Functional Forms for the Production Technology

in Electric Power Generation

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Functional Forms for the Production Technology in Electric Power Generation

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A generalized Box-Cox cost function is used to estimate the prouction technology in U.S. electric power generation. The functional form formulated in this paper enables us to test the validity of a generalized Leontief, translog, generalized quadratic square rooted and more restricted forms of cost function. We find that the translog is the only form which cannot be rejected on statistical ground.

In past a variety of functional forms has been introduced and applied to estimate the production technology of U.S. electric power generation. The forms commonly used in recent studies include Cobb-Douglas (CD), log linear (LL), generalized Leontief (GL) (Diewert 1971),and translog (TLOG) (Christensen, Jorgenson and Lau 1971) functions[1]. The question of how to discriminate among various functional forms for the specific application is the central issue of this paper.

The adaptations of functional forms which do not allow input substitutions are believed to be useful for estimating ex-post technology at the levels of generating unit or plant. However if the estimating technology is ex-ante[2] or the unit of observation is at firm level, suitable forms are the ones which allow substitutions among inputs, especially those which do not impose a priori restrictions on the elasticities of substitution. Flexible forms such as the GL and TLOG forms are in this sense more suitable than the CD and LL specifications for our purposes. It is however much more difficult to discriminate among the flexible forms on theoritical ground, for each of the forms can provide a second order local approximation to an arbitrary twice continuously differentiable function. On econometric and computational grounds, the TLOG form is preferable as it yields a system of input share equations which are linear in parameters. Other flexible forms such as the GL, quadratic square rooted (QSR) (Diewert 1974) and generalized quadratic (GQ) (Denny 1974), generate share equations which are nonlinear in parameters. The desired convergence with the system of equations which are nonlinear in paramters are considerably more diffcult to achieve.

Our approach is to employ statistical test procedures to choose among different functional forms. The testing requires a specification of general functional form which can embody other forms as special or limiting cases. The functional form of our choice is a generelized form of Box-Cox (1964) transformation (GBC), although the GQ form is equally satisfactory in terms of meeting the required level of generality. The GBC form is chosen because it is slightly more convenient than the GQ in deriving the TLOG form as a limiting case. In the context of cost function, the GBC form becomes equivalent to the GQ form when the restriction of linear homogeneity in prices is imposed.

In Section 2, we formulate the GBC cost function which can nest the CD, LL, constant elasticities of substitution (CES), QSR, GL and TLOG as special or limiting cases. In Section 3, we set up a model in a GBC framework for the production system in electricity generation. In Section 4, we present empirical results. We find that the GBC cost function (with λ =.05) is a proper form for representing the production technology. Among other forms, the TLOG is the only acceptable alternative to the GBC form, but it underestimates substitutability of labor and fuel for capital and the extent of scale economies.

I. A Generalized Box-Cox Function and Alternative Forms

According to duality theorems, a production technology can be represented, under certain regularity conditions, by a production function or a cost function. In the context of econometric work, it implies that a direct specification and estimation of a cost function is equivalent to estimating the underlying production function. The required regularity conditions include the existence of a production function which is a continuously twice differentiable nondecreasing quasiconcave function. Given the technology described by such a production function, if producers minimize the total cost of production subject to given levels of output and input prices, the technology can be equally represented by its dual cost function. The cost function satisfies nondecreasing monotonicity in output and input prices, and positivity, linear homogeneity and concavity in input prices.

For the specification of the cost function, we first formulate the most general form of the Box-Cox cost function in which the constraint of linear homogeneity in prices can be imposed. Later, we will trade a portion of generality for easier estimation. The most general form of a GBC cost function can be written as

$$C = \{1 + \lambda(a_0 + \sum_i a_i P_i(\tfrac{\lambda}{2}) + \tfrac{1}{2}\sum_i\sum_j a_{ij} P_i(\tfrac{\lambda}{2}) P_j(\tfrac{\lambda}{2}) + \sum_i b_i P_i(\lambda) Y(\lambda_Y))\}^{\frac{1}{\lambda}} \cdot Y^{(g + \frac{d}{2}\ln Y)} \tag{1}$$

where

$$P_i(\tfrac{\lambda}{2}) = \frac{P_i^{\frac{\lambda}{2}} - 1}{\frac{\lambda}{2}}, \quad P_i(\lambda) = \frac{P_i^{\lambda} - 1}{\lambda}, \quad Y(\lambda_Y) = \frac{Y^{\lambda_Y} - 1}{\lambda_Y} \quad \forall i \tag{2}$$

