RISK MEASUREMENT AND THE COST OF EQUITY
CAPITAL: THE CAPM APPROACH

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THE CAPM APPROACH*

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The purpose of this paper is to present some of the elements of a new approach to estimating risk and relating it to the cost of equity capital-- the capital asset pricing model (CAPM) approach. Although the CAPM has not been widely used in rate cases at the state level, it has been used at the federal level and in civil cases, and it is today the most popular approach in academic circles. Although the CAPM will not surplant the more traditional approaches, I anticipate that it will gain increasing recognition and be used, along with the traditional approaches, to measure risk in rate case cost of capital studies.

BASIC ASSUMPTIONS OF THE CAPM

Like all financial theories, a number of assumptions were made in the development of the CAPM; these were summarized by Jensen (1972, Bell Journal) as follows:

1. All investors are single-period expected utility of terminal wealth maximizers who choose among alternative portfolios on the basis of mean and variance (or standard deviation) of return.

2. All investors can borrow or lend an unlimited amount at an exogeneously given risk-free rate of interest, R_f, and there are no restrictions on short sales of any asset.

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*This paper draws heavily upon J. F. Weston and E. F. Brigham, Managerial Finance, 4th edition (New York: Holt, Rinehart & Winston, 1972), and upon materials being developed for the 5th edition of the book, scheduled for publication in March 1975.

3. All investors have identical subjective estimates of the means, variances, and covariances of return among all assets.

4. All assets are perfectly divisible, perfectly liquid (i.e., marketable at the going price), and there are no transactions costs.

5. There are no taxes.

6. All investors are price takers.

7. The quantities of all assets are given.

While the assumptions may appear to be limiting, they are similar to those made in the standard economic theory of the firm and in the basic models of Modigliani-Miller, Gordon, and others. Further, extensions in the literature that seek to relax the basic CAPM assumptions yield results that are generally consistent with the basic theory. Also, statistical investigations of the model derived from the basic CAPM formulation have obtained results reasonably consistent with these other models. Finally, the CAPM has been used in several rate cases and civil court cases, and its advocates have stood up quite well under intense and expert cross-examination.

SECURITY RISK VERSUS PORTFOLIO RISK

It can be shown that the riskiness of a portfolio of assets as measured by its standard deviation of returns is generally less than the average of the risks of the individual assets as measured by their standard deviations.[1] This phenomenon, in turn, has direct implications for the cost of capital: since investors generally hold portfolios of securities, not just one security, it is reasonable to consider the riskiness of a security in terms of its contribution to the riskiness of the portfolio rather than in terms of its riskiness if held in isolation. The significant contribution of the capital asset

[1] See Managerial Finance, 4th edition, Appendix C to Chapter 8.

pricing model (CAPM) is that it provides a measure of the risk of a security in the portfolio sense.

An empirical study by Wagner and Lau can be used to demonstrate the effects of diversification.[2] They divided a sample of 200 NYSE stocks into six subgroups based on the Standard and Poor's quality ratings as of June 1960. Then they constructed portfolios from each of the subgroups, using 1 to 20 randomly selected securities and applying equal weights to each security. For the first subgroup (A+ quality stocks), Table 1 summarizes some effects of diversification. As the number of securities in the portfolio increases, the standard deviation of portfolio return decreases, but at a decreasing rate, with further reductions in risk being relatively small after about ten securities are included in the portfolio. More will be said about the third column, correlation with the market, shortly.

Table 1. Reduction in Portfolio Risk Through Diversification

No. of Securities in Portfolio	Standard Deviation of Portfolio Returns (σ_p) (% per month)	Correlation with Return on Market Index*
1	7.0	.54
2	5.0	.63
3	4.8	.75
4	4.6	.77
5	4.6	.79
10	4.2	.85
15	4.0	.88
20	3.9	.89

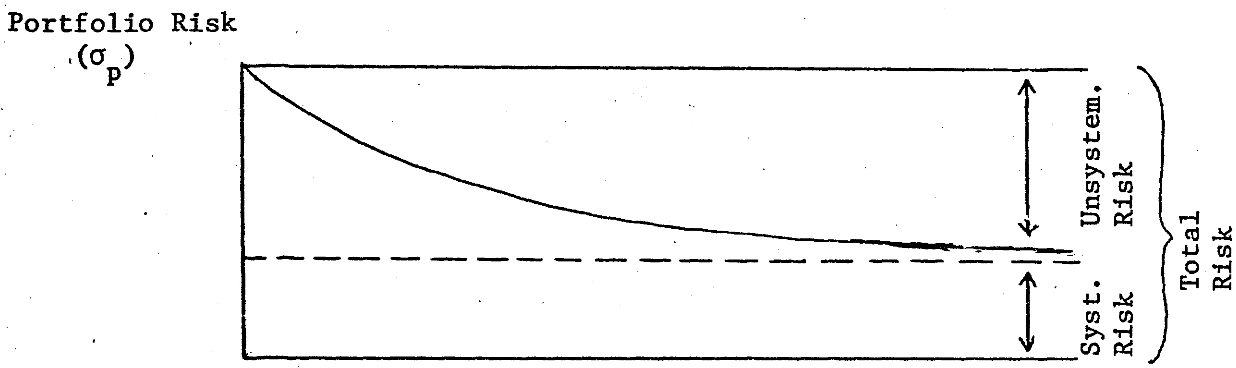
*The market here refers to an unweighted index of all NYSE stocks.

The data in Table 1 indicate that even well-diversified portfolios possess some level of risk that cannot be diversified away. Indeed, this is

[2] W. H. Wagner and S. C. Lau, "The Effect of Diversification on Risk," Financial Analysts Journal, November-December 1971, pp. 48-53.

exactly the case, and the general situation is illustrated graphically in Figure 1. The risk of the portfolio, σ_p, has been divided into two parts. The part that can be reduced through diversification is defined as unsystematic risk, while the part that cannot be eliminated is defined as systematic, or market-related risk. Unsystematic risk can be diversified away, but systematic risk cannot be eliminated no matter how well the portfolio is diversified.[3]

Figure 1



Now refer back to the third column of Table 1. Notice that as the number of securities in each portfolio increases, and as the standard deviation decreases, the correlation between the return on the portfolio and the return on the market index increases. Thus, a broadly diversified portfolio is highly correlated with the market, and its risk is (1) largely systematic and (2) arises because of general market movements.

We can summarize our analysis of risk to this point as follows:

1. The risk of a portfolio can be measured by the standard deviation of its rate of return, σ_p.
2. The risk of an individual security is its contribution to the portfolio's risk.
3. The standard deviation of a stock's return, σ_i, is the relevant measure of risk for an undiversified investor who holds only stock i.

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[3] In the real world, it is extremely difficult to find stocks with zero or negative correlations; hence, some risk is inherent in any stock portfolio.

