THE RELATIONSHIP BETWEEN A UTILITY COMPANY'S MARKET PRICE AND BOOK VALUE

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The relationship between a utility stock's market price and book value is important for several reasons. First, if the market/book ratio (M/B) exceeds 1.0, then selling new shares will tend to increase earnings per share, while if new stock is issued when the ratio is below 1.0, this will tend to drive earnings per share down. Second, the relationship between market and book values, coupled with information about the allowed rate of return, the dividend payout ratio, and the rate of capital expansion, can be used to generate an estimate of the DCF cost of capital. Third, the market/book ratio is frequently cited in rate cases as one indication of whether a particular allowed rate of return meets the Hope Case fairness criterion; if the actual M/B ratio is different from the "fair" ratio, then the allowed rate of return should be adjusted to move the M/B ratio to the "fair" ratio.

This third issue--using the M/B ratio in rate cases--involves two separate considerations: (1) determining a "fair" or reasonable target M/B ratio and (2) determining the rate of return on book equity that will cause the actual M/B ratio to move to the target level. This paper is addressed primarily to the second of these tasks, i.e., given a target M/B ratio, what rate of return should be allowed on book equity?[1] We show that the M/B ratio is influenced

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*University of Florida. The authors wish to acknowledge the support of the Public Utility Research Center, University of Florida, in preparing this paper.

[1]The first question--what is a proper M/B ratio--is perhaps more interesting than the second one. A research project currently underway at the University of Florida is studying this subject.

by the allowed rate of return, the DCF cost of capital, the payout ratio, and the asset growth rate; based on statements made in recent rate cases, it is clear that these relationships are quite important, yet not at all well understood.

I. THE DCF COST OF CAPITAL VERSUS THE FAIR RATE OF RETURN ON BOOK EQUITY

It is necessary to distinguish at the outset the difference between the cost of capital, or the rate of return that investors require on equity investments, and a fair rate of return on book equity. The DCF required rate of return, which we define as k, is determined by competition in the marketplace. k consists of a riskless rate of return (R_F), generally taken as the current yield on high grade long-term bonds, plus a risk premium (P):

$$k = R_F + P. \tag{1}$$

For common stocks, investors expect to receive their required return in the form of dividends plus capital gains, and, if growth is expected to be constant,

$$k = \frac{D_1}{P_o} + g, \tag{2}$$

where D_1 is the dividend expected during the year, P_o is the current price of the stock, and g is the expected growth rate in earnings and dividends. Equation (2) is the familiar "DCF cost of capital formula" that often appears in the literature on the cost of capital.

The fair rate of return on book equity is defined as r; if utility commissions permit companies to set prices sufficient to earn the fair return, r is also the allowed, or actual, rate of return on equity.[2] Assuming that the DCF

[2] Because of regulatory lag or other problems, the actual rate of return may turn out to be quite different from the commission-determined fair return.

cost of capital is known, should it be used as the fair rate of return utility companies are allowed to earn on book equity? I.e., should r be set equal to k? If regulators do set r = k, then the common stock will sell at book value and the M/B ratio will be 1.0. This conclusion is derived as follows:

1. The constant growth model for stock prices from which $k = \frac{D_1}{P_o} + g$ is derived is stated as follows:[3]

$$P_o = \frac{D_1}{k - g}. \tag{3}$$

2. Earnings per share, E_t, is equal to the allowed rate of return, r, times the book value at the beginning of the period, B_{t-1}:

$$E_t = r\,B_{t-1}. \tag{4}$$

3. If a constant percentage of earnings, b, is retained, then the dividend payout ratio is (1 - b), and dividends per share may be determined as follows:

$$D_t = (1 - b)\,E_t = (1 - b)\,r\,B_{t-1}. \tag{5}$$

4. The growth rate in earnings, dividends, and share prices, assuming k, b, and r are constant, is:

$$g = br. \tag{6}$$

5. Letting t = 1, substituting equations (5) and (6) into (3), and then setting r = k, we see that $P_o = B_o$; i.e.,

[3]See Weston and Brigham [5], Ch. 10, for a derivation of this model.

market value is equal to book value:

$$P_o = \frac{D_1}{k - g} = \frac{(1 - b)\ r\ B_o}{k - b\ r} = \frac{k\ (1 - b)B_o}{k\ (1 - b)} = B_o. \tag{7}$$

Thus, if an accurate estimate of the DCF cost of capital is obtained and used as the allowed rate of return on book equity, this will force the market value of the common stock to equal the book value per share.[4]

II. A MODEL OF MARKET/BOOK RATIOS

A regulated utility company's market/book ratio is fundamentally dependent upon the following factors: r, the rate of return on book equity; b, the retention rate; k, the DCF required rate of return; G, the capital expansion rate; and s, the percentage of new equity obtained by selling common stock.[5] In this section we first examine the model when s is zero, then go on to show the modifications necessary when expansion is so rapid that new stock financing is required.