The cost function (1) may be transformed into a more apparent Box-Cox form.

$$C^*(\lambda) = a_0 + \sum_i a_i P_i(\tfrac{\lambda}{2}) + \tfrac{1}{2}\sum_i\sum_j a_{ij} P_i(\tfrac{\lambda}{2}) P_j(\tfrac{\lambda}{2}) + \sum_i b_i P_i(\lambda) Y(\lambda_Y) \tag{3}$$

where

$$C^*(\lambda) = \frac{(C/Y^{(g+\frac{d}{2}\ln Y)})^{\lambda} - 1}{\lambda}$$

The conditions of symmetry ($a_{ij} = a_{ji}$, $\forall$ i,j) and linear homogeneity in prices impose the restrictions on the parameters as

$$\sum_i a_i = 1 + \lambda a_0, \quad \sum_j a_{ij} = \frac{\lambda}{2} a_i \quad \forall i, \quad \sum_i b_i = 0. \tag{4}$$

Incorporating the above constraints into Equation (1), we obtain

$$C=\{\frac{2}{\lambda}\sum_i\sum_j a_{ij}P_i^{\frac{\lambda}{2}}P_j^{\frac{\lambda}{2}}+\sum_i b_i P_i^{\lambda}Y(\lambda_Y)\}^{\frac{1}{\lambda}}Y^{(g+\frac{d}{2}\ln Y)} \qquad (5)$$

By fixing th values of λ and λ_Y at different levels, a wide variety of functional forms can be generated.

A. Restricting the value of λ to 2 in Equation (5) produces a QSR cost function.

$$C=\{\sum_i\sum_j a_{ij}P_iP_j+\sum_i b_i P_i^2 Y(\lambda_Y)\}^{\frac{1}{2}}Y^{(g+\frac{d}{2}\ln Y)} \qquad (6)$$

B1. When $\lambda = 1$, Equation (5) reduces to a GL[3] form,

$$C=\{2\sum_i\sum_j a_{ij}P_i^{\frac{1}{2}}P_j^{\frac{1}{2}}+\sum_i b_i P_i^{\frac{1}{2}}P_i^{\frac{1}{2}}Y(\lambda_Y)\}Y^{(g+\frac{d}{2}\ln Y)} \qquad (7)$$

B2. A homothetic Leontief function can be derived by imposing additional constraints, $a_{ij} = 0$ $(i \neq j)$ and $b_i = 0\ \forall\ i$, in the above GL form[3].

$$C=(2\sum_i a_{ii}P_i)Y^{(g+\frac{d}{2}\ln Y)} \qquad (8)$$

C1. As $\lambda \to 0$ and $\lambda_Y \to 0$, Equation (3) approaches to its limiting form, a TLOG cost function.

$$\ln C=a_0+\sum_i a_i\ln P_i+\frac{1}{2}\sum_i\sum_j a_{ij}\ln P_i\ln P_j+\sum_i b_i\ln P_i\ln Y+(g+\frac{d}{2}\ln Y)\ln Y. \qquad (9)$$

C2. A log linear form can be obtained from the TLOG with constraints, $a_{ij} = 0\ \forall\ i,j$, $b_i = 0\ \forall\ i$ and $d = 0$.

$$\ln C=a_0+\sum_i a_i\ln P_i+g\ln Y \qquad (10)$$

C3. A CD form requires additional constraint, $g = 1$,

$$\ln C=a_0+\sum_i a_i\ln P_i+\ln Y \qquad (11)$$

or

$$C=e^{a_0}\prod_i P_i^{a_i}Y$$

D. With no restriction on the values of λ but imposing the restrictions, $a_{ij} = 0\ \forall i,j$, $b_i = 0\ \forall\ i$ and $d = 0$, Equation (5) reduces to a CES form,

$$C=\{\frac{2}{\lambda}\sum_i a_{ii}P_i^{\lambda}\}^{\frac{1}{\lambda}}Y^g \qquad (12)$$

II. Application to Electric Power Generation

The electric power industry contends with substantial regulation by public authorities. The firms are obliged to meet the demand in their franchised areas at the prices consistent with the rate-of-return regulation. In this sense, a firm's production level is exogenously determined[5]. The major factors of production such as capital, labor and fuel are purchased competitively so that their prices are also exogenously determined. Under these conditions, we can use a cost function to describe the underlying production technology.

The most general form of the GBC cost function developed in the previous section, despite of its generality and usefulness, will not be employed in this study. The cost of estimating a nonlinear model with more than one transformation parameter[7] is extremely high when the model involves a large number of parameters to be estimated. We instead use a less general form of a Box-Cox function. By imposing certain restrictions on the parameters, we obtain the following GBC forms which can maintain the required level of generality.

By replacing λ_Y with λ in Equation (5), the number of transformation parameter can be reduced to one. The corresponding cost fuction is,

$$C = \{\frac{2}{\lambda}\sum_i\sum_j a_{ij} P_i^{\frac{\lambda}{2}} P_j^{\frac{\lambda}{2}} + \sum_i b_i P_i^{\lambda} Y(\lambda)\}^{\frac{1}{\lambda}} Y^{(g+\frac{d}{2}\ln Y)} \qquad (13)$$

Another way of reduceing the number of transformation parameter is to impose the restriction, $\lambda_Y \to 0$. The resulting cost function is,

$$C = \{\frac{2}{\lambda}\sum_i\sum_j a_{ij} P_i^{\frac{\lambda}{2}} P_j^{\frac{\lambda}{2}} + \sum_i b_i P_i^{\lambda} \ln Y\}^{\frac{1}{\lambda}} Y^{(g+\frac{d}{2}\ln Y)} \qquad (14)$$

The GBC cost function employed by Berndt and Khaled(1979) is a modified form of Equation (~~19~~ 14).

$$C = \{\frac{2}{\lambda}\sum_i\sum_j a_{ij} P_i^{\frac{\lambda}{2}} P_j^{\frac{\lambda}{2}}\}^{\frac{1}{\lambda}} Y^{(g+\frac{d}{2}\ln Y + \sum_i b_i \ln P_i \ln Y)} \qquad (15)$$

The forms (~~18~~ 13) and (~~19~~ 14) have some computational advantage over (~~20~~ 15). As the

Box-Cox transformation transforms observations into power forms, it is likely to encounter the problem of floating-point overflow before the desired convergence is achieved. With the use of Form (13) or (14), the problem of overflow is less likely to occur than with (15). On the other hand, Form (15) has an convenient feature which the other two do not have. It does eliminate nonlinearlity in the parameters of the ~~derived demand eequations.~~ equations for output elasticity of cost, and thereby making the interpretation of the scale parameters more straight forward. ~~The input share equations of this form are thus all linear in parameters~~.

For this reason, we adopt the GBC form (15) to describe the production structure of U.S. electric power generation.

Modeling the Production System With a GBC Cost Function

In the absence of the restrictions of symmetry and linear homogeneity in prices (4), the GBC cost function can be written

$$C = \{1 + \lambda\{a_0 + \sum_i a_i P_i(\tfrac{\lambda}{2}) + \tfrac{1}{2}\sum_i\sum_j a_{ij} P_i(\tfrac{\lambda}{2}) P_j(\tfrac{\lambda}{2})\}\}^{\frac{1}{\lambda}} Y^{(g + \frac{d}{2}\ln Y + \sum_i b_i \ln P_i)} \qquad (16)$$

The derived demand functions for the factors of production can be computed from the cost function (15) by partially differentiating the cost function with respect to each input price. Using the result known as the Shephard's lemma(1953), the ith input share equation can be derived by taking the partial derivative in logarithmic form.

$$S_i = \frac{\sum_j a_{ij} P_i^{\frac{\lambda}{2}} P_j^{\frac{\lambda}{2}}}{\sum_i\sum_j a_{ij} P_i^{\frac{\lambda}{2}} P_j^{\frac{\lambda}{2}}} + b_i \ln Y \qquad (17)$$

The Allen-Uzawa partial elasticeties of substitution E_{ij} for the GBC cost function are calculated as

$$E_{ij} = 1 - \frac{\lambda S_i S_j}{(S_i + b_i \ln Y)(S_j + b_j \ln Y)} + \frac{\lambda a_{ij} (P_i P_j)^{\frac{\lambda}{2}}}{2\sum_i\sum_j a_{ij}(P_i P_j)^{\frac{\lambda}{2}}(S_i + b_i \ln Y)(S_j + b_j \ln Y)} \qquad (18)$$

The elasticity of cost with respect to output along a cost-minimizing expansion path is given by

$$\frac{\partial \ln C}{\partial \ln Y} = g + d\ln Y + \sum_i b_i \ln P_i \qquad (19)$$

The measurement of scale economies is defined[8] as

$$SCE = 1 - \frac{\partial \ln C}{\partial \ln Y} \tag{20}$$

The positive values of SCE indicate economies of scale and the negative values for disceconomies of scale.

Production technology is homothetic if the cost function is separable in output and input prices. For the GBC cost function, this requires

$$b_i = 0 \;\; \forall\, i \tag{21}$$

Production technology is homogeneous of degree g^{-1} if, in addition to homothetic restrictions, the elasticity of total cost with respect to output is constant, that is,

$$b_i = 0 \;\; \forall\, i \quad \text{and } d = 0. \tag{22}$$

When the value of g is restricted to one, production is called "homogeneous of degree one" or "constant returns to scale".

Scale effect on ith input share can be measured by

$$\frac{\partial S_i}{\partial \ln Y} = b_i \tag{23}$$

For a homothetic production technology, a change in scale, holding input prices constant, does not alter the proportion of total cost spent on each input. Homotheticity also implicates that the input-output ratios are not altered as the level of output changes. This can be seen from the following expression for $\partial S_i / \partial \ln Y$.

$$\frac{\partial S_i}{\partial \ln Y} = S_i \left(\frac{\partial \left(\frac{X_i}{Y}\right)}{\left(\frac{X_i}{Y}\right)} \Big/ \left(\frac{\partial Y}{Y}\right) - \frac{\partial AC}{AC} \Big/ \left(\frac{\partial Y}{Y}\right) \right) \tag{24}$$

where AC is unit cost of production. If scale economies exist $\frac{\left(\frac{\partial AC}{AC}\right)}{\left(\frac{\partial Y}{Y}\right)} < 0$. So, for homothetic technology, each input-output ratio must decline at the same proportion as AC. If $b_i < 0$, the scale effect is nonhomothetic, and an increase in output brings about a decline in the cost share of ith input. If scale economies exist, a greater scale induces a greater proportionate decline in the input requirement of ith factor per unit of output. An anlogous statement can be made for the case, $b_i > 0$.

III. Estimation and Results

We assume additive disturbaces for the scale adjusted cost function.

$$\frac{C^{*\lambda}}{\lambda} = \left\{ \frac{2}{\lambda^2} \sum_i \sum_j a_{ij} (P_i P_j)^{\frac{\lambda}{2}} \right\} + \epsilon \tag{25}$$

where $C^* = C \,/\, Y^{g + d/2 \ln Y + \sum_i b_i \ln P_i \ln Y}$ and for each of the input share equations,

$$S_i = \frac{\sum_j a_{ij} (P_i P_j)^{\frac{\lambda}{2}}}{\sum_i \sum_j a_{ij} (P_i P_j)^{\frac{\lambda}{2}}} + b_i \ln Y + e_i \tag{26}$$

The disturbances are assumed to have a joint normal distribution, allowing correlations for an individual firm but no correlations across firms. The scale adjusted specification of the cost function ~~(30)~~ 25 yield a continous likelihood function which comprises the likelihood functions of other forms nested in the GBC form. Since the disturbances on the share equations add up to zero for each firm, they are not independent. One of the share equations must be deleted from the system. It is known that maximum likelihood estimates are invariant to which input share equation to be deleted. To obtain maximum likelihood estemates, we use the iteration of the seemingly unrelated estimation procedure until convergence.

The likelihood ratio tests are used to test for homotheticity, homogeneity and constant returns to scale. Once the form of returns to scale is determined, we test for the validity of the various functional forms. The test statistic is $2(L_U - L_R)$ where L_U and L_R are the maximum log likelihood values of the unrestricted and restricted models, respectively. This statistic is distributed asymptotically as chi-squared with the degree of freedom equal to the number of restrictions in restricted model minus that in unrestricted model.

To provide a link with the results obtained by Christensen and Greene (1976), we use their 1970 cross-section data on individual firms.

Table 1 shows the estimated parameters for nonhomothetic (NH), homothetic (HT) and homogeneous (HG) models.

TABLE 1

Estimated Coefficients of the GBC Cost Function

(t-Ratios in Parentheses)

Models

Parameters	NH	HT	HG
g	.900 (54.09)	.900 (47.96)	.901 (76.59)
d	.013 (1.19)	.012 (0.99)	
b_K	-.002 (-0.58)		
b_L	-.013 (-4.02)		
b_F	.015 (3.07)		
	.052 (1.74)	.036 (0.99)	.036 (0.84)
a_{KK}	.070 (3.85)	.070 (4.06)	.070 (4.06)
a_{LL}	.034 (2.28)	.034 (1.86)	.034 (1.80)
a_{FF}	.182 (5.98)	.177 (5.71)	.177 (5.72)
a_{KL}	.032 (2.38)	.031 (2.16)	.031 (2.14)
a_{KF}	-.099 (-6.12)	-.099 (-6.39)	-.099 (-6.36)
a_{LF}	-.064 (-4.73)	-.063 (-4.05)	-.064 (-3.92)
Log L	536.333	500.622	486.873

As for the properties of the estimated function, symmetry of input substitution and linear homogeneity in prices are imposed as the maintained hypothesis. Other regularity conditions which our GBC cost function must satisfy are nondecreasing monotonicity in prices and output, and positivity and concavity in prices.

C is a positive function in prices if the matrix $[a_{ij}]$ is positive definite $\forall \lambda > 0$, and nondecreasing in prices if $\frac{\partial C}{\partial P} > 0$. Using the Shephard's lemma, the positivity of each input share equation is sufficient to insure the latter condition.

$$S_i = \frac{P_i}{C} \frac{\partial C}{\partial P_i} > 0 \quad \forall\ i.$$

We find positive values for all of the fitted cost and input share equations at every observation. Therefore, the positivity and monotonicity conditions in prices are satisfied in all our models.

C is nondecreasing in output if $\frac{\partial C}{\partial Y} > 0$ or elasticities of cost with respect to output is nonnegative.

$$\frac{\partial \ln C}{\partial \ln Y} = g + d \ln Y + \sum_i b_i \ln P_i > 0$$

This condition is also satisfied at every point of observation.

Finally, C is concave in prices iff its Hessian matrix $[\ C_{ij}\ ]$ is negetive semidefinite. The condition for concavity is also satisfied for all of the models we estimated. Consequently, our estimated cost functions correspond to their dual production functions, and thereby describing the underlying production technology.

For the testing of the validity of homothetic and homogeneous restrictions, we calculate the chi-square statistic, defined as $2(L_U - L_R)$. For the homothetic restrictions, the chi-square statistic is 71.422 whereas the 1 % chi-square critical value for 2 degrees of freedom is 9.21. The homothetic hypothesis is decisively rejected. Similarly, we find, based on the same testing procedure, homogeneity restrictions are inconsistent with the data set. These findings support our contention that we need a nonhomothetic model to describe the production technology of U.S. electric power generation.

Among the functional forms used in recent studies, nonhomotheticity is allowed only in the GL form of Fuss (1978) and the TLOG forms of Christensen and Greene (1976) and Stevenson (1980). For testing the validity of these forms, we list in Table 2 the maximum log likelihood values corresponding to the values of λ which generate these forms.

TABLE 2

Log Likelihood Values for Different Functional Forms

Functional Form	λ	Log L
GBC	.052 (Estemated)	536.333
TLOG	0	534.303
GL	1	349.641
QRS	2	113.429

We summarize in Table 3 the chi-square statistics for alternative functional forms based on the log likelihood values presented in Table 2.

TABLE 3

Chi-Square Statistics for Alternative Functional Forms

Functional Form	$2(L_U - L_R)$
TLOG	4.06
GL	373.38
QSR	845.81

The TLOG is the only form that cannot be rejected at 1% level of significance. Serious specification errors would result in with the use of the GL, QSR, homothetic or homogeneous specification. These functional forms bring about significant loss of fit as compared with the GBC or TLOG forms.

The TLOG however can be rejected at 5% level of significance as the critical chi-square value for one degree of freedom is 3.841. To compare the performance of the TLOG with that of the GBC, we present, along with our results, the sample averages of the elasticities of substitution in Table 4 for the TLOG model[9].

TABLE 4

Estimated Elasticities of Substitution

for GBC and TLOG Cost Functions

	Capital-Labor	Capital-Fuel	Labor-Fuel
GBC	1.687	0.325	0.134
TLOG	0.639	0.218	0.165

The results based on the GBC form show much stronger substitutabilities of labor and fuel for capital than with the TLOG form. As noted by Nerlove (1963),"at the firm level,there are many possibilities for substitution that may go unnoticed at the plant level". Firms can substitute labor for capital by using less efficient plants more intensively. Our results support such substitution possibilities, much more so than those based on the TLOG.