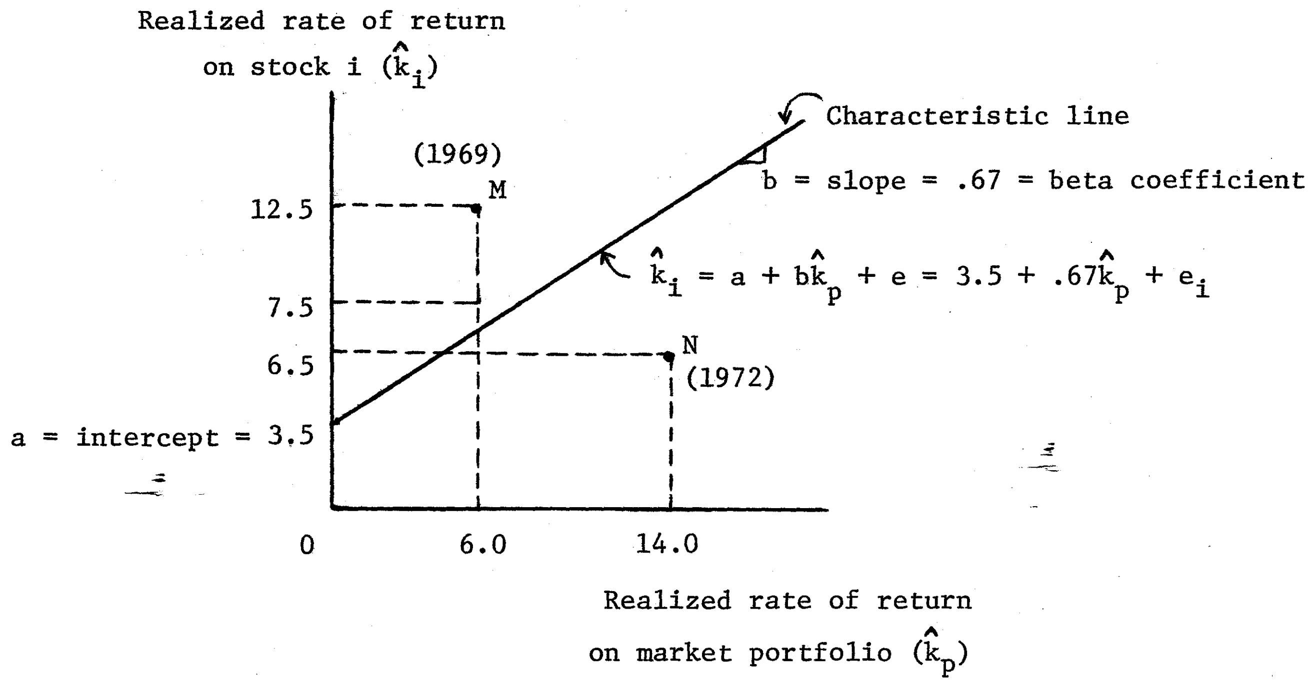
4. A stock's standard deviation reflects both unsystematic risk that can be eliminated by diversification and systematic or market-related risk; only the systematic component of security risk is relevant to the well-diversified investor, so only this element is reflected in the risk premium.

5. A stock's sytematic risk is measured by its volatility in relation to the general market. This factor is analyzed next.

THE SHARPE MODEL: BETA COEFFICIENTS

A procedure has been developed to separate the standard deviation of returns on an individual stock into its systematic and unsystematic risk components (Sharpe, 1963). The systematic risk of a given stock can be measured by its tendency to move with the general market, using the procedure illustrated in Figure 2.[4] The returns on any stock i are presumed to bear a linear relationship

Figure 2. Correlation of Stock i with the "Market"



[4]The Dow-Jones Industrials, S&P 500, New York Stock Exchange Index, or any other market indicator could be used as "the portfolio."

of the following form to those of the market portfolio:

$$k_i = a_i + b_i k_p + \varepsilon_i. \tag{1}$$

The line shown in Figure 2, which is simply a graph of equation (1) for our hypothetical stock, is called the security's <u>characteristic line</u>. The return on the i^{th} stock (k_i) is equal to an intercept term (a), plus a regression coefficient (b) times the market portfolio return (k_p), plus a random error term (ε_i). The term (b), often called the <u>beta coefficient</u>, is generally positive, indicating that when the market return goes up, the return on the stock in question also goes up, and conversely.

However, any firm i also faces events that are peculiar to it and independent of the general economic climate, and such events tend to cause the returns on firm i's stock to move independently from those for the market as a whole. For example, Ethyl Corporation, which makes gasoline additives, suffered a sharp price decline despite a rising market when reports of the harmful effects of lead in auto exhausts began coming out. Such an event is accounted for in equation (1) by the "shock" term, ε_i.[5]

To illustrate the nature of the relationship, consider point M in Figure 2, which represents the observation for 1969. The market in that year provided an average return of 6 percent, which would lead us to expect a 7.5 percent return on stock i according to our fitted regression line in Figure 2: [3.5 + .67 (6) = 7.5]. However, something favorable happened to firm i in 1969 apart from what happened to the rest of the market, causing its stock to yield 12.5 percent; in 1969 the shock term is positive and is equal to 5 percent: [3.5 + .67 (6) + ε = 12.5, so ε = 5 percent]. At point N, which might represent the observation for 1972, we have the opposite situation: the market return was 14 per-

[5]The shock or error term is assumed to be independent from one period to the next. The error terms for any two stocks are also assumed to be independent, and independent of what happens to the "market."

cent, and i's return was only 6.5 percent, so ε is equal to -6.38 percent.

Systematic, or Nondiversifiable, Risk

The regression coefficient b, or the beta coefficient, measures the extent to which the return on firm i's stock moves with the market. If b = 1.0, then on the average we expect i's rate of return to rise or fall in direct proportion to changes in market returns. Thus, if the market return falls one year by 2 percentage points, say from a 10-percent return to an 8-percent return, we can expect i's rate of return to fall by 2 percentage points. This tendency of an individual stock to move with the market constitutes a risk, because the market does fluctuate, and these fluctuations cannot be diversified away. This component of total risk is the stock's systematic, or nondiversifiable, risk.

A beta of 1.0 indicates a stock with "average" systematic risk--on the average, such a stock rises and falls by the same percentage as the market. What does b = .5 indicate?[6] This means that if the market return rises or falls by X percentage points, firm i's rate of return rises or falls by only .5X, so this stock has less systematic risk than the average stock. Conversely, if beta = 2.0, then the stock's rate of return fluctuates twice as much as the market rate of return, so it has more than the average systematic risk. From this it follows that the size of beta is an indicator of systematic risk: the larger the value of beta, the greater a stock's systematic, or nondiversifiable, risk.

Unsystematic Risk

The actual observations in Figure 2 can lie close to the regression line or be widely scattered around it. As we have seen, this scatter is due to events

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[6]A technical point should be noted here. In general, if a stock has b = 1.0, then its intercept, a, should approximate zero; if b > 1.0, it will tend to have a < 0; and if b < 1.0, then a > 0. This is logical, for if b > 1.0 and a > 0, this would imply that investors expect the stock to do better than the average stock in "good markets" and in most bad markets. This would be a disequilibrium, so the stock's price would be bid up and its expected yield driven down.

that affect firm i but not "the market." Since these events are random and are no more likely to be positive than negative, the expected effect of the shocks is zero. Through diversification, the effect of random events for all stocks in the portfolio can be made to approach zero. The plusses and minuses for the individual firms will, if the sample is large enough, cancel out, and the portfolio will simply move with the market. Thus, the scatter around the regression line is termed "unsystematic," or diversifiable, risk.