Market/Book Ratios When No Stock is Sold

The basic dividend growth model developed by Gordon [4] and used so often in cost of capital studies is based on the assumption that no external equity financing is required. Under this assumption, the growth rate is equal to br, the product of the retention rate times the rate of return on retained earnings, and the M/B ratio is a function of k, b, and r. This may be seen by letting $t = 1$, substituting equations (5) and (6) into (3) to obtain (8), and then solving for P_o/B_o, which is the same as M/B:

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[4]As noted above, we have underway another research project examining the proper levels of M/B ratios. We might note at this point that most authorities feel that M/B ratios should exceed 1.0, hence r should exceed k.

[5]Leverage, earnings stability, covariance with other stocks, and so forth influence stock prices through k; hence, we may abstract from these variables in this analysis. Also, we shall assume that k is independent of dividend policy, and that the company finances in a manner that keeps the debt/equity ratio constant.

$$P_o = \frac{D_1}{k - g} = \frac{(1 - b)\ r\ B_o}{k - b\ r}, \tag{8}$$

$$\frac{P_o}{B_o} = \frac{(1 - b)\ r}{k - b\ r}. \tag{9}$$

Some illustrative M/B ratios are calculated with equation (9) and plotted in Figure 1. The significant features of this exhibit may be summarized as follows:

1. For any given b and k, the higher the value of r, the higher the M/B ratio.

2. If b is zero--i.e., if the firm pays all its earnings out as dividends--then the M/B ratio rises linearly with r. Otherwise, the relationship between M/B and r is not linear.

3. For any specified r and b, the M/B ratio is higher for lower values of k.

4. Over most of the curves, the higher the value of b, the steeper the slope of the curve relating M/B to r; i.e., M/B is most sensitive to changes in r for high values of b.

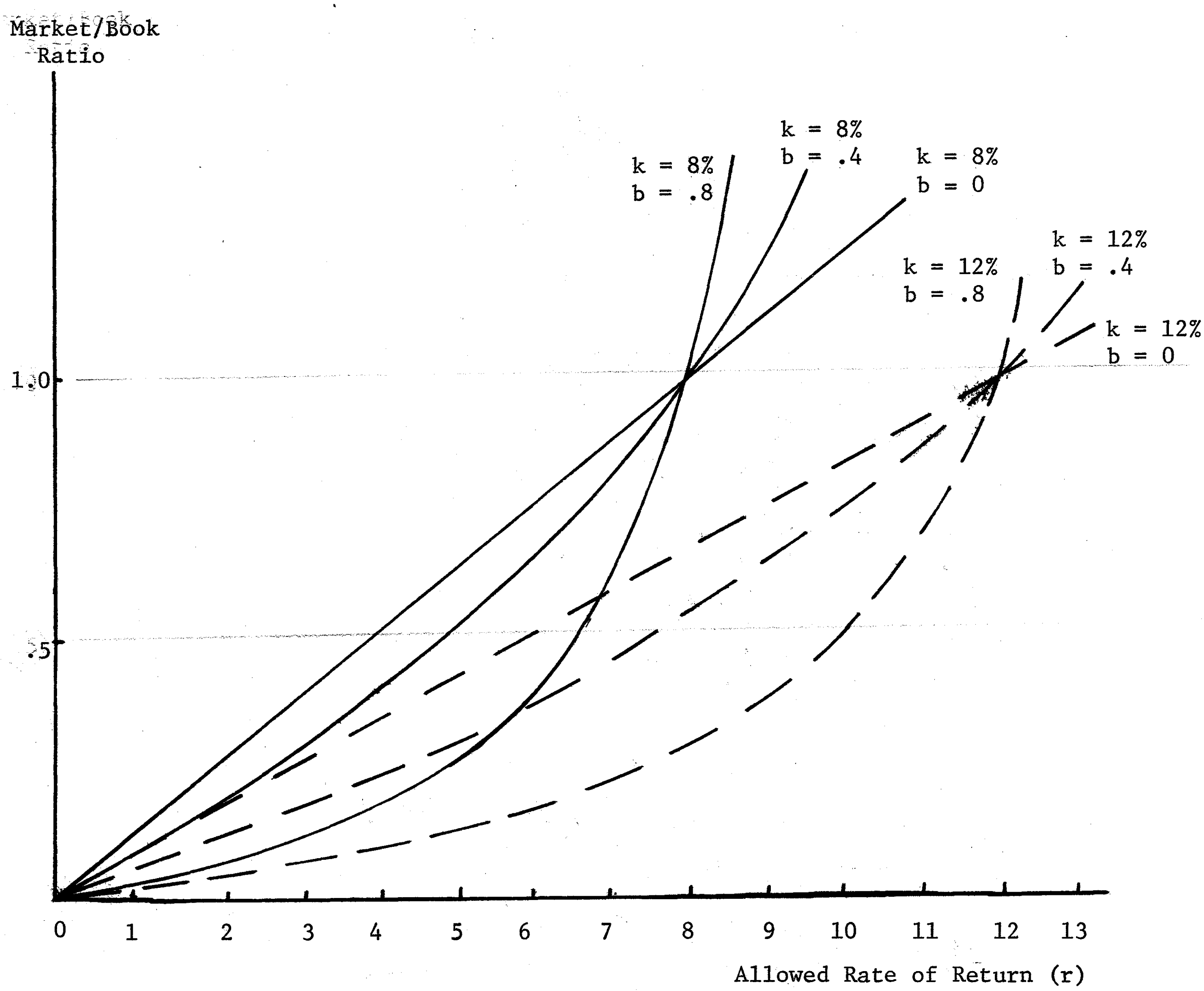
5. When $r = k$, $M/B = 1.0$ regardless of the level of b. If $r > k$, then $M/B > 1.0$, and conversely if $r < k$.