An estimate of returns to scale for individual firm can be derived by evaluating the fomula (25) at each observation. Following Christensen and Greene, we derive the average cost curve facing a typical firm by evaluating the cost function for a range of outputs while holding the factor prices fixed at the sample means. Declining protion of the average cost curve[10] is the indication of the existence of scale economies. Christensen and Greene reproted that, based on the TLOG model, the exhaustion of scale economies was obserbed at the output level of 19.8 billion kwh. Our results however indicate statistically significant scale economies up to 25.8 billion kwh.

We present, in Table 5, our estimates of scale economies along with the estimates obtained by Christensen and Greene for the selected firms.

TABLE 5

Company	Output (Million kwh)	SCE TLOG	SCE GBC
Community Public Service	183	.247	.261
Iowa Southern Utilities	1,328	.160	.176
Missouri Public Service	1,886	.143	.160
Rochester Gas & Electric	2,020	.136	.152
Iowa Electric Light & Power	2,445	.133	.151
Central La. Gas & Electric	2,689	.127	.145
Wisconsin Public Service	3,571	.103	.120
Atlantic City Electric	4,187	.094	.111
Central Illinois Public Service	5,316	.097	.116
Kansas Gas & Electric	5,785	.094	.113
Northern Indiana Public Service	6,837	.079	.098
Indianapolis Power & Light	7,484	.080	.099
Oklahoma Gas & Electric	10,149	.066	.086
Niagara Mohawk Power	11,667	.049	.068
Potomac Electric Power	13,864	.037	.057
Gulf State Utilities	17,875	.036	.057
Virginia Electric Power	23,217	.015	.035
Consolidated Edison	29,613	-.003	.017
Detroit Edison	30,958	-.004	.016
Duke Power	34,212	-.012	.008
Commonwealth Edison	46,871	-.014	.007
Southern	53,918	-.028	-.002

It is apparent that the TLOG model leads to an underestimation of scale economies as compared with the GBC model.

The scale effects are found to be non-neutral, affecting labor and fuel requirements more than capital. The negative value of b_L indicates significant scale economies in labor input. A 1% increase in output brings about a decline in the cost share of labor by the magnitude of .013. This implies a greater proportionate decline in labor requirement per unit of output as output increases. The last statement can be made because of our finding that none of the firms in our sample faces statistically significant scale diseconomies.

The scale effect in electricity generation in 1970 is fuel using. We find that a 1% increase in output increases the cost share of fuel by .015. As for the scale effect on capital, we check the estimated value of b_K which is found to be not significantly different from zero. It appears that an increase in output does not alter the cost share of capital input.

Concluding Remarks

The production technologh of the U.S. electric power generation in 1970 can be represented by the GBC cost function with $\lambda = .05$. Among other alternative functional forms, the TLOG is the only form which cannot be rejected on statistical ground. The TLOG model however underestimates the substitutability of labor and fuel for capital and also the extent of scale economies.

From computational point of view, the TLOG is an attractive alternative to the GBC form. We find that it is considerably more difficult to achieve convergence with the GBC form than with the TLOG form.

Footnotes.

1. The CD form appeared in the studies of Komiya (1962). But Komiya eventually turned to a limitational model. The LL forms were employed by Nerlove (1963), Courville (1974) and Cowing (1974). Fuss (1978) used the GL, and Christensen and Greene (1976) and Stevenson (1980) employed the TLOG forms.

2. An example is Komiya's limitational model.

3. Diewert's original GL form is homothetic. The GL form developed here is nonhomothetic.

4. The Leontief can be also derived from the CES by setting $\lambda = 1$.

5. As pointed out in Nerlove's paper, the output of power pool members may not be truly exogenous.

6. There may be objections to the assumption of static cost minimization. See Nerlove for details.

7. The GBC form (1) contains two transformation parameters, λ and λ_Y.

8. This definition was used by Christensen and Greene. SCE can be defined by using output elasticity of cost fromula (25). Definition (25) is convenient for the comparison with their results.

9. These are the estimates obtained by Christensen and Greene.

10. Christensen and Greene define the "flat" portion of the average cost curve ~~as~~: "any point on the average cost curve is considered to be in the flat region of the curve if its corresponding SCE is less than 1.96 times its standard error".

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