Mathematical Treatment

The discussion up to this point can be treated mathematically, separating the i^{th} stock's variance, σ_i^2, into systematic and unsystematic components. Once this has been done, we can see what determines i's ultimate riskiness to an investor.[7]

Step 1. Define the variance of returns on stock i, σ_i^2, as follows:

$$\sigma_i^2 = \sum_{j=1}^{n} P_{ij} (k_{ij} - k_i)^2. \tag{2}$$

Here k_{ij} is the j^{th} possible return on stock i; P_{ij} is the probability of occurrence of the j^{th} return; and k_i is the expected value of the i^{th} firm's rate of return. Note that we use k, not k* or $\hat{k}$, because we are now referring to expected returns.

Step 2. The j^{th} return on stock i is found as follows:

$$k_{ij} = a + bk_{pj} + e_{ij}. \tag{3}$$

Here k_{pj} is the j^{th} possible return on the market portfolio.

Step 3. Under the assumption that the expected value of the shock term is zero, the expected return on stock i is found as follows:

[7] One can skip the derivations in Steps 1 through 6 of this section, going directly to Step 7, the interpretation, without loss of continuity.

$$k_i = a + bk_p. \tag{4}$$

Here k_p is the expected return on the market portfolio.

Step 4. Substituting equations (3) and (4) for k_{ij} and k_i in equation (2), we obtain:

$$\sigma_i^2 = \sum_{j=1}^{n} P_{ij} (a + bk_{pj} + e_{ij} - a - bk_p)^2. \tag{5}$$

Step 5. Canceling and rearranging terms, we form the following expression:

$$\begin{aligned}
\sigma_i^2 &= \sum_{j=1}^{n} P_{ij} [(bk_{pj} - bk_p) + e_{ij}]^2 \\
&= \sum_{j=1}^{n} P_{ij} [(bk_{pj} - bk_p)^2 + 2 (bk_{pj} - bk_p) e_{ij} + e_{ij}^2] \\
&= \sum_{j=1}^{n} P_{ij} [b^2 (k_{pj} - k_p)^2 + 2b (k_{pj} - k_p) e_{ij} + e_{ij}^2] \\
&= \sum_{j=1}^{n} P_{ij} [b^2 (k_{pj} - k_p)^2] + \sum_{j=1}^{n} P_{ij} [2b (k_{pj} - k_p) e_{ij}] \\
&\quad + \sum_{j=1}^{n} P_{ij} e_{ij}^2 \\
&= b^2 \sum_{j=1}^{n} P_{ij} (k_{pj} - k_p)^2 + 2b \sum_{j=1}^{n} P_{ij} [e_{ij} (k_{pj} - k_p)] \\
&\quad + \sum_{j=1}^{n} P_{ij} e_{ij}^2.
\end{aligned}$$

Since $\sum_{j=1}^{n} P_{ij} e_{ij} = 0$, i.e., the expected value of the error term equals zero, the middle term drops out, and we are left with

$$\sigma_i^2 = b^2 \sum_{j=1}^{n} P_{ij} (k_{pj} - k_p)^2 + \sum_{j=1}^{n} P_{ij} e_{ij}^2. \tag{6}$$

Step 6. The summation part of the first term to the right of the equal sign in equation (6) is, by definition, the variance of returns on the market portfolio, σ_{kp}^2. Further, the second term is the variance of stock i's error term, σ_e^2:

$$\sigma_e^2 = \sum_{j=1}^{n} P_{ij} [e_{ij} - E(e_{ij})]^2$$

where E (e_{ij}) is the expected value of the error term. But since E $(e_{ij}) = 0$, then

$$\sigma_e^2 = \sum_{j=1}^{n} P_{ij} e_{ij}^2.$$

Thus, we may rewrite equation (6) as follows:

$$\begin{aligned} \sigma_i^2 &= b^2\sigma_{kp}^2 + \sigma_e^2 \\ &= (b\sigma_{kp})^2 + \sigma_e^2. \end{aligned} \tag{7}$$

Step 7: Interpretation. Equation (7) divides the variance of stock i's return into two parts, the systematic risk component, $(b\sigma_{kp})^2$, which is dependent on the beta coefficient as well as the variability of market returns, and the unsystematic residual risk component, σ_e^2. The unsystematic component can be eliminated through diversification, but the systematic component can only be reduced by altering the firm's correlation with the "market," that is, by attempting to change its beta coefficient through a change in investment or financial policy.

The logical conclusion of all this is that, if investors think in portfolio terms, then they should not worry about the unsystematic risk because it can be diversified away. Thus, investors should consider only systematic risk, $(b\sigma_{kp})^2$ is equation (7). Since the variance of the market is a given, the determinant of relative riskiness among stocks is the beta coefficient. Accordingly, in the equation $k^*_i = R_F + \rho_i$, where k* is the cost of equity, R_F is the riskless rate of return, and ρ_i is the i^{th} asset's risk premium, the key determinant of ρ_i for any given security is its beta coefficient. Thus, if a particular firm's stock has a beta of 1.5, it is twice as risky as another stock with a beta of .75, and its risk premium should be twice as large.

THE TRADEOFF BETWEEN RISK AND RETURN

Since investors as a group are averse to risk, the higher the risk of a stock, the higher its required rate of return. Figure 3 illustrates this concept. Here the required rate of return is plotted on the veritcal axis and risk is shown on the horizontal axis. The line showing the relationship between risk and rate of return is defined as the capital market line (CML).[8] The intercept of the capital market line, R_F, is the riskless rate of return, generally taken as the return on U. S. Treasury securities. Riskless securities have beta coefficients equal to zero, indicating that they do not move at all with changes in the market. An "average" stock has a beta of 1.0, and such a stock has a required rate of return, k^*_M, equal to the market average return. A relatively low-risk stock might have a beta of .6 and a required rate of return equal to k^*_L, while a relatively high-risk stock might have a beta of 1.4 and a required return equal to k^*_H.