6. If $r > k$, then the higher the level of b, the larger the value of M/B, and conversely if $r < k$.

The Asset Growth Rate. In the calculations used to construct Figure 1, we implicitly assumed that the growth rate in total assets, G, increases with r. Notice that b is a constant for each of the curves, and with $s = 0$ and the debt ratio constant, then $G = br$; i.e., an increase in the allowed rate of return implies a faster rate of growth in assets.

Is it reasonable to assume that G increases with r? Ordinarily, G is probably independent of r. The demand for most utility services is relatively inelastic, so the price changes necessitated by changes in r will not greatly affect demand. And since growth in demand is the primary determinant of the asset growth rate, G would seem to be relatively independent of r. Accordingly, it

Figure 1. Relationship Between Market/Book Ratios, Allowed Rates of Return (r), Required Rates of Return (k), and Retention Rates (b)



would seem preferable to evaluate equation (8) under the assumption that G is a constant.[7]

If we assume that G is a constant, then increases in r must be offset by decreases in b so as to keep the product br = G constant. Under these conditions, the M/B ratio is a linear function of r. Figure 2 plots some illustrative M/B lines under the constant growth assumption. The significant features of this figure may be summarized as follows:

1. M/B is linearly related to r.
2. M/B = 1.0 if r = k.
3. The X-axis intercept is r = G; if r < G, negative M/B ratios, which are nonsense, arise.
4. For any value of k, M/B is more sensitive to changes in r at higher growth rates.

The points noted above hold if all new equity financing is expected to come from retained earnings, or if new equity is sold at book value. These conditions certainly do not hold for all utilities; accordingly, in the next section we broaden the analysis to permit the sale of stock at prices different from book values.

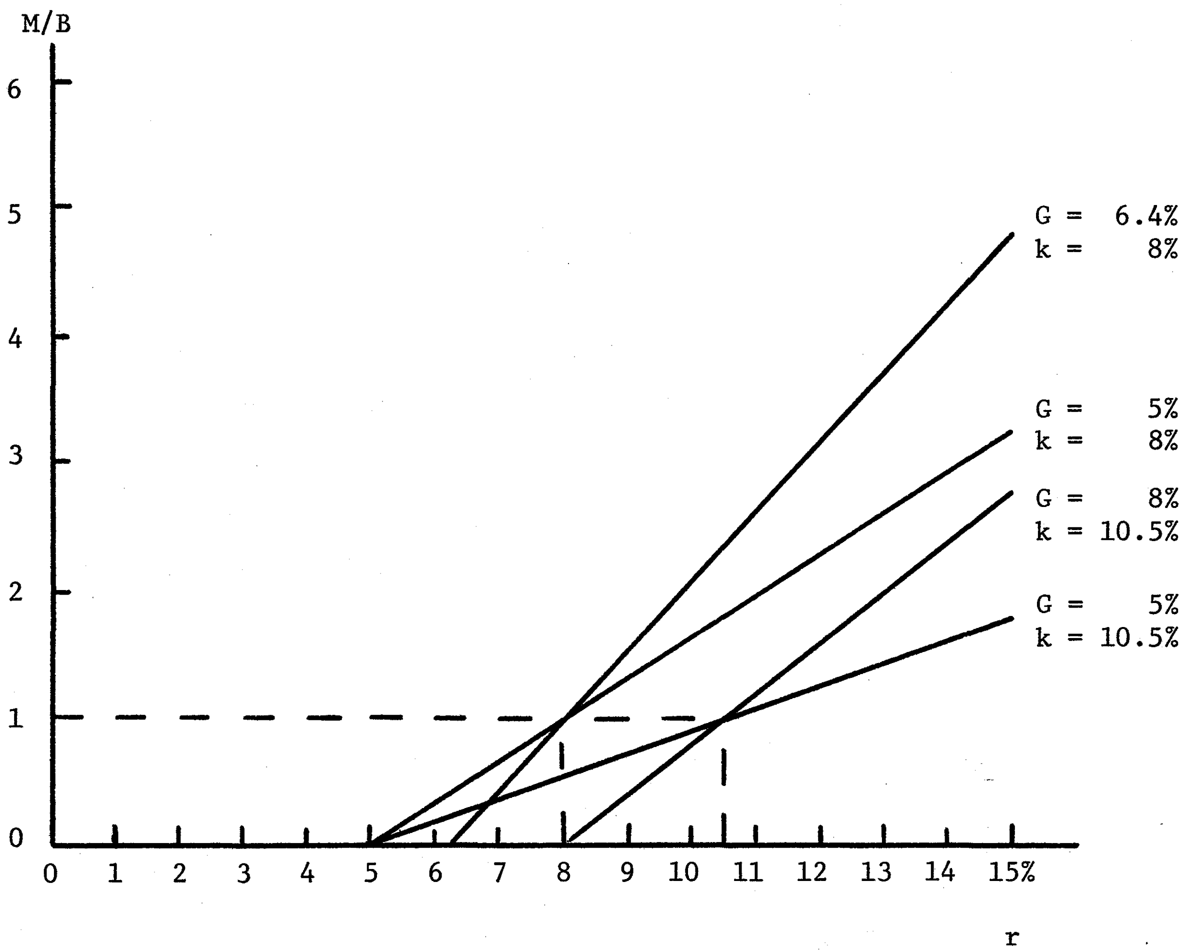
Market/Book Ratios With External Equity Financing

In the no-external-equity model, growth occurs only through reinvestment of retained earnings. However, if equity is raised externally, an additional element

[7] The whole question of utility investment policy has been studied in depth by Averch and Johnson [1], Baumol and Kelvorick [2], and others. All the issues have not been resolved, but it is reasonably clear that utility companies have an incentive to step up their investment rate the higher the value of r. Further, if r < k, they encounter difficulties in financing construction programs, and stock prices are lower the larger the value of G. If r < k, studies (Brigham and Pettway [3]) suggest that utility companies postpone investments, make less capital intensive investments, and the like, and conversely if r > k. However, if r is "reasonably close" to k, then investment policy, hence G, is not likely to be greatly affected by changes in r. All things considered, we consider it valid as a first approximation to assume that G is a constant.

Figure 2. <u>M/B Ratios When Growth is Held Constant:</u>

$$M/B = \frac{(1 - b)r}{k - br} \text{ and } br = G$$



of growth will occur if the stock is sold at prices above its book value, while growth will be retarded if the sales price is below book.[8] A utility's earnings are dependent upon its book equity, and if stock is sold at prices different from book value, the difference between M and B accrues to the old stockholders. Thus, book value, hence earnings, dividends, and stock prices, will grow if stock is sold at prices above book value. In this section, we extend the M/B ratio equation to encompass the sale of common stock.

The Two Components of Growth. There are two elements of growth for a utility stock: (1) growth from retention, defined as $g_1 = br$, and growth from sale of stock, defined as g_2 and determined in accordance with the following equation:[9]

$$g_2 = \frac{P_o\ (1 + s)}{P_o + sB_o} - 1.0. \tag{10}$$

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[8]If a rights offering is used, then the stock split effect of underpricing must be taken into account. The relevant book value is B_o after adjusting for this stock split effect.

[9]Equation (10) is derived as follows:

Step 1. ΔS = number of shares of stock sold
$= [s\ S\ B_o] \div P_o$,

where S = original shares outstanding and s = rate of growth in total equity from sale of stock.

Step 2. B_{oa} = new book value per share after sale of stock

$$= \frac{SB_o + \Delta SP_o}{S + \Delta S}$$

$$= \frac{SB_o + [(s\ S\ B_o)/P_o](P_o)}{S + (s\ S\ B_o)/P_o}$$

$$= \frac{B_o\ P_o\ (1 + s)}{P_o + s\ B_o}$$

Step 3.

$$g_2 = \frac{B_{oa}}{B_o} - 1 = \frac{P_o\ (1 + s)}{P_o + sB_o} - 1.0. \tag{10}$$

In this derivation we assume no stock of flotation costs; hence, new shares are sold at a price P_o. This assumption is relaