}{4})^4 - 1.0 = 13.65\%.$$

If the analyst compared these securities using simple annual rates, he or she would conclude that they both yield 13 percent. However, on an APR basis, the effective return on the preferred stock exceeds that on the bond by 13.65% - 13.42% = 0.23%, or 23 basis points. Thus, when comparing the yields on different types of securities, it is important to put all returns on an APR basis to avoid an apples-to-oranges comparison.

The quarterly DCF model which we develop in this paper provides a method for calculating the APR on a share of stock. Still, the primary benefit of our model is that it provides a generalized DCF framework, which properly accounts for dividend payment patterns and for the actual timing of dividend receipts.

The paper is divided into three sections. First, we discuss DCF models in general. Next, we develop constant and nonconstant growth DCF models, and we demonstrate how they can be used. Finally, we provide a brief summary and restatement of our conclusions.

DCF Models in General

For common stock, the general DCF model may be defined as follows:

$$\text{Value} = P_0 = \frac{D_1}{(1 + k)^1} + \frac{D_2}{(1 + k)^2} + \cdots + \frac{D_\infty}{(1 + k)^\infty}, \tag{1}$$

where P_0 is the current price of the stock, D_t is the expected periodic dividend, and k is the required rate of return. Under a certain set of assumptions, Equation 1 reduces to Equation 2:[1]

$$P_0 = \frac{D_1}{k - g}, \tag{2}$$

which may be solved for k to produce Equation 3, the constant growth/annual payment DCF equation:

$$k = \frac{D_1}{P_0} + g, \tag{3}$$

where D_1 is the dividend expected at the end of the next period and g is the constant expected growth rate for earnings, dividends, and the stock price.

Equations 2 and 3 are correct if and only if (1) the price, P_0, is determined on a dividend payment date, (2) the next dividend, D_1, will be received exactly one period hence, (3) each dividend will increase at the rate g, and (4) g is constant, which implies that both the return on

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[1]For a derivation of this model, see E. F. Brigham, [1, Chapter 5].

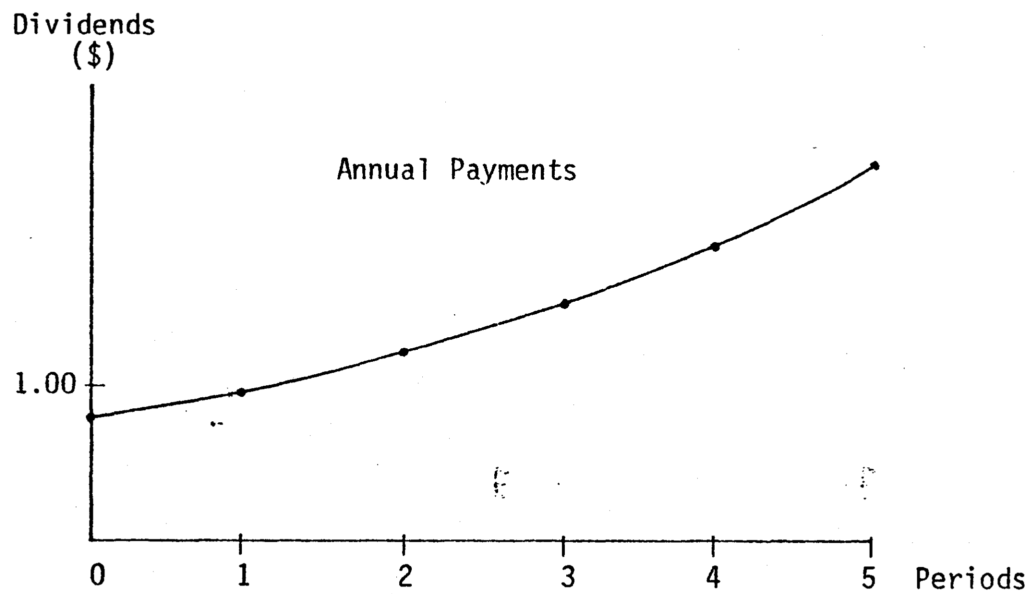
equity (ROE) and the payout ratio are expected to remain constant. The top panel of Figure 1 shows the dividend payment pattern assumed in the constant growth/annual payment model inherent in Equations 2 and 3.

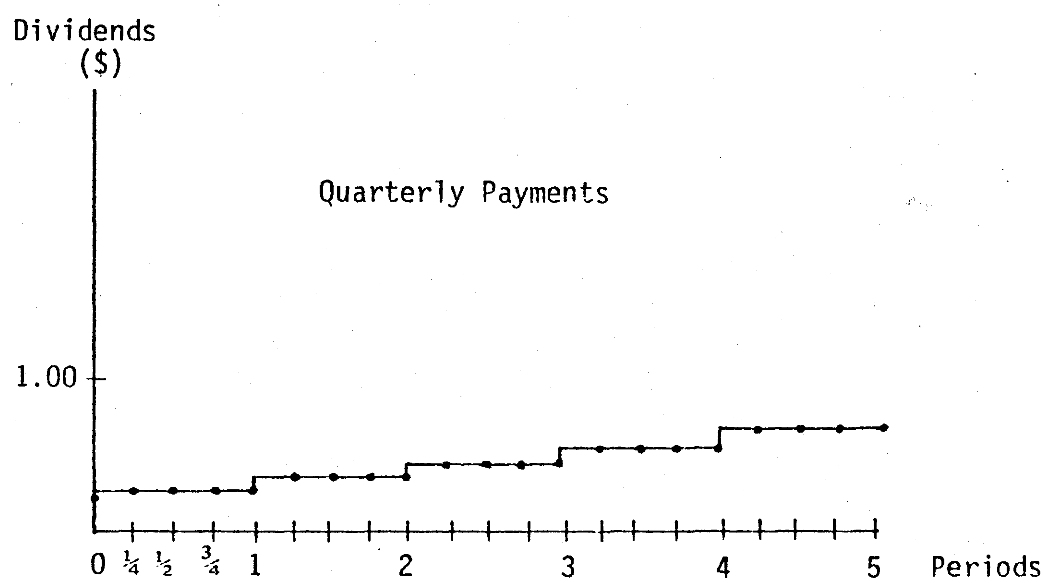
In the real world, we know that dividends are normally paid quarterly and increased once a year as shown in the lower panel of Figure 1. Further, the analysis could take place at times other than a dividend payment date, so the length of time until the next dividend is received, and also until the dividend is increased, could vary from case to case. Finally, the dividend growth rate is not always expected to be constant. For example, a firm in the early stages of its life cycle, or a firm in an industry that is experiencing rapid technological changes, may have an expected growth rate in the short-run that is either higher or lower than the average growth rate expected over the long run. For such a firm, a multi-stage (or nonconstant) DCF model of this form should be used:

$$P_0 = \frac{D_1}{(1 + k)^1} + \frac{D_2}{(1 + k)^2} + \cdots + \frac{D_n}{(1 + k)^n} + \left(\frac{D_n(1 + g)}{k - g}\right)\left(\frac{1}{1 + k}\right)^n. \quad (4)$$

Dividends during Periods 1 to n could be either constant (no growth), growing at a constant rate different from the terminal growth rate, declining, or growing at different rates from year to year. However, the model does assume that all dividends received after Period n will grow at a constant, long-run growth rate, g, so the last term in Equation 4 is the present value of all dividends expected beyond Year n as determined by the constant growth model. To use the nonconstant model, one must obtain specific dividend forecasts for Years 1 to n, as well as

Figure 1


Dividends
($)
Annual Payments
1.00
0
1
2
3
4
5
Periods


Dividends
($)
Quarterly Payments
1.00
0
¼
½
¾
1
2
3
4
5
Periods

an estimate of the long-run, steady-state growth rate, g, and then solve for k, the DCF cost of equity.[1]

Equation 4 is conceptually superior to Equation 2 in situations where there is reason to believe that investors do not expect a constant growth rate, but it is still based on two incorrect assumptions, namely, that dividends are paid annually and that the analysis always occurs on a dividend payment date. These incorrect assumptions can lead to incorrect estimates of the cost of equity.

In terms of practical applications, the annual model is somewhat easier to derive and to use than the quarterly model. Further, even though the quarterly model is theoretically correct, it cannot be easily evaluated except by the use of a computer model. However, although these computer models used to be hard to develop, with a personal computer and an electronic spreadsheet such as Lotus 1-2-3, it is now relatively easy to set up and solve either the constant or the nonconstant quarterly DCF model, and to adjust for pricing at dates other than the dividend payment date. Thus, the procedures we discuss below are not at all difficult to implement in actual practice.

Constant Growth Quarterly DCF Models

Equation 1 above assumes that the next dividend will be paid exactly one period from the analysis date. If the analysis is done between payment dates, with f_1 being the fraction of a period before the

[1]We illustrate Equation 4 later in the paper.

next payment will be made, then we can transform Equation 1 into Equation 5:

$$P_0 = \frac{D_1}{(1 + k)^{f_1}} + \frac{D_2}{(1 + k)^{f_2}} + \cdots + \frac{D_\infty}{(1 + k)^{f_\infty}}. \qquad (5)$$

This equation is completely general, so it can be applied to the case of a company which pays quarterly dividends and increases them once a year. For example, suppose a company's stock sells for \$29.25; it is expected to pay an annual dividend of \$2.60, or \$0.65 per quarter over the coming year; dividends are growing at 7 percent per year; and the company has just paid the last of the quarterly dividends at the old rate. The valuation equation then becomes:

$$\$29.25 = \frac{0.65}{(1 + k)^{.25}} + \frac{0.65}{(1 + k)^{.50}} + \frac{0.65}{(1 + k)^{.75}} + \frac{0.65}{(1 + k)^{1}} + \frac{0.65(1.07)}{(1 + k)^{1.25}} + \cdots.$$

The first exponent, f_1, represents the fraction of the year before a purchaser of the stock will receive his or her first dividend, and in this example we assume that the first dividend will be received at the end of one quarter. Each subsequent exponent is increased by 0.25, that is, $f_{t+1} = f_t + 0.25$.

From a practical viewpoint, there are two problems with Equation 5. First, it is obviously not feasible to extend it on out to infinity and then to solve for k. Second, we might want to determine a company's cost of capital on some date other than the dividend payment date. Fortunately, both problems can be solved by use of the following model, which is derived in Appendix A:

$$k = \frac{D_1(1+k)^{1-f_1} + D_2(1+k)^{1-f_2} + D_3(1+k)^{1-f_3} + D_4(1+k)^{1-f_4}}{P_0} + g. \quad (6)$$

Here

D_1 through D_4 are the next four quarterly dividends expected to be received over the coming year. If the analysis is done during the first quarter of the firm's "dividend year" and before the ex-dividend date, then D_1 through D_4 will be equal. However, if the analysis is done during the second, third, or fourth quarters, then two different quarterly dividend amounts will be reflected in the numerator of Equation 6.

f_1 is the fraction of a year before the first dividend is received, so $1 - f_1$ is the fraction of the year between the time the first dividend is received and the end of the year, the end of the year being 12 months after the analysis date. When we consider ex-dividend dates, f_1 will normally be in the range of 0.10 to 0.35, so $1 - f_1$ will normally range from 0.90 to 0.65.

f_2, f_3, and f_4 are the fractions of the year before the second, third, and fourth dividends are received.[1]

Several points should be noted regarding Equation 6:

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[1]To be exactly correct, this model should use a 365-day year, and the actual number of days between each payment date should be calculated. However, it is much easier to apply the model using a 360-day year and 90-day quarters. Further, our studies indicate that this simplification generally results in an error of less than one basis point.

1. The derivation of Equation 6 from Equation 5, as set forth in Appendix A, is similar to the derivation of Equations 2 and 3 from 1, but there are many more terms in the quarterly model, and hence both the derivation and the final model (Equation 6) are more complex than with the annual model.

2. The annual model assumes that one dividend is paid at the end of the year, so D_1 in the annual model is the sum of the four dividends expected over the next four quarters. However, these four dividends do not have to have the same value. For example, a company might be currently paying an annual dividend of \$2.60, or \$0.65 per quarter, with two of the 65 cent dividends having already been paid. If its expected growth rate is 7 percent, then its annual dividend rate should go to \$2.60(1.07) = \$2.78, which then implies that its quarterly dividend should increase to approximately \$0.70 two quarters hence. Therefore, D_1 = \$0.65 + \$0.65 + \$0.70 + \$0.70 = \$2.70.[1]

3. The quarterly model recognizes that dividends are received quarterly, that they can be reinvested in the market to earn a return of k percent, and therefore that the compounded value of the dividends at the end of the year will be greater than the simple sum of the quarterly dividends. Thus, the dividend yield in the quarterly model exceeds that found with the annual model as it is generally applied. For example, suppose a stock sells for \$29.25, is expected to pay \$2.60 over the next four quarters (\$0.65 per quarter), and has its first payment due in 90 days. If the growth rate is constant at 7 percent, then the annual model produces a cost of 15.89%:

$$k = \frac{D_1}{P_0} + g = \frac{\$\,2.60}{\$29.25} + 7\%$$

$$= 0.0889 + 0.070 = 15.89\%.$$

Thus, according to the traditional DCF model, an investor who bought the stock would expect to earn a return of 15.89 percent. However, with the quarterly model, we insert the known values into Equation 6,

$$k = \frac{\$0.65(1+k)^{0.75} + \$0.65(1+k)^{0.50} + \$0.65(1+k)^{0.25} + \$0.65(1+k)^{0}}{\$29.25} + 0.07,$$

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[1] Some security analysts use as D_1 the latest quarterly dividend times 4. This understates D_1 except just after a dividend increase has been announced but before the stock goes ex for the first of the new dividends. Other analysts find D_1 as the latest quarterly dividend times 4, multiplied by (1 + g). This procedure overstates the expected dividend except just after the stock has gone ex dividend for the last of the old, lower dividends.

and then solve for k. Using an iterative Lotus 1-2-3 model on an IBM PC, we find the value of k to be 16.42 percent. Thus, the quarterly model produces a return that is 16.42% - 15.89% = 0.53 percent above that produced by the annual model.

In effect, the quarterly model converts the four quarterly dividends into their equivalent future value. If investors receive a dividend of $0.65 each quarter, and if they reinvest these dividends at their required rate of return (16.42%), then these dividends will have a value at the end of the year of approximately $2.76 instead of the $2.60 assumed by the annual model, and this, in turn, simultaneously implies an expected return of 16.42 percent. Thus, the quarterly model is equivalent in general form to the annual model, since it implies

$$k \simeq \frac{\$\,2.76}{\$29.25} + 0.07$$

$$\simeq 0.0942 + 0.070 = 16.42\%.$$

The difference between the quarterly and annual models depends on the relative contributions of the dividend yield and the capital gains yield (or growth rate) to the total required rate of return--for any given price, as the value of dividends paid increases, assuming that the required return remains the same, the dividend yield increases and the growth rate (that is, percent price appreciation) decreases. To illustrate, let us start with our "base case," where the dividends paid during the year were equal to $2.60 (with a compounded terminal value of $2.76), and the price appreciation was equal to $2.05:

$$\text{Price appreciation} = (\$29.25)(0.07) = \$2.05.$$

This implies that earnings per share for the year must be equal to $4.65:

$$\text{Earnings per share} = \$2.05 + \$2.60 = \$4.65,$$

which in turn implies a dividend payout rate of 55.91 percent:

$$\text{Payout rate} = \$2.60/\$4.65 = 55.91\%.$$

We showed earlier that under these conditions, the annual model produces a DCF k of 15.89 percent versus a quarterly DCF k of 16.42 percent.

Now assume that the firm decides to change its dividend payout rate, and that the "true" required rate of return remains constant at 16.42 percent.[1] If the firm pays no dividends (payout rate = 0%), then the dividend yield will be zero and the 16.42 percent required rate of return will come entirely from growth in the stock's price. Further, since we are assuming that the stock price will increase dollar for dollar with retained earnings, earnings per share for the year must equal $4.80:

$$\text{Earnings per share} = (\$29.25)(0.1642) = \$4.80.$$

Under the zero payout scenario, both the annual and quarterly DCF models will produce the same required rate of return: by definition, both have a zero dividend yield and a 16.42 percent growth rate. Thus, the two models produce the same results if the payout is zero.

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[1]If Miller and Modigliani [4] are correct, then the payout of dividends versus the retention of earnings should have no effect either on the value of the firm or on investors' required rate of return. We assume that Miller and Modigliani are correct for purposes of illustration.

On the other hand, suppose the firm pays out all earnings each quarter as dividends. If the required rate of return remains constant at 16.42 percent, then the firm must achieve a 3.87401 percent quarterly rate of return:

$$(1.1642)^{\frac{1}{4}} - 1.0 = 0.0387401 = 3.87401\%.$$

This implies quarterly earnings per share of $1.13:

$$\text{Quarterly earnings} = (\$29.25)(0.0387401) = \$1.13.$$

Note that the total earnings for the year will now be ($1.13)(4) = $4.52 versus $4.80 under the assumption of a zero dividend payout rate. This is to be expected, since under a 100 percent payout rate, the firm no longer has the opportunity to earn a return on incremental retained earnings each quarter. Using the quarterly model as expressed in Equation 6, we would calculate a dividend yield of 16.42 percent and a zero growth rate. Under the annual model, the firm would still have a zero growth rate, but its calculated dividend yield would be only 15.50%:

$$\text{Dividend yield} = \frac{(\$1.13)(4)}{\$29.25} = 15.50\%.$$

Therefore, the difference between the quarterly and annual DCF models is 16.42% - 15.50% = 0.92%, or 92 basis points.

Intuitively, we recognize that when a firm retains earnings, it has the opportunity to reinvest these earnings at its cost of equity. On the other hand, if the firm pays out the earnings as dividends, then it is now the investor who has the chance to reinvest these earnings at the cost of equity (required rate of return). The quarterly model takes reinvestment into account, whereas the annual model does not. Therefore, the higher the percentage of earnings paid out, the greater the difference between the annual and quarterly models.

Table 1 shows the differences between the two models at varying payout rates. The higher the payout, the larger the difference between the two models, and the greater the annual model's inherent downward bias. This suggests that the quarterly adjustment is particularly important in industries where payout ratios tend to be high, such as the utility industry.

Payment Date Adjustment

Thus far we have assumed that the analysis occurs, and P_0 is observed, on a dividend payment date. If the analysis actually takes place before or after a payment date, then this fact should be taken into account. The quarterly model, as we use it, already reflects the time until the next dividend, but the annual model requires the following adjustment:

$$P_0 = \frac{D_1}{k - g}(1 + k)^t, \tag{7}$$

which implies

$$k = \frac{D_1}{P_0}(1 + k)^t + g. \tag{8}$$

Here t is equal to (90 - days to receipt of next dividend)/360.

Table 1
Differences between the Annual and
Quarterly DCF Models at Various Payout Rates

% Payout	Annual DCF k	Quarterly DCF k	Difference in k
0	16.42%	16.42%	0.00%
25	16.18	16.42	0.24
50	15.94	16.42	0.48
75	15.72	16.42	0.70
100	15.50	16.42	0.92

Table 2 shows how changing the number of days until the dividend is received affects the calculated DCF cost of equity, assuming an annual dividend of \$2.60, P_0 = \$29.25, and g = 7%.[1] The difference between the k based on the adjusted annual and the quarterly models does not vary significantly as the number of days is changed, but the absolute value of the DCF cost of equity does change significantly for each model. Thus, Table 2 demonstrates the importance of properly accounting for the length of time until the next dividend payment date.

Nonconstant Growth

For many firms, the assumption of constant growth of dividends is not realistic over the near term, and for them the constant growth DCF model cannot be used to calculate the true DCF cost of equity. To illustrate, suppose the market consensus view for a firm is as shown in Table 3. As we can see, during Years 1 to 4, the company's dividend growth rate is high and nonconstant, but in Year 5 and thereafter, dividends are expected to grow at a constant rate of 7 percent.

Using the annual model, as defined by Equation 4, we would set up the basic problem as follows:

$$\$29.25 = \frac{\$2.60}{(1+k)^1} + \frac{\$3.00}{(1+k)^2} + \frac{\$3.40}{(1+k)^3} + \frac{\$3.68}{(1+k)^4} + \left(\frac{(\$3.68)(1.07)}{k - 0.07}\right)\left(\frac{1}{1+k}\right)^4 . \quad (9)$$

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[1]The quarterly model's k value must be different from 16.42 percent if the \$29.25 stock price is observed at any time other than 90 days before the next dividend is received. In other words, a \$2.60 annual dividend, a growth rate of 7 percent, and a stock price of \$29.25 is consistent with different k values--the closer the next dividend, the larger the implied k.

Table 2
Effect of Days to Dividend Payment on DCF k

	Days until First Dividend	Annual DCF k	Quarterly DCF k	Difference
	130	15.75%	16.26%	0.51%
Payment Date	90	15.89	16.42	0.53
	50	16.04	16.59	0.55
	10	16.19	16.76	0.57

Note: The annual DCF k was developed using Equation 8. Had this correction not been made, the differences would have been much larger, because the annual DCF k would have remained constant at 15.89 percent and would then have included both the quarterly payment error and the dividend payment date error. For example, if there were 10 days until the next dividend payment date, the unadjusted annual model would produce k = 15.89 percent versus the correct 16.76 percent, for an error of 0.87 percentage points.

Table 3
Assumed Conditions in the Nonconstant Growth Case

Year	Expected Annual Dividend	Expected Quarterly Dividend	Expected Growth Rate	
1	$2.60	$0.650	--	Nonconstant growth
2	3.00	0.750	15.38%	
3	3.40	0.850	11.76	
4	3.68	0.920	8.24	
5	3.94	0.985	7.00	Constant growth
6	4.22	1.054	7.00	
.	.	.	.	
.	.	.	.	
.	.	.	.	

This model can be solved for k using an iterative process; the solution value is k = 17.09 percent.[1]

Appendix A shows the derivation of a nonconstant quarterly growth DCF model. However, the resulting equation is extremely complicated, and the longer the period of nonconstant growth, the "messier" the equation. Fortunately, we can analyze the nonconstant quarterly growth case easily with an electronic spreadsheet and a personal computer. These are the steps to be followed:

1. Specify the dividends to be received, on a quarterly basis, during the nonconstant growth period. Key these values into the spreadsheet model.

2. Determine the fraction of the year before the first dividend is received. Each subsequent dividend will be received 0.25 periods later.

3. Once the nonconstant growth years have ended, dividends will grow every fourth period at the rate g. (This situation is easy to model using a spreadsheet such as Lotus 1-2-3.)

4. The spreadsheet model thus defines this equation:

$$P_0 = \frac{D_1}{(1 + k)^{f_1}} + \frac{D_2}{(1 + k)^{f_2}} + \cdots + \frac{D_N}{(1 + k)^{f_N}}, \tag{10}$$

where f_1 is the fraction of a year before the first dividend is received, k is the cost of equity, and N is the number of quarters

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[1]The cost of equity in this case, 17.09 percent, is higher than the 15.89 percent found in the 7 percent constant growth case because, in the new case, growth is much higher in the early years. The average growth rate is

$$g = k - D_1/P_0 = 0.1709 - \$2.60/\$29.25 = 17.09\% - 8.89\% = 8.20\%,$$

where the 8.20 percent is a weighted average of the relatively high short-run growth rates and the 7 percent long-run growth rate.

the model is to run. Theoretically, N should be set equal to infinity, but as a practical matter, the present value of dividends out beyond 200 years is virtually zero, so we can set N = 800 quarters.

5. With all variables except k specified in the spreadsheet model, we have the computer solve iteratively for k, setting the present value of the future dividend stream equal to the current price.

We can illustrate the nonconstant quarterly model with the data and conditions set forth in Table 3 above. To simplify the explanation, we assume that the analysis takes place on a dividend payment date, so there is one full quarter before the next dividend will be received. Under these assumptions, the DCF cost of equity is found to be 17.73 percent, and the solution values for Equation 10, as generated by the electronic spreadsheet, are shown in Table 4.

The cost of equity, as calculated earlier by the annual payment model (Equation 9), 17.09 percent, may be compared to the 17.73 percent found using the quarterly model. Thus, in this nonconstant growth situation, the annual payment model involves an error of 17.73% - 17.09% = 0.64%, or 64 basis points.

Summary and Conclusions

The primary purpose of this paper was to extend the traditional DCF model to encompass the realistic case of quarterly dividends that are increased annually. In addition, we demonstrate that the annual model, if it is to be used, should be adjusted whenever the analysis occurs on a date other than the dividend payment date. Both adjustments are especially important for high payout firms such as the electric, gas, and telephone companies.

Table 4
Data Calculated from the Quarterly Payment, Nonconstant Growth Spreadsheet Model, Where k is Equal to 17.73 Percent

Quarter (t)	Time to Payment: f_t	Quarterly Dividend: D_t	PV Factor: $\frac{1}{(1+k)^{f_t}}$	PV of D_t: $\frac{D_t}{(1+k)^{f_t}}$	Cumulative PV of D_t
1	0.25	$ 0.65	0.9600	0.6240	$ 0.62
2	0.50	0.65	0.9216	0.5991	1.22
3	0.75	0.65	0.8848	0.5751	1.80
4	1.00	0.65	0.8494	0.5521	2.35
5	1.25	0.75	0.8155	0.6116	2.96
6	1.50	0.75	0.7829	0.5872	3.55
7	1.75	0.75	0.7516	0.5637	4.11
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
18	4.50	0.98	0.4798	0.4723	9.87
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
799	199.75	528,455.29	0.0000	0.0000	29.25
800	200.00	528,455.29	0.0000	0.0000	29.25

We are not sure what the implications of our study are for capital budgeting. In capital budgeting, people often assume that cash flows occur at the end of the year. If cash flows actually occur all during the year, then perhaps this fact should be taken into account if a quarterly adjusted DCF cost of equity is to be used in capital budgeting.[1]

One could argue that, given the uncertainty inherent in the basic data required for a DCF analysis of common stock, the refinements we suggest are not worth the effort. We have two responses. First, the differences in calculated rates of return are not trivial, and the annual model always understates the "true" annual return on a stock which pays dividends quarterly; therefore, to avoid biases one should make the quarterly adjustment. Second, with a relatively inexpensive personal computer, the analysis is really quite easy.

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[1]In some respects, this is similar to the application of a quarterly DCF k to a utility's rate base in commission rate hearings.

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Appendix A

Definition of Variables

D_t = the dividend to be paid at the end of Quarter t. Dividends are paid quarterly but are assumed to change annually.

g = the constant annual growth rate of dividends, such that $(D_{t-4})(1 + g) = D_t$.

P_0 = the current stock price as of the analysis date, where the analysis date is not required to be a dividend payment date.

f_t = the fraction of a year until dividend D_t is to be received. Technically, a 365-day year should be used, but we have found that using the convention of a 360-day year and 90-day quarters introduces an insignificant error.

k = the quarterly DCF cost of equity, which is also the investors' effective annual required rate of return on common equity.

Derivation of a Quarterly Constant Growth DCF Model

We start with the assumption that the current price of a share of common stock is equal to the present value of all dividends to be received, or

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1 + k)^{f_t}}. \tag{1}$$

This equation may be written in the following expanded form:

$$P_0 = \frac{D_1}{(1 + k)^{f_1}} + \frac{D_2}{(1 + k)^{f_2}} + \frac{D_3}{(1 + k)^{f_3}} + \frac{D_4}{(1 + k)^{f_4}} + \cdots + \frac{D_{\infty-1}}{(1 + k)^{f_{\infty-1}}} + \frac{D_\infty}{(1 + k)^{f_\infty}}. \tag{2}$$

Keeping in mind that $D_t = D_{t-4}(1 + g)$, so $D_{t-4} = D_t/(1 + g)$, we can multiply both sides of Equation 2 by $(1 + k)/(1 + g)$ to obtain

$$P_0 \left(\frac{1 + k}{1 + g}\right) = D_{-3}(1 + k)^{1-f_1} + D_{-2}(1 + k)^{1-f_2} + D_{-1}(1 + k)^{1-f_3}$$

$$+ D_0(1 + k)^{1-f_4} + \ldots + D_{\infty-5}(1 + k)^{1-f_{\infty-1}}$$

$$+ D_{\infty-4}(1 + k)^{1-f_\infty}, \tag{3}$$

where D_{-3} through D_0 are the quarterly dividends already paid in the previous year. If we now subtract Equation 2 from Equation 3, we will obtain

$$P_0 \left(\frac{1 + k}{1 + g}\right) - P_0 = D_{-3}(1 + k)^{1-f_1} + D_{-2}(1 + k)^{1-f_2}$$

$$+ D_{-1}(1 + k)^{1-f_3} + D_0(1 + k)^{1-f_4}$$

$$- D_{\infty-3}(1 + k)^{1-f_{\infty-3}} - D_{\infty-2}(1 + k)^{1-f_{\infty-2}}$$

$$- D_{\infty-1}(1 + k)^{1-f_{\infty-1}} - D_\infty(1 + k)^{1-f_\infty} \tag{4}$$

If we assume that dividends are growing on an annual basis at a rate, g, which is less than the investors' required rate of return, k, then the last four terms on the right hand side of Equation 4 will approach zero in the limit. Thus, Equation 4 may be reduced to

$$P_0\left(\frac{k - g}{1 + g}\right) = D_{-3}(1 + k)^{1-f_1} + D_{-2}(1 + k)^{1-f_2} + D_{-1}(1 + k)^{1-f_3}$$

$$+ D_0(1 + k)^{1-f_4}. \tag{5}$$

If we now multiply both sides of Equation 5 by (1 + g)/(k - g), we will obtain

$$P_0 = \frac{D_1(1+k)^{1-f_1} + D_2(1+k)^{1-f_2} + D_3(1+k)^{1-f_3} + D_4(1+k)^{1-f_4}}{k-g}. \quad (6)$$

Solving Equation 6 for k, we then obtain

$$k = \frac{D_1(1+k)^{1-f_1} + D_2(1+k)^{1-f_2} + D_3(1+k)^{1-f_3} + D_4(1+k)^{1-f_4}}{P_0} + g, \quad (7)$$

where D_1 through D_4 are the quarterly dividends expected to be received over the coming 12-month period. Note that the numerator of Equation 7 is simply the terminal (future) value of the quarterly dividends to be received over the coming year, compounded forward to the end of the year by the investors' required rate of return. As such, it takes into consideration that investors will have the opportunity to reinvest these dividends as they are received. If we allow the terminal value of these quarterly dividends to be represented by ${}_TD_1$, then Equation 7 may be restated as

$$k = \frac{{}_TD_1}{P_0} + g, \quad (8)$$

which is nothing more than a quarterly representation of the annual DCF model.

If pricing takes place at a dividend payment date, where f_1 will be equal to 0.25, then Equation 7 reduces to

$$k = \frac{D_1(1+k)^{0.75} + D_2(1+k)^{0.50} + D_3(1+k)^{0.25} + D_4(1+k)^{0}}{P_0} + g. \quad (9)$$

Derivation of a Quarterly Nonconstant Growth DCF Model

We will assume that there are n periods of nonconstant growth of dividends, after which dividends will grow at the constant annual growth rate of g percent. From Equation 6 we know that the price of a share of stock at the beginning of this constant growth period (right after the last, nonconstant growth dividend had been paid) will be equal to

$$P_n = \frac{D_{n+1}(1 + k)^{0.75} + D_{n+2}(1 + k)^{0.50} + D_{n+3}(1 + k)^{0.25} + D_{n+4}(1 + k)^{0}}{k - g}. \quad (10)$$

Therefore, the price as of the beginning of the nonconstant growth period (time period zero) will be equal to

$$P_0 = \frac{D_1}{(1 + k)^{f_1}} + \frac{D_2}{(1 + k)^{f_2}} + \cdots + \frac{D_n}{(1 + k)^{f_n}}$$

$$+ \left(\frac{D_{n+1}(1 + k)^{0.75} + D_{n+2}(1 + k)^{0.50} + D_{n+3}(1 + k)^{0.25} + D_{n+4}(1 + k)^{0}}{k - g}\right)$$

$$\times \left(\frac{1}{1 + k}\right)^{f_n}. \quad (11)$$

Solving Equation 10 for k, we then obtain:

$$k = \left\{\left(\frac{D_1}{(1 + k)^{f_1}} + \frac{D_2}{(1 + k)^{f_2}} + \cdots + \frac{D_n}{(1 + k)^{f_n}}\right)(k - g)\right.$$

$$+ \left(D_{n+1}(1 + k)^{0.75} + D_{n+2}(1 + k)^{0.50} + D_{n+3}(1 + k)^{0.25} + D_{n+4}(1 + k)^{0}\right)$$

$$\left.\times \left(\frac{1}{1 + k}\right)^{f_n}\right\} \Big/ \left\{P_0\right\} + g, \quad (12)$$

which may also be expressed as

$$k = \frac{\left(\sum_{t=1}^{n} \frac{D_t}{(1+k)^{f_t}}\right)(k-g) + \left(\sum_{t=1}^{4} D_{n+t}(1+k)^{1-(t)(0.25)}\right)\left(\frac{1}{1+k}\right)^{f_n}}{P_0} + g. \quad (13)$$