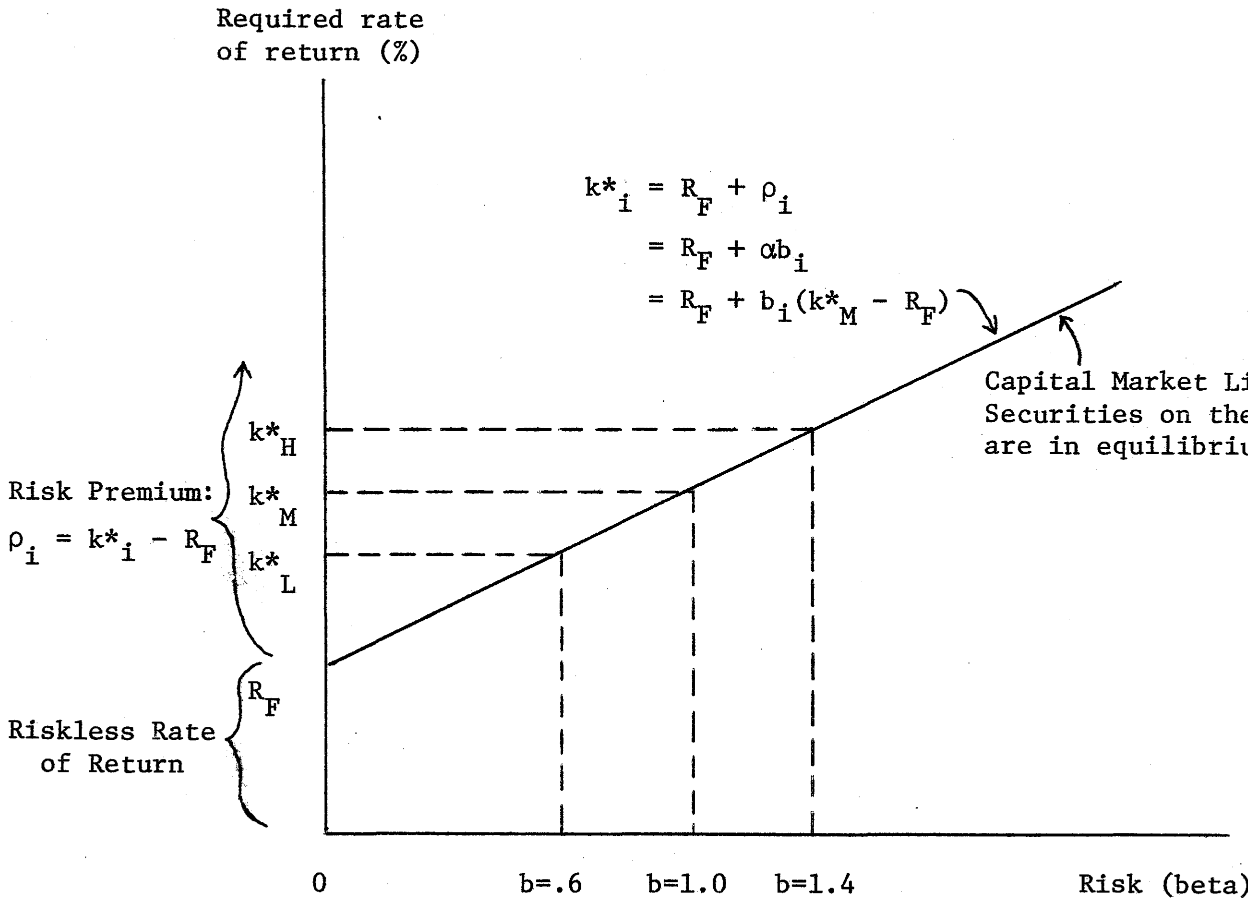
BETAS OF PORTFOLIOS

It should be noted that a portfolio made up of low beta securities will itself have a low beta, as the beta of any set of securities is a weighted average of the individual securities:

$$b_p = \sum_{i=1}^{n} x_i b_i .$$

[8]The reason why the CML is linear is explained in the appendix. Also, it should be noted that in formal developments of the CAPM a distinction is made between the CML and the SML, or "security market line." We ignore this distinction here.

Figure 3. The Tradeoff Between Risk and Return: The Capital Market Line (CML)



Here b_p is the beta of the portfolio, which reflects how volatile the portfolio is in relation to the market index; x_i is the percentage of the portfolio invested in the i^{th} stock; and b_i is the beta coefficient of the i^{th} stock.

The beta coefficients of mutual funds, pension funds, and other large portfolios are today being calculated and used to judge the riskiness of the portfolio, and mutual funds are actually being constructed to provide investors with specified degrees of riskiness. It is too early to judge how well betas will work as a measure of long-term risk, but the financial community is actually using the concept in security selection and portfolio construction.

Cost of Capital Dynamics

The required return on any stock i is equal to the riskless rate of return plus a risk premium, $k_i^* = R_F + \rho_i$. Since the risk premium for the entire market is equal to $(k^*_M - R_F)$, we can develop the following equation for any individual stock i:[9]

$$k_i^* = R_F + b_i\ (k^*_M - R_F). \tag{8}$$

In words, the expected return on any stock is equal to the sum of the riskless rate of return plus the product of the stock's beta coefficient times the risk premium on the market as a whole. If beta is less than 1.0, then the stock has a smaller than average risk premium, while if beta is larger than 1.0, the converse holds.

Interest rates change violently over time, and when they do, the change in R_F is reflected in the cost of equity both for the "average" stock, k^*_M, and for any individual stock, k^*_i. Such changes cause the capital market line in Figure 3 to shift. The intercept term, R_F, goes up or down, while the slope of the line could remain constant, increase, or decrease.[10]

Risk premiums, which are reflected in the slope of the market line, may also change over time. When investors are pessimistic and worried, the market

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[9]Equation (8) is developed as follows: First, the risk premium is a linear function of the stock's beta coefficient, i.e., $\rho_i = f(b_i) = \alpha b_i$. Accordingly, $k_i^* = R_F + \alpha b_i$. By definition, the market as a whole has a beta of 1.0--the average stock must move in proportion to the market, and if this is so, then beta is 1.0. Therefore, this average or market risk premium must be equal to α, i.e., $\alpha = \rho_m$. Thus, $k_i^* = R_F + b_i\rho_m$. But the market risk premium is equal to $(k^*_M - R_F)$, so for any stock i we have the following equation:

$$k_i^* = R_F + b_i\ (k^*_M - R_F).$$

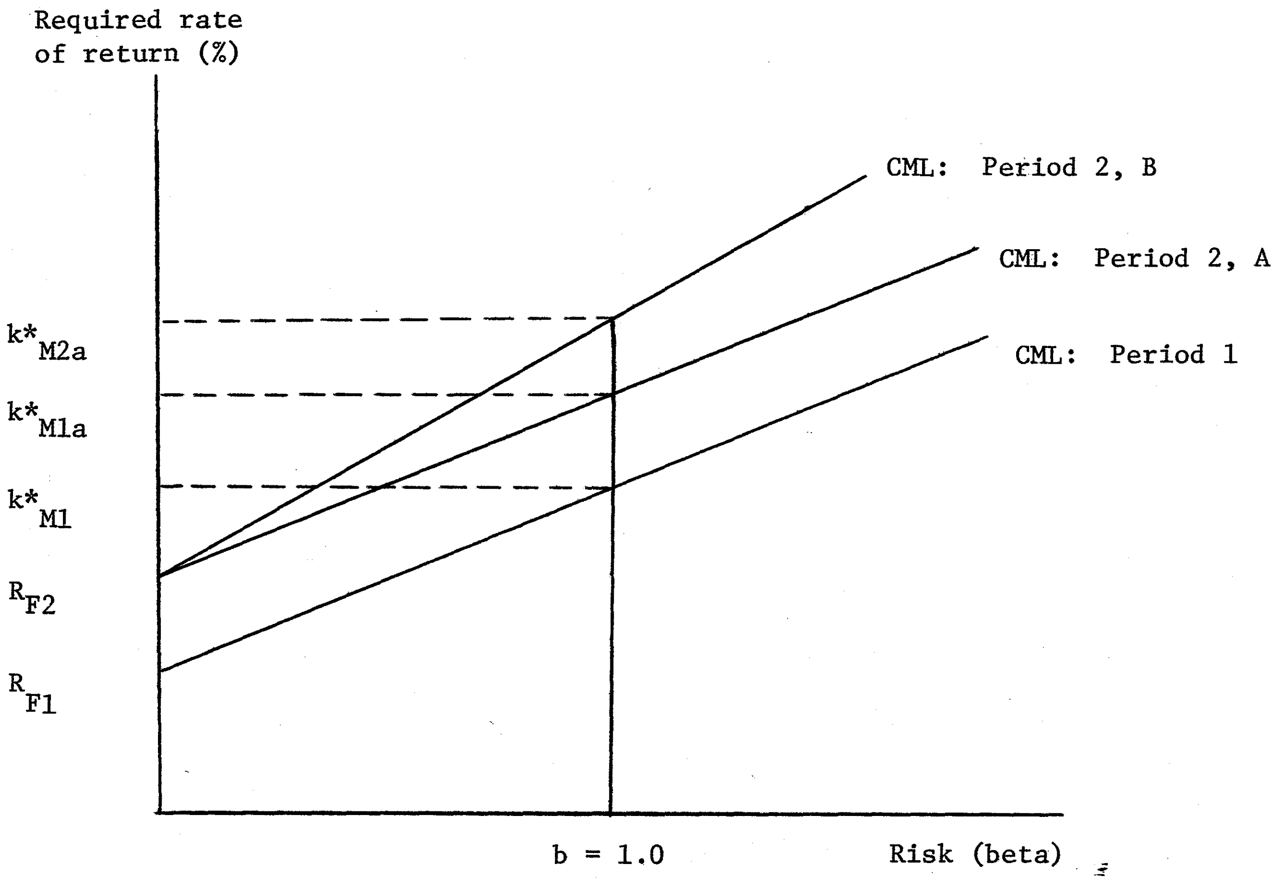
[10]R. H. Litzenberger and A. P. Budd, "Secular Trends in Risk Premiums," Journal of Finance, September 1972, pp. 857-864.

line of Figure 3 will tend to be steep, implying a high "price of risk," whereas when investors are less risk averse, the price of risk declines and the market line is less steeply inclined.

Graph of Market Dynamics[11]

Figure 4 shows several possible capital market lines. The first is for a

Figure 4. Possible Relationships Between Risk and Rates of Return



[11] Most of the formal literature on the CAPM is in a single period context, not a dynamic context. Researchers are working on the dynamics of the model, but our discussion at this point is based on our intuitive judgment, rather than on formal CAPM theory, and as a result, our discussion is highly simplified.

period of low interest rates, when R_F is down at R_{F1}. Now suppose conditions change, and interest rates rise. This will cause R_F to increase, say to R_{F2}. The market line will rise, but how much? It could make a parallel upward shift, with the slope coefficient of the market line remaining constant, as in line A for period 2. In this case, risk premiums would not change. The slope could also increase, implying an increase in risk premiums, as in line B for period 2, or it could be less steep in period 2.

Empirical studies suggest that the slope of the capital market line varies over time, but no consistent relationship has been found between the level of interest rates and the slope of the line. Accordingly, it is necessary either to analyze the slope coefficient to determine if it has changed or, if not enough data are available to undertake such an analysis, to assume that the market line undergoes a parallel shift when R_F shifts.

ESTIMATING THE CAPITAL MARKET LINE

Three steps are involved in using the capital asset pricing model to determine a firm's cost of equity capital: (1) estimate the capital market line, (2) estimate the firm's beta coefficient, and (3) read off the firm's cost of equity capital. The first of these steps is described in this section, while the next two steps, using Comsat as an example, are described in later sections.[12]

A number of careful studies confirm that required rates of return rise with risk. However, the empirical tests do not show neat, stable relationships; rather, depending on the test period analyzed and the methodology used, many

[12] Much of the data and analysis in this section are drawn from a study prepared by S. C. Myers and G. A. Pogue, An Evaluation of the Risk of Comsat's Common Stock, August 1973, submitted to the FCC in connection with Comsat's rate case (FCC Docket 16070). Myers also used the same procedures in the 1971 AT&T rate case before the FCC, as did Professors Robert Haugen and Howard Thompson to estimate Armco Steel and Republic Steel's cost of capital in the Reserve Mining case (U.S. vs. Reserve Mining, U.S. District Court, Minneapolis, No. 5-72, Civil 19, State of Wisconsin Exhibit #3; this was a landmark civil case in the pollution area involving Reserve's dumping of wastes into Lake Superior). In both the AT&T and Reserve cases, the beta estimate of the cost of capital was accepted, while other estimates were rejected. The Comsat case has not yet (July 1974) been decided.

market lines could be generated. This instability is to be expected for two reasons: First, we would expect the market line to change over time as both interest rates and investors' outlooks change, so a stable market line over time would be strange indeed. Second, we are forced to estimate the market line on the basis of imperfect data, and where errors in the data exist, estimating problems are bound to arise.[13]

How can we actually estimate the market line at a given point in time? One procedure is outlined below:

1. Determine the rate of return on risk-free securities, and use this rate as the intercept of the market line.

2. Estimate the required rate of return on "the market," or k^*_M. Note that k^*_M has beta = 1.0.

3. Connect R_F and k^*_M to construct the capital market line.

These steps sound simple, but in fact they are filled with problems, some of which are discussed below.

<u>The Riskless Rate</u>

The basic CAPM is a single-period theory, and it is concerned with R_F, a one-period, riskless rate. For practical applications, the short-term government bond rate would appear to be most consistent with the theory, but this question has not been completely resolved. Further, if there is a significant spread between long- and short-term rates, the choice of the rate used for R_F can seriously affect the estimated cost of capital for a firm; this is especially true for firms whose beta is materially different from zero. This point is illustrated in Figure 5. Here k^*_M is the required market return, assumed for the moment to be 11 percent; 4.4 percent is the riskless rate of return on short-term Treasury

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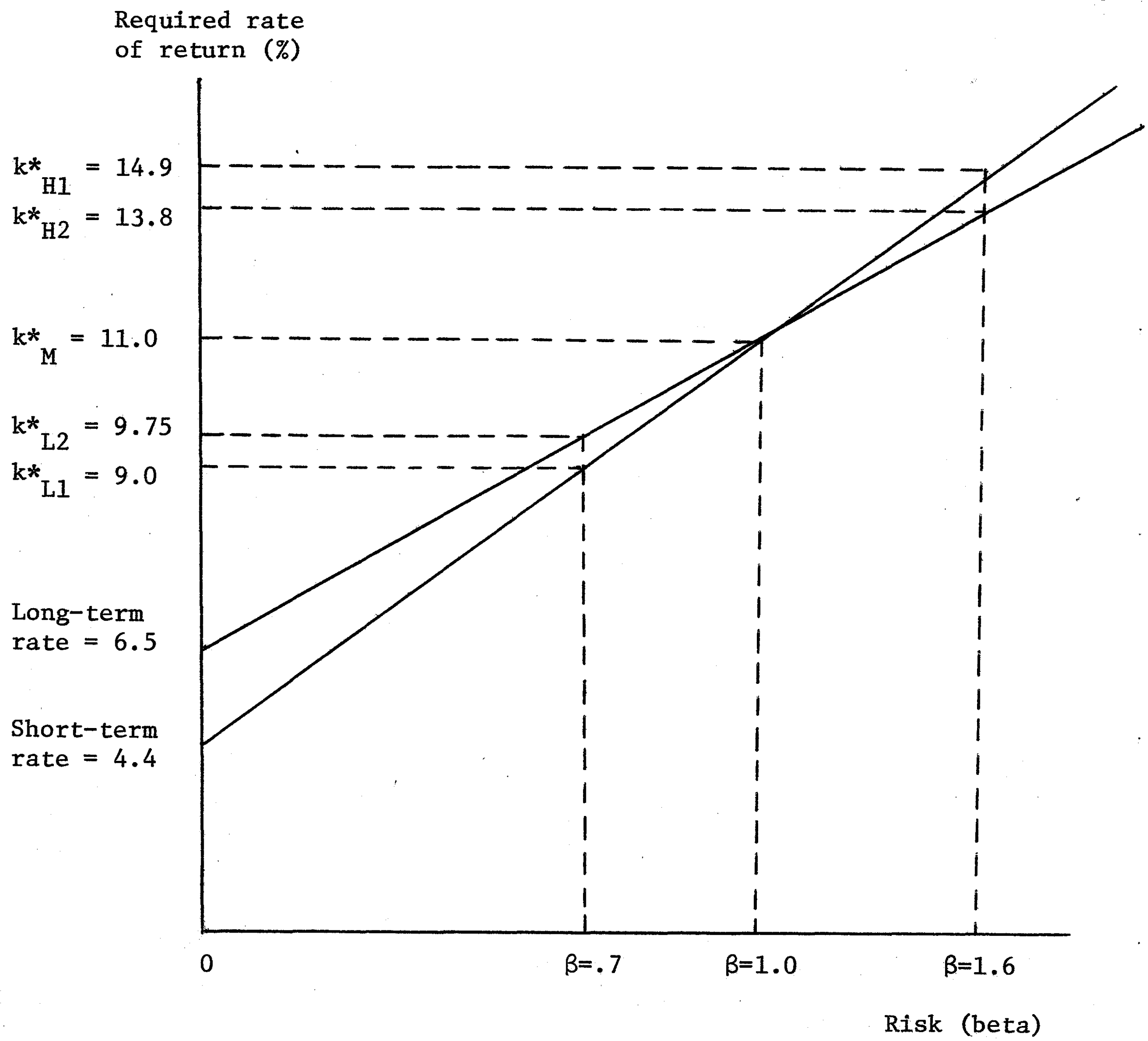
[13]However, these problems are no more severe in the capital asset pricing model approach to cost of capital determination than are the problems encountered using other approaches.

Figure 5. Effects on Estimated Cost of Capital of Using Different Proxies for R_F



securities in October 1972; 6.5 percent is the riskless rate of return on long-term Treasury bonds in October 1972; and k^*_H and k^*_L are the estimated required rates of return on high- and low-risk stocks. Since the market line is estimated by connecting the riskless rate with the point (k^*_M = 11.0, b = 1.0), then extending the line, it is obvious that the choice of a short-term or long-term riskless rate affects the slope of the line and, hence, the estimates of individual firms' costs of capital.

Under the assumptions used in Figure 5, the cost of equity for a low-risk firm, one with a beta coefficient of .7, is 9 percent if the short-term rate is used as a proxy for R_F, but the estimate of k^*_b rises to 9.75 if the long-term rate is used for R_F. For the high-risk stock with b = 1.4, the cost of capital estimate based on a long-term riskless rate is 13.8 percent versus 14.96 percent based on a short-term rate. As a general rule, during periods when long-term rates exceed short-term rates, high-risk stocks (those with $b > 1.0$) will have a higher estimated cost of equity capital if we use short-term rates as the intercept than if we use long-term rates. The converse holds for low beta stocks. As is obvious from Figure 5, the difference in the estimated cost of capital is not great for stocks with betas close to 1.0, and no problem arises if long- and short-term rates are close together.

A Key Benchmark: The Market Rate of Return (k^*_M)

Capital asset pricing model theory suggests that the CML is linear, and the available evidence generally supports this contention. Accordingly, if we know the intercept, R_F, and if we have a reasonably accurate fix on only one additional point, we will be able to estimate the CML. One possible point that comes to mind immediately is k^*_M, the required rate of return on "the market portfolio," a portfolio composed of all stocks, weighted in accordance with their market values.[14]

Ordinarily, it is reasonable to assume that the stock market is in equilibrium, hence that the required rate of return is equal to the expected rate of

[14]An "average" stock will have b = 1.0; this is true by definition, for the average stock must move with the market, which is what b = 1 implies. If only one b = 1 stock is held, then this one-asset portfolio will be highly variable--it will tend to move with the market, but its returns will also vary randomly as a result of the firm's unsystematic risk. However, the unsystematic risk will not matter to diversified investors, so a b = 1 stock, or an average stock, will command a risk premium that reflects only its systematic risk. This means that an average stock, one with b = 1, will have the same risk premium and required rate of return as "the market." Therefore, $k^*_i = k^*_M$ if $b_i = 1.0$.

return, i.e., that $k^*_M = k_M$. But how do we measure k_M, the expected rate of return on the market? One approach is to assume that investors expect to earn in the future rates of return similar to those earned in the past, i.e., to assume that $k_M = \hat{k}_M$. We could then take data on past market returns and use them as an estimate of $k_M = k^*_M$.

Assuming that $k_M = \hat{k}_M$ has an obvious flaw: $\hat{k}_M$ varies widely depending on the holding period examined. If the base year happens to be in a market low such as 1949, then calculated returns will be high to almost any terminal period. On the other hand, if the base year is a stock market high, such as 1965, and the terminal year a low market, such as 1970, then $\hat{k}_M$ will be low, perhaps even negative. The importance of the base year is reduced if long holding periods are used, but using a very long holding period means using possibly irrelevant data. After all, we are seeking to estimate k^*_M, the expected future rate of return, and surely investors will give results experienced in more recent years a higher weight than those experienced in the distant past.[15]

We shall not dwell further on procedures for estimating k^*_M, but assuming that a reasonable estimate has been obtained, then this value lies on the CML at the value b = 1.

Another Key Benchmark: AT&T's Allowed Rate of Return

In their estimate of Comsat's cost of capital, Myers and Pogue relied heavily on earlier estimates of AT&T's values of k* and b. No matter how careful a job one does in estimating k^*_M, it is clear that this estimate is subject to error.

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[15]One potential solution to this problem involves decomposing the past realized rate of return into several components, then excluding those components that investors are not likely to project into the future. Stock yields arise both from dividends and from capital gains, and capital gains (or losses), in turn, result from growth in earnings and from changes in capitalization rates. Since some sources of capital gains are more likely to continue into the future than others, it seems reasonable to project a continuation of some elements of past yield and to remove others when projecting expected future returns. For a discussion of a procedure for decomposing security returns, see Brigham and Pappas, "Rates of Return on Common Stocks," Journal of Business, July 1969. This methodology was used by E. F. Brigham to estimate Comsat's cost of capital as of 1964 (testimony in FCC Docket No. 16070, November 1972).

Accordingly, it is useful, indeed imperative, to establish other benchmarks on the CML, and Myers and Pogue regarded one such benchmark as the estimated cost of capital for AT&T and other utilities. In a widely publicized rate case decision announced in November 1972, the Federal Communications Commission indicated that it believed AT&T's cost of equity capital (k*) to be approximately 10 1/2 percent, and it stated an intention to allow the company to earn this rate of return on its book equity. The stock was selling at approximately its book value at the time, so investors had reason to believe that their own future rate of return, if they purchased AT&T stock, would be approximately equal to the company's rate of return on equity.[16]

Stated differently, investors had strong reason to expect AT&T to earn about 10.5 percent on book value, and the fact that the stock was selling at close to book value indicated that investors would also, over the long run, receive about 10 1/2 percent on their own investment. The stock price was stable after the decision was announced, which suggests that 10 1/2 was indeed a reasonably good approximation to AT&T's cost of equity.[17]

AT&T's beta coefficient has been estimated to be approximately .7. Combining this value with the estimated 10 1/2 percent cost of equity, Myers and Pogue derived a point on the CML for 1971.

FINDING A FIRM'S COST OF CAPITAL

Having established, or at least estimated, the CML, the remaining steps are (1) to estimate the firm in question's beta coefficient and (2) to read its

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[16]If a company earns 10 percent on its book equity, but the stock sells for twice book value, then the rate of return on an investor's purchase price is only 5 percent. But AT&T was selling at about book value, so this complication did not arise.

[17]The question of whether or not a utility company's stock should sell at about its book value is another matter, and one that goes beyond the scope of this paper.

cost of capital from the CML.

The Computer Research and Applications Department of Merrill Lynch, Pierce, Fenner, and Smith estimated Comsat's beta coefficient in 1972 to be 1.63. Merrill Lynch calculates betas as a weighted average of the actual beta over the past 60 months and 1.0, with 70 percent of the weight given to the calculated beta and 30 percent to 1.0. Comsat's unadjusted beta for the five-year period was 1.9, so its adjusted beta is calculated as follows:

$$b = .7(1.9) + .3(1.0) = 1.63.$$

In general, the Merrill Lynch analysts believe that future betas--the betas that should be used by investors--are better approximated by the adjusted than the unadjusted beta coefficient.[18] However, they readily admit that estimating betas is an inexact science!

Myers and Pogue made independent estimates of Comsat's beta, and they calculated betas over several different subperiods. Myers and Pogue concluded (1) that Comsat's beta is approximately 1.7; (2) that it is relatively stable over time; and (3) that in view of the standard error of the beta (.30), there is only a 16 percent probability that the "true" beta is less than 1.4.

Comsat's cost of equity may be estimated using the following equation:

$$k^*_{Comsat} = R_F + b_{Comsat}\,(k^*_M - R_F).$$

At the time Myers and Pogue conducted their study (March 1972), the long-term riskless rate was 7.75, while the short-term rate was 8.50 (the yield curve was downward-sloping). They recognized that any number used for k^*_M would only be an estimate, but, based on various pieces of evidence including the 10.5 percent estimate for AT&T's cost of equity, they concluded that k^*_M was in the range of 10 to 13 percent, with a midpoint of 11.5 percent.

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[18]Professor Sharpe is Merrill Lynch's technical consultant, and the beta estimation process used by that company was developed by Sharpe. His rationale is largely empirical--betas determined as described are better estimates of future betas than are betas determined simply on the basis of historical data.

Myers and Pogue generated a number of other cost of capital estimates based on alternative assumptions about the appropriate R_F, the proper value of k^*_M, and whether the slope of the CML is less than the CAPM would predict, and they obtained k* values ranging from 10.9 percent to 17.2 percent, with a midpoint of 14 percent. Myers and Pogue did not pick a single point estimate of Comsat's cost of equity, but a careful reading of their study suggests that they would probably have picked 14 percent if forced to choose a single number.[19]

RELATIONSHIP BETWEEN LEVERAGE AND BETA

Stocks in general tend to rise when the economy is strong, and conversely. Some individual stocks perform better than the average under boom conditions, but worse than average when the economy is weak--these are the high beta stocks. The converse holds for low beta stocks. Now suppose a leverage-free firm has b = 1.0, meaning its returns tend to move up or down in direct proportion to the general market. If this firm changes its capital structure to include debt, what effect will this have on its beta coefficient? First, note that interest on the debt constitutes a fixed charge, so a leveraged firm with a given level of business risk (variability of EBIT) will exhibit larger fluctuations in EPS than an otherwise identical leverage-free firm. If earnings are more sensitive

[19]We should note that Comsat had earlier argued, based on a "traditional" cost of capital study by another witness (E. F. Brigham), that its k* was 12 percent. Myers and Pogue's study was designed to determine if this 12 percent was "too high," not if it was exactly right. In testifying on the Myers-Pogue study before the FCC, Myers concluded that "Comsat's cost of equity capital is at least as large as a typical industrial firm's and well in excess of AT&T's. Twelve percent is a reasonable estimate of Comsat's cost of capital--if anything, it is conservative."

We should note, however, that this conclusion was not shared by all. Professor Willard T. Carleton testified that Comsat's stock is less risky than AT&T's, and that Comsat's k* is below that of AT&T. The Commission has not yet (March 1974) reached a judgment on Comsat's cost of capital.

to the business cycle, then so will be the firm's stock price and beta coefficient. Thus, it is intuitively clear that beta should increase with the debt ratio. In general, we would expect the beta of a leveraged firm (b_L) to bear a relationship of the following form to the beta of an otherwise identical unleveraged firm (b_U):[20]

$$b_L = b_U + b_U\left(\frac{D}{S}\right), \tag{9}$$

where D and S are the market value of the firm's debt and equity. Empirical studies have shown a positive relationship between beta and leverage, but the empirical tests do not prove conclusively the existence of the precise form indicated in equation (9).[21]

USING THE CAPM TO DETERMINE RISK-ADJUSTED DISCOUNT RATES

Thus far we have discussed the use of portfolio theory and the capital asset pricing model to determine the firm's cost of equity capital. This same body of theory can be used to develop risk-adjusted discount rates needed in capital budgeting.

Under the assumptions of the Sharpe model, the beta for a portfolio is simply a weighted average of the betas of the individual stocks included in the portfolio. The firm itself may be considered as a portfolio of assets, each with a beta coefficient of its own, so the firm's overall beta is simply the

[20] Different equations can be developed depending upon the assumptions one makes about investors' reaction to corporate leverage. Equation (9) is based upon the Modigliani-Miller assumptions (Hamada, 1972).

[21] Ibid.

average of the betas of its individual assets.[22] In other words, the firm's beta coefficient, which determines its cost of equity capital, is a linear function of the betas of its individual assets.[23]

To illustrate this, suppose the following conditions hold: (1) the CML has been estimated with a "reasonable" degree of accuracy and is plotted in Figure 6; (2) the firm's beta is estimated to be .85; (3) the firm uses only internally generated equity capital;[24] and (4) this information had been used to calculate the required rate of return for the firm, or its cost of equity capital, which is 10.4 percent.

The firm should use 12 percent as its cost of capital when evaluating "average" projects. However, if it is considering a new project with a subjectively estimated beta of less than .85, then it should use a lower cost of capital, and the

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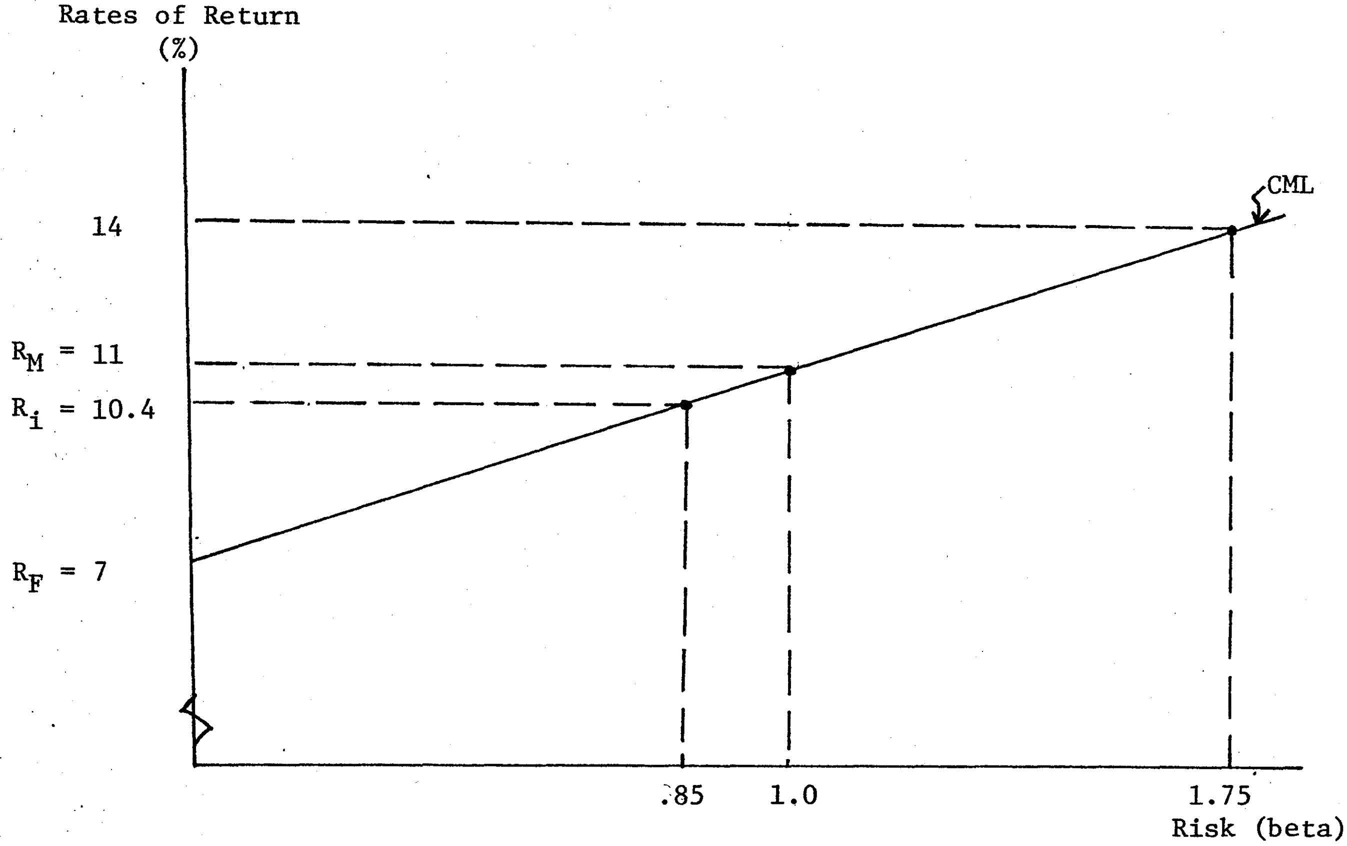
[22]We do not analyze here the implications and assumptions inherent in this statement in rigorous detail. The capital asset pricing model, as developed by Sharpe, assumes that lending and borrowing rates are the same, that all investors have homogeneous expectations, and so on. Further, although the various companies are all affected to a greater or lesser degree by a "common factor," the various companies whose stocks are contained in portfolios are probably less related, in general, than are the component parts of most businesses. Another potential difficulty in making the extension of the capital asset pricing model to the level of the firm relates to the number and divisibility of stocks in the market versus projects for the firm, and the potential purchasers of these assets. Stocks are available in virtually unlimited quantities, and they are completely divisible for all intents and purposes. Further, there are many buyers and sellers operating in the stock market, making it relatively perfect. These same conditions do not hold for asset investments at the firm level--profitable projects are <u>not</u> available in unlimited numbers to any single firm; many projects are lumpy; and, in many cases, no market other than the firm in question exists for a specific investment proposal. These differences complicate things, but they do not rule out the suggested procedure.

[23]In an analysis of the type proposed, it would be necessary to lump assets to a considerable extent, perhaps treating entire plants, or even product lines or divisions, as projects, rather than dealing with specific assets such as individual machine tools. However, we see no reason to think that the proposed analysis would be significantly more difficult than the type of risk analysis generally recommended today.

[24]If the firm used debt capital, then the procedure described here would be followed to determine its cost of equity, which would be averaged with its debt to determine an average cost of capital.

Figure 6. Using the Market Line to Estimate Risk-Adjusted Discount Rates



converse is true if the new project is more risky than the average. Suppose, for example, that our hypothetical firm is a combination electric-gas utility thinking of investing in an oil exploration venture. The present beta is .85, but management estimates that the proposed oil venture will have b = 1.75.[25] Assuming that it goes ahead with the exploration program, management expects the firm to end up with a new beta, one that lies between .85 and 1.75, with the exact position depending upon the relative size of the oil investment. At any rate, the oil venture should be evaluated using a 14 percent cost of capital; this figure is simply read off the market line in Figure 6.[26]

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[25] Perhaps its analysts determine that exploration companies, on average, have b = 1.75.

[26] Any "synergistic" effects of the new operation should be allocated to the cash flows arising from the new petrochemical operation.

CONCLUSIONS

The use of beta coefficients to help determine a firm's cost of equity capital in utility rate cases is not widespread, although the CAPM approach has been used with good results in several key cases. The CAPM is a relatively new concept, and it typically takes time for new concepts to gain acceptance in practice. In many respects, the CAPM is today where the discounted cash flow (DCF, or $k^* = D/P + g$) approach was about ten years ago. Our guess is that the CAPM will gain increasing acceptance in rate work, and that in the not-too-distant future beta coefficients will be widely used, along with other estimating procedures, to measure risk in cost of capital studies.