Peak Load Pricing and Random Demand

Roger D. Blair
University of Florida

and

Patricia L. Pacey*
University of Colorado
Colorado Springs

Peak load pricing opportunities exist whenever the firm is faced with sequential demands for output, which is produced from some sort of fixed capital. The demand for electricity during several periods of the day provides the classic example. But there are many other examples: airport landing privileges, urban transportation services, computer time, personal income tax advice, and tourist hotels to name just a few. In each instance, one could apply peak load pricing principles. Although there had been a substantial continuing interest in this issue,[1] much greater attention was sparked by the fundamental contribution of Peter Steiner (1957). Most of the ensuing analysis were confined to the welfare question of determining the socially optimal prices. An interesting exception was provided by Bailey and White (1974) when they considered alternative objective functions. In this paper, we shall generalize some of the Bailey and White argument to include the effects of uncertainty. In particular, we shall consider the influence of random demand upon a firm that attempts to maximize the expected utility of profit.[2] The solutions for the regulated and unregulated firms in the firm peak and shifting peak cases will be examined.

I. The Unregulated Case.

In this model we assume that there are two independent (sequential) demands of equal length.[3] When we deal with the firm peak case, we assume that distinct peak and off peak periods exist. In the presence of uncertainty, this requires that the probability distributions are disjoint. Without this assumption, random shifts in demand could cause the firm peak case to become a shifting peak problem.

Uncertainty is introduced by making demand a random variable for each period. In particular, demand in the i-th period is specified as:

(1) $P_i = P_i(Q_i, u_i)$

where u_i is a random variable. Of course, the values that u_i may assume are limited so that prices cannot be negative. Otherwise, there are no constraints on the u_i. Adopting familiar notation, we let b represent the marginal operating cost, which is assumed to be constant. Further, we let β represent the (constant) cost of a unit of capacity that is just large enough to produce one unit of output per period.

Firm Peak Case. For the firm peak case in the presence of random demand, the random profit function is

(2) $\Pi = P_1(u_1)Q_1 + P_2(u_2)Q_2 - b(Q_1 + Q_2) - \beta Q_2$

where period 2 is assumed to impose the peak demand upon capacity. In order to allow nonlinear risk preferences, we assume that the firm attempts to maximize the expected utility of profits:

(3) $E[U(\Pi)] = E[U(P_1(u_1)Q_1 + P_2(u_2)Q_2 - b(Q_1 + Q_2) - \beta Q_2)]$

where U is a von Neumann-Morgenstern utility index and E is the expectations operator.[4]

The first-order conditions for an extremum require that the partial derivatives of (3) vanish:

(4) $\partial E[U(\Pi)]/\partial Q_1 = E[U'(\Pi)(MR_1 - b)] = 0$

and

$$(5)\quad \partial E[U(\Pi)]/\partial Q_2 = E[U'(\Pi)(MR_2 - b - \beta)] = 0.$$

By using the definition of covariance, we may write (4) and (5) in an analytically more convenient form:

$$(6)\quad E[MR_1] = b - \mathrm{cov}[U'(\Pi),MR_1]/E[U'(\Pi)]$$

and

$$(7)\quad E[MR_2] = b + \beta - \mathrm{cov}\ [U'(\Pi),MR_1]/E[U'(\Pi)].$$

In general, the qualitative nature of these conditions will depend upon the covariance terms. First, suppose the firm exhibits a risk-neutral attitude. In that event, the utility function will be linear and consequently the marginal utility of profit, $U'(\Pi)$, will be constant. This means that the covariance term will be zero. The case of risk neutrality then provides the stochastic analog for the certainty case as can be seen by comparing the present results with those of Bailey and White (1974, p. 78).

Second, suppose the firm expresses risk aversion (preference). In that case, the firm's utility function is concave (convex) and therefore the marginal utility of profit will vary with profit. In particular, the marginal utility of profit will decline (increase) as profit increases. Generally, when demand shifts randomly, the marginal revenue will move in the same direction. But as marginal revenue increases, profit will increase and the marginal utility of profit will decrease (increase). Thus, for a risk-averse (-preferring) firm, the covariance between the marginal utility of profit and marginal revenue will be negative (positive). Since the expected marginal utility of profit $E[U'(\Pi)]$ is always positive, the second term on the righthand side of (6) is negative (positive). Consequently, for a risk averter, optimality requires a smaller output than for a risk-neutral firm because production will stop short of the point where expected marginal revenue equals marginal cost, which is b in period 1 and $b + \beta$ in period 2.

Bailey and White (1974) examined the conditions under which a pricing reversal could occur, i.e., a situation where the peak consumer would pay a lower price than the off peak consumer. In the presence of uncertainty, the conditions are complicated somewhat. If we assume that random shifts in demand change only the intercept and not the slope of the demand functions, we can easily solve equations (6) and (7) for the expected prices:

$$(8)\quad E[P_1] = \left\{ b - \frac{\mathrm{cov}[U'(\Pi), MR_1]}{E[U'(\Pi)]} \right\} \frac{1}{1 - \dfrac{1}{E[\eta_2]}}$$

and

$$(9)\quad E[P_2] = \left\{ b + \beta - \frac{\mathrm{cov}[U'(\Pi), MR_2]}{E[U'(\Pi)]} \right\} \cdot \frac{1}{1 - \dfrac{1}{E[\eta_2]}}$$

where $E[\eta_1]$ and $E[\eta_2]$ represent the expected price elasticities of demand for the offpeak and peak consumers, respectively.

Since the covariance term is zero for the risk neutral firm, its output and expected prices will equal those in the deterministic model. Thus, on average, the uncertain price will equal the deterministic price of Bailey and White's model. In contrast, if the firm is a risk averter, the sign of the covariance will be negative and the desired capacity will be less than the capacity required in the risk-neutral case because $E[MR_2] > b + \beta$ from condition (7). From (9), it is apparent that the expected price will be higher for the risk-averse case than for that of risk neutrality.[5]

Off peak prices are higher on average than peak prices for the unregulated firm attempting to maximize the expected utility of profit whenever the right-hand side of (8) exceeds the right-hand side of (9). This is the stochastic analog of the Bailey and White condition for a pricing reversal. These results are depicted in Figure 1 where $E[P_1]$ exceeds $E[P_2]$ for the risk-neutral case. The expected prices are higher in the risk-averse case due to the influence of the covariance terms. As an example, these are depicted as $E[\hat{P}_1]$ and $E[\hat{P}_2]$.

It should be noted that there is an ambiguity in the final result under uncertainty that does not exist in the nonstochastic case. Its source can be found in conditions (6) and (7); namely, in the covariance terms. It makes most sense to view the covariance term as an addition to marginal cost due to having some risk to bear.[6] There is, however, no reason to suppose that the covariance terms in (6) and (7) should be equal. Consequently, one cannot tell on an a priori basis whether the right-hand side of (7) exceeds that of (6) or the opposite is true. Thus, one may not conclude that if the expected elasticities of peak and offpeak demand were equal, then the offpeak prices would always be lower then the peak prices. This follows because $-\mathrm{cov}[U'(\Pi),MR_1]/E[U'(\Pi)]$ may be sufficiently larger than $-\mathrm{cov}[U'(\Pi),MR_2]/E[U'(\Pi)]$ to cause a pricing reversal. In other words, in the event of a pricing reversal, we do not know whether the cause is differential risk that affects marginal costs or sufficiently different demand elasticities.

Shifting Peak Case. In the shifting peak case, the demands of customers in both periods press on capacity. Consequently, both groups should bear some of the capacity costs. Retaining the assumption of independent, random, equal-length demands, the random profit function is given by

$$(10)\quad \Pi = P_1(u_1)Q_1 + P_2(u_2)Q_2 - b(Q_1 + Q_2) - bQ^*$$

where $Q^* = Q_1 = Q_2$. The objective function of the firm becomes

$$(11)\quad E[U(\Pi)] = E[U(P_1(u_1)Q_1 + P_2(u_2)Q_2 - b(Q_1 + Q_2) - \beta Q^*).$$

The resultant first-order condition is obtained by differentiating (11) with respect to Q^*. Noting that $\partial Q_1/\partial Q^* = \partial Q_2/\partial Q^* = 1$ and using the definition of covariance, we may write the first-order condition as

$$(12)\quad E[MR_1] + E[MR_2] =$$

$$2b + \beta - \frac{\text{cov}[U'(\Pi), MR_1] + \text{cov}[U'(\Pi), MR_2]}{E[U'(\Pi)]}.$$

If the firm is risk neutral, the covariances in (12) are zero and the condition becomes

$$(13)\quad E[MR_1] + E[MR_2] = 2b + \beta,$$

which is the stochastic analog of the certainty case. The pricing solution can be depicted as in Figure 2. Optimal output Q^* is set where the aggregated or combined expected marginal revenue curve intersects the combined operating and capacity cost curve denoted as $2b + \beta$. Thus, the capacity required in both periods is Q^*, but period 1 customers pay only P_1 on average.

If the firm is risk averse, the covariances in (12) are negative. On our earlier interpretation, this means that <u>perceived</u> marginal costs exceed $2b + \beta$ because some payment must be made for risk bearing. Consequently, optimal capacity will be smaller than Q^*, say $\hat{Q}$. As a result, the prices for period 1 and period 2 users will be $\hat{P}_1$ and $\hat{P}_2$, respectively.[7]

II. Impact of Rate of Return Regulation.

The impact of a rate of-return constraint upon the peak load pricing problem was analyzed in a nonstochastic setting by Bailey (1972). In this section, we shall allow the demand functions to be random to examine the influence of uncertainty. Since revenue is random, it is not obvious how one should formulate a rate of return constraint. We shall assume that the constraint requires that the expected return on capital investment not exceed some value.[8]

The Firm Peak Case. We shall specify the rate of return constraint as

$$(14)\quad E[P_1(u_1)Q_1 + P_2(u_2)Q_2 - b(Q_1 + Q_2) - \gamma Q_2] < 0$$

where γ is the allowed return on investment in capacity. Following the usual convention, we assume that $\gamma > \beta$. The random profit in (2) is constrained on average by (14). Consequently, we formulate the firm's objective function as the expected utility of constrained profit, i.e., the firm attempts to maximize

$$(15)\quad E[U(\cdot)] = E[U(P_1(u_1)Q_1 + P_2(u_2)Q_2$$
$$-b(Q_1 + Q_2) - \beta Q_2 - \lambda(\bar{P}_1Q_1 + \bar{P}_2Q_2 - b(Q_1 + Q_2) = \gamma Q_2))]$$

The first-order conditions are obtained from the partial derivatives of (15) with respect to Q_1, Q_2, and λ. The first two can be written as

$$(16)\quad E[MR_1] = b - \frac{\mathrm{cov}[U'(\cdot), MR_1]}{E[U'(\cdot)]}$$

and

$$(17) \quad E[MR_2] = b + \beta - \frac{\lambda}{1-\lambda}(\gamma - \beta) - \frac{cov[U'(\cdot)MR_2]}{E[U'(\cdot)]} .$$

It is clear that risk neutrality yields the stochastic analog to the results under uncertainty. For the risk-neutral firm, the offpeak price is identical to the deterministic model's offpeak price and its output is set where MR = b since demand does not press on capacity. The peak price and output (capacity) are derived in the same manner as in the deterministic case. In Figure 3, we see that the expected surplus profits, $\bar{P}_1AQ_1b$, accrued in the offpeak period can subsidize a deficit in the peak period equal to that amount. The expected peak price in this case would be $\bar{P}_2$ while the output (capacity) would be Q^*. Because the offpeak surplus profits are greater on average than the profits allowed, the firm is induced to price their peak period output below its cost and expand its capacity to take advantage of the offpeak surplus.

If the firm were risk-averse, however, the output for the offpeak demand would be something less than Q_1, say $\hat{Q}_1$, since $E[MR_1] > b$. The expected offpeak price then would rise and the offpeak surplus profit would fall on average. (This assumes that the expected marginal revenue and marginal cost curves for offpeak demand intersect on the elastic portion of its expected demand curve, which is a reasonable assumption for our purposes.) With a reduced offpeak surplus, the peak deficit correspondingly must be reduced. This will mean an increase in the expected peak price, $\bar{P}_2$ to $\hat{P}_2$ and a decrease in the output (capacity) from Q^* to $\hat{Q}_2$.

<u>The Shifting Peak Case</u>. A similar analysis may be applied to the shifting peak case. The firm's objective function (15) is basically the same except that Q_1 and Q_2 are equal. As a result the first-order condition is given by

$$(16)\quad E[MR_1] + E[MR_2] = 2b + \beta - \frac{\lambda}{1-\lambda}(\gamma - \beta) - \frac{cov[U'(\cdot),MR_1] + cov[U'(\cdot),MR_2]}{E[U'(\cdot)]} .$$

The result for the regulated, risk-neutral firm is the stochastic analog to the deterministic case. Since the covariances will be negative for the risk-averse firm, it follows that $E[MR_1] + E[MR_2] > 2b + \beta - \frac{\lambda}{1-\lambda}(\gamma - \beta)$. As a result, the desired capacity will be less for the risk-averse firm than for the risk-neutral firm. The pricing solutions may be examined in Figure 4 for the shifting peak case.

In the shifting peak case, offpeak surpluses will not exist and the regulated deterministic firm (or the regulated risk-neutral firm) will set capacity where the aggregate (expected) price level will generate the allowed rate of return. In Figure 4, this will be where $\bar{P} = 2b + \gamma$ where period 1 users will pay an expected price of $\bar{P}_1$ for their output Q^* and period 2 users pay only $\bar{P}_2$ on average for the same output Q^*. Intuitively, we can see that the area $\bar{P}_c AQ^*b$ allows the firm to recover the market cost of its capital as well as an additional $\bar{P}_c\bar{P}_1$ per unit of capital due to γ being larger than β.

The pricing differential between the two peak demands corresponds to their intensity and elasticity of demand. If the marginal capacity costs were higher and the difference between γ and β remained constant, then both $2b + \gamma$ and $2b + \beta$ would shift upward and the difference between the peak prices would be reduced.

If the firm were risk-averse, we know that it will choose to produce at a capacity somewhere below the risk-neutral firms level Q^*, say $\hat{Q}$. Figure 4 shows that expected prices then rise to $\hat{P}_1$ and $\hat{P}_2$ where $\hat{P}_2$ experiences a more substantial increase because of the difference in demand elasticity.

III. Concluding Remarks.

For both the regulated and unregulated case, we have examined the influence of random demand upon the peak load pricing solution in an expected utility framework. The results for the case of linear risk preferences provide the stochastic analog of the deterministic model. When risk aversion prevails, we should find that the firm acts cautiously. In the context of the models we have analyzed, this generally means that optimal capacity is reduced as is the optimal output. Consequently, expected prices rise. Of course, it is not surprising to find that the risk-averse firm acts cautiously, but it is of some interest to know what this means in the peak load setting.

Eootnotes

*. The authors are indebted to the Public Utility Research Center at the University of FLorida for financial support.

1. See the brief, historical overview in Steiner (1971).

2. The problem of maximizing social welfare was addressed by Brown and Johnson (1969) and further refined by Crew and Kleindorfer (1978) and Sherman and Visscher (1978).

3. The Steiner (1957) assumption of equal length periods is employed in this analysis. Although we recognize that Williamson (1966) provides a somewhat more general formulation, the equal length periods facilitate our graphcial presentation.

4. Meyer (1976) examines a similar problem in the framework provided by the capital asset pricing model. This is an interesting approach to the problem.

5. The reverse holds for the risk-preferring firm. In other words, capacity will be higher and prices lower than in the risk-neutral case.

6. Baron (1970) shows that the covariance term can usefully be interpreted as the marginal change in the risk premium as output changes. Since the risk premium is the payment that must be made to induce the firm to bear a given risk, the cost interpretation has an obvious appeal.

7. For the risk seeker, optimal capacity will be larger and prices will be lower than for the risk neutral case.

8. This is the approach adopted by Meyer (1979).

References

Bailey, E.E. "Peak-Load Pricing under REgulatory Constraint," Journal of Political Economy, Vol. 80 (July/August 1972), pp. 662-679.

Bailey, E.E. and White, L.J., "Reversals in Peak and Offpeak Prices," Bell Journal of Economics and Management Science, Vol. 5 (Spring 1974), pp. 75-92.

Baron, D.P. "Price Uncertainty, Utility, and INdustry Equilibrium in Pure Competition," International Economic Review, Vol. 11 (October 1970), pp. 463-480.

Blair, R.D., "Random Input Prices and the THeory of the FIrm," Economic Inquiry, Vol. 12 (June 1974), pp. 214-226.

Boiteux, M. "Peak Load Pricing," in Marginal Cost Pricing Practice, ed. by J.R. Nelson, Englewood Cliffs, N.J.: Prentice-Hall, 1964.

Brown, G. and Johnson, M.B. "Public Utility Pricing and Output under Risk," American Economic Review, Vol. 59 (March 1969), pp. 119-128.

Crew, M.A. and Kleindorfer, P.R., "Reliability and Public Utility Pricing," American Economic Review, Vol. 68 (March 1978), pp. 31-40.

Leland, H., "Theory of the Firm Facing Uncertain Demand," American Economic Review, Vol. 62 (June 1972), pp. 278-291.

Meyer, R.A., "Risk-Efficient Monopoly Pricing for the Multiproduct Firm," Quarterly Journal of Economics, Vol. 90 (August 1976), pp. 461-474.

Meyer, R.A., "Regulated Monopoly Under Uncertainty," Southern Economic Journal, Vol. 45 (April 1979), pp. 1121-1129.

Sherman, R. and Visscher, M., "Second Best Pricing with Stochastic Demand," American Economic Review, Vol. 68 (March 1978), pp. 41-53.

Steiner, P.O., "Peak Loads and Efficient Pricing," Quarterly Journal of Economics, Vol. 71 (November 1957), pp. 585-610.

Steiner, P.O., "Peak Load Pricing Revisited," in Harry M. Trebing, ed. Essays on Public Utility Pricing and Regulation, East Lansing: Michigan State University Press, 1971.

Williamson, O.E., "Peak Load Pricing and Optimal Capacity under Indivisibility Constraints," American Economic Review, Vol. 56 (September 1966), pp. 810-827.

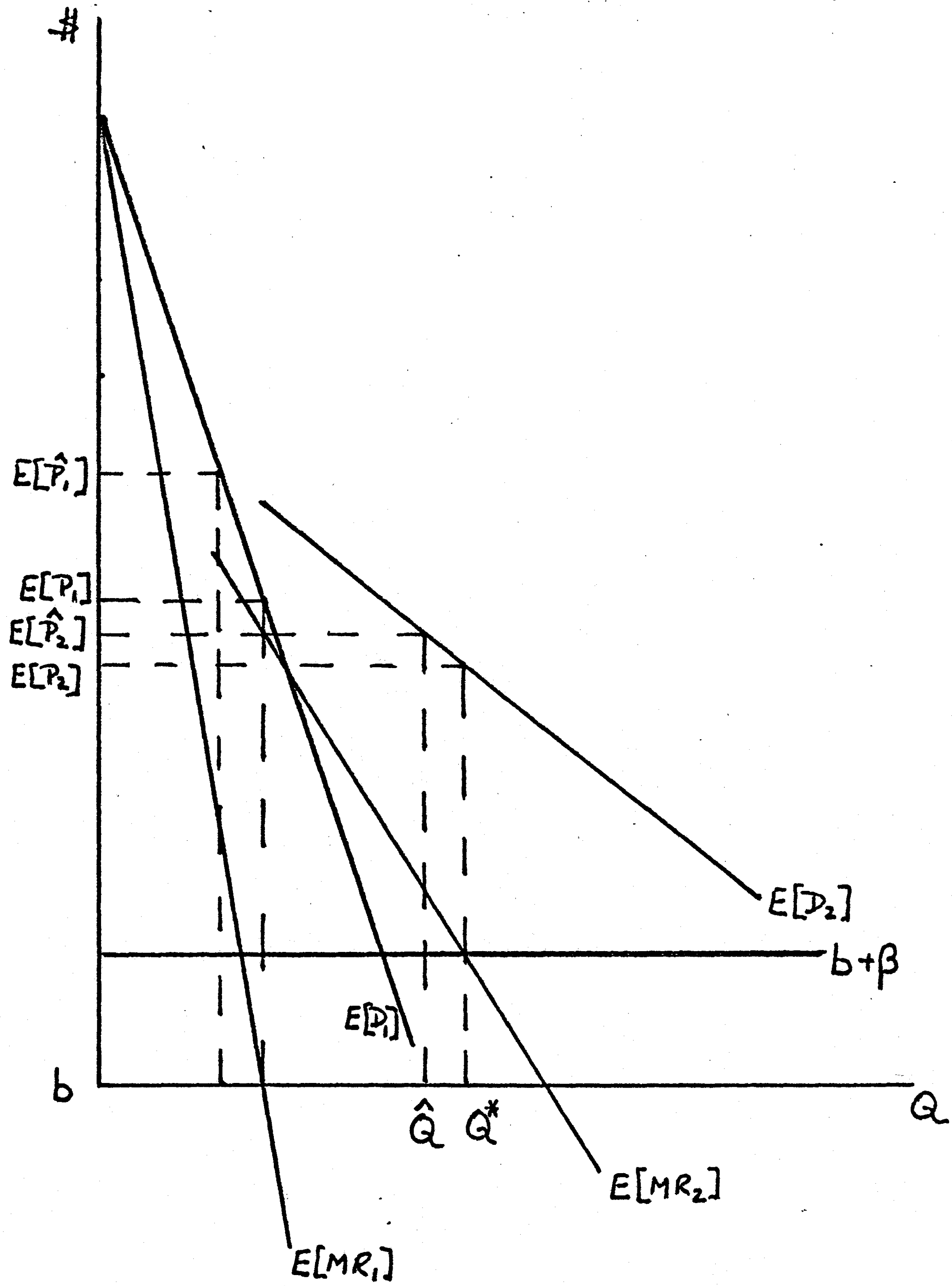

FIGURE 1

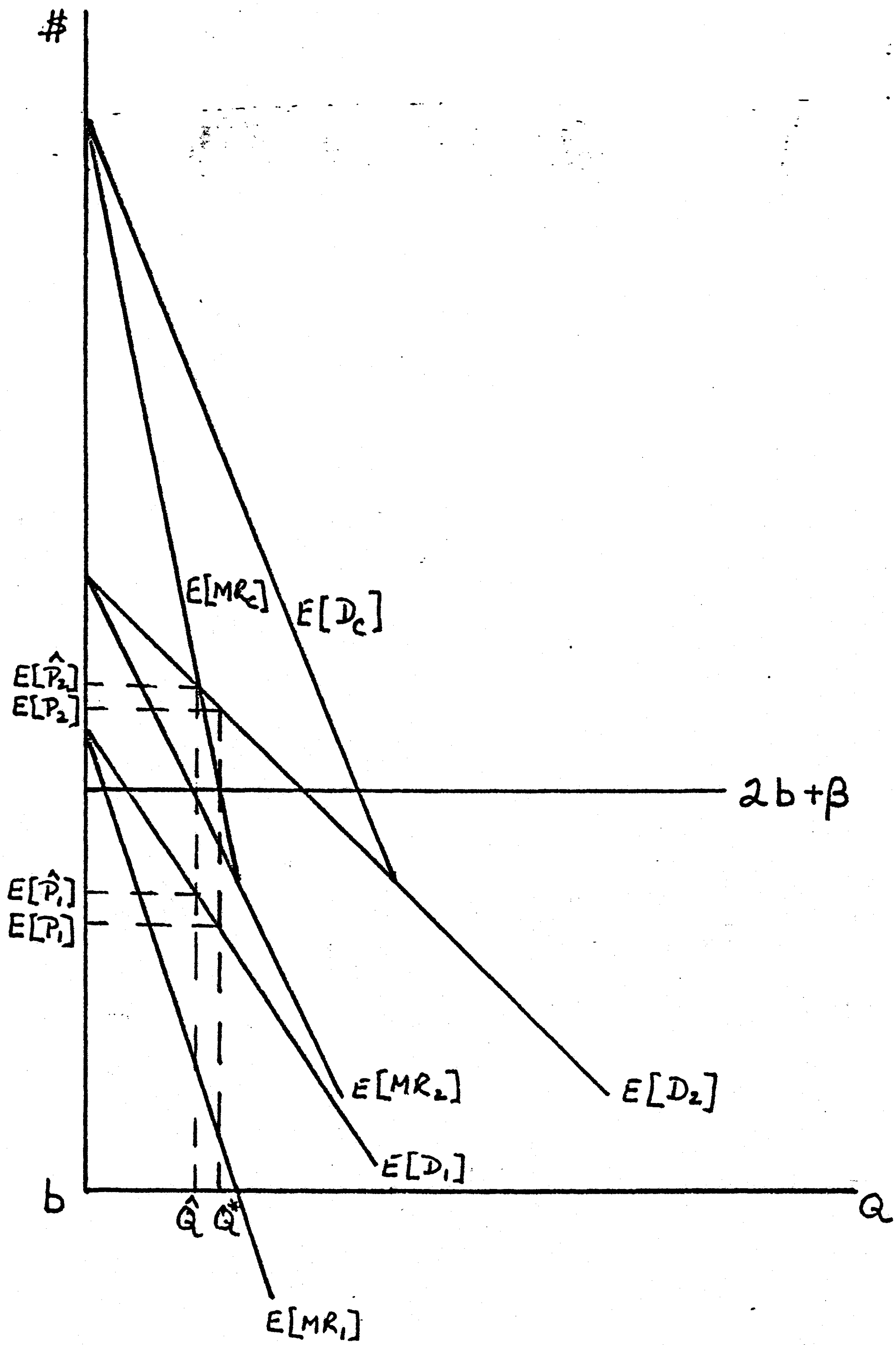

FIGURE 2

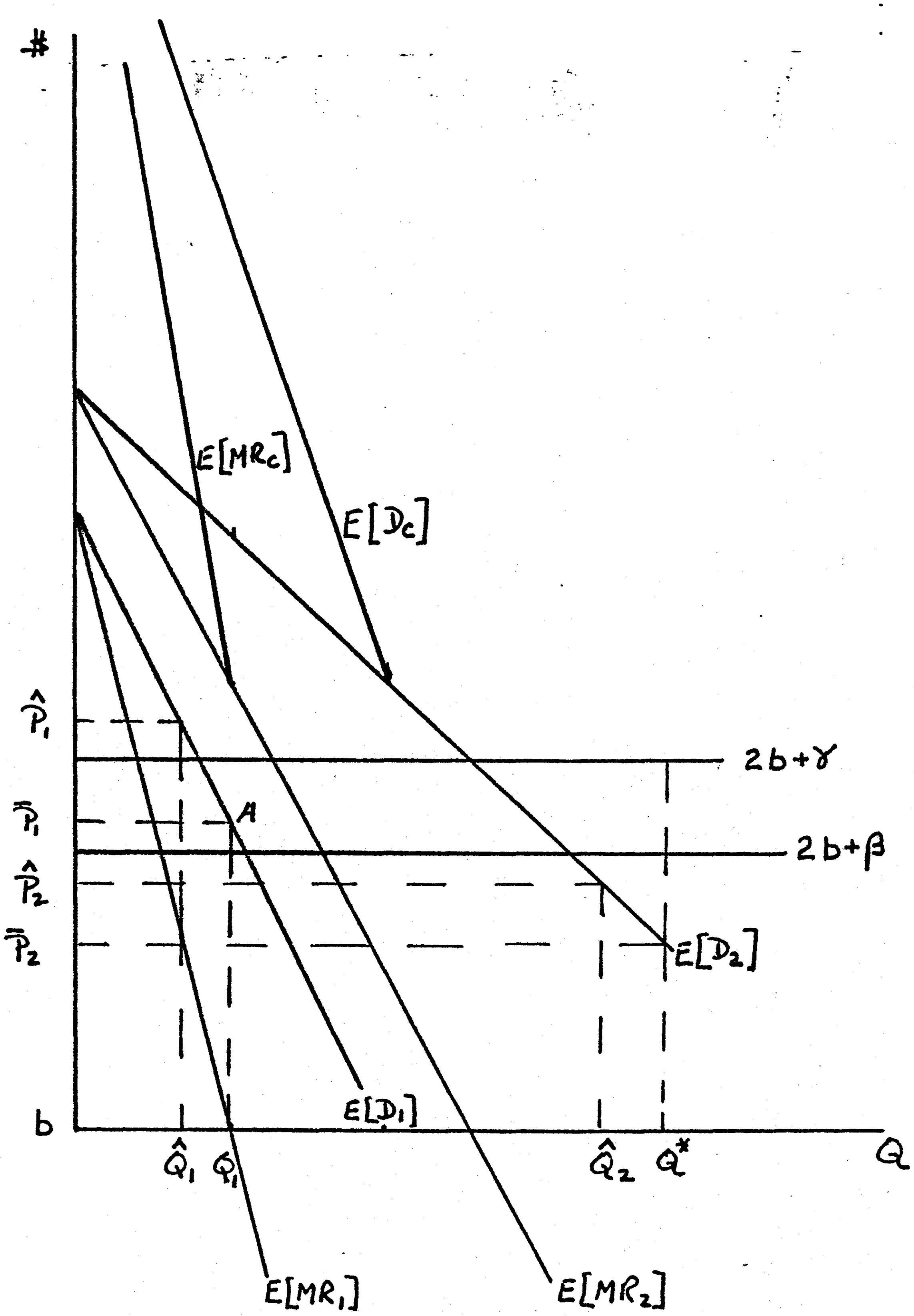

FIGURE 3

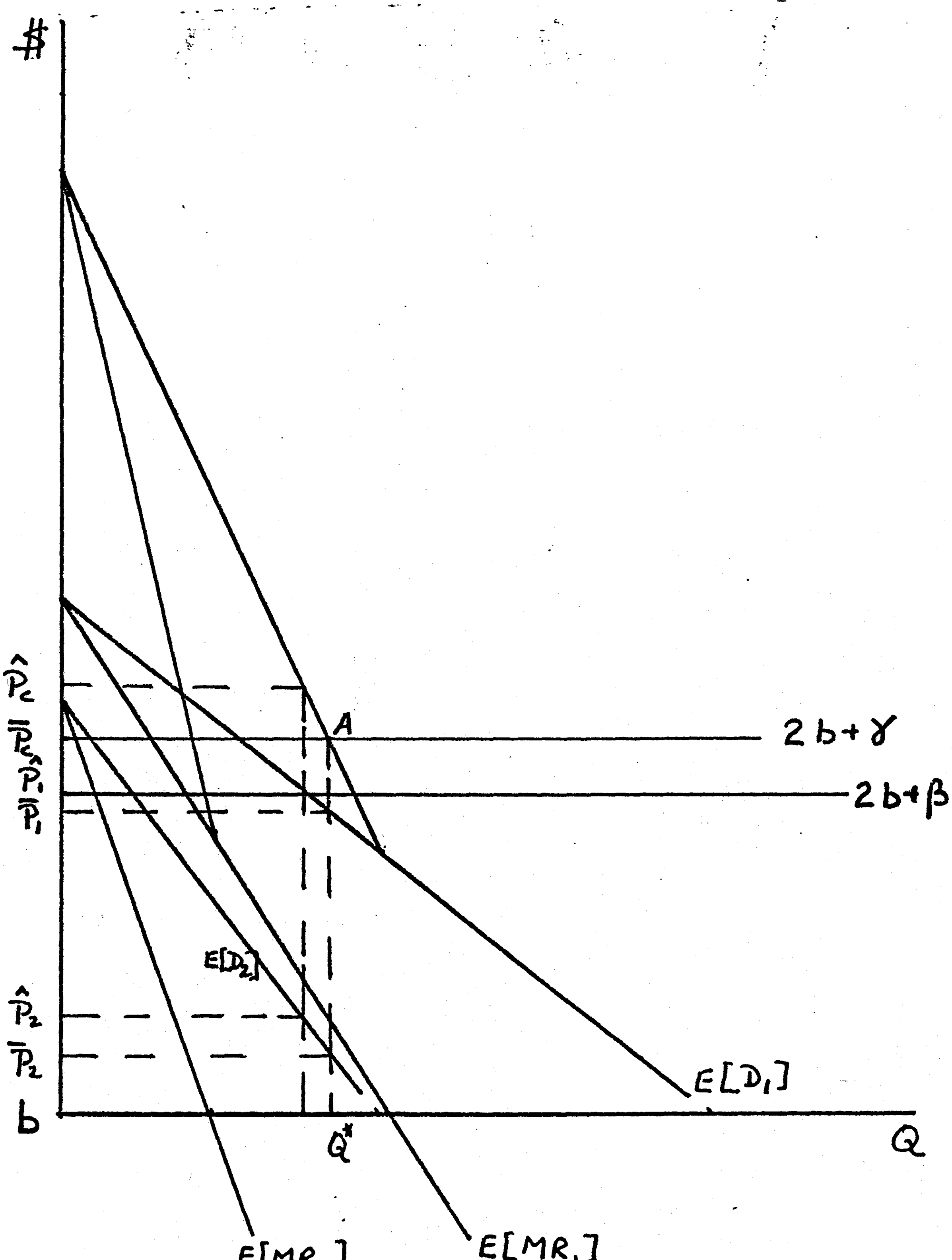
$
$\hat{P}_c$
$\bar{P}_c$
$\hat{P}_1$
$\bar{P}_1$
A
$2b+\gamma$
$2b+\beta$
$E[D_2]$
$\hat{P}_2$
$\bar{P}_2$
$E[D_1]$
b
Q^*
Q
$E[MR_2]$
$E[MR_1]$

FIGURE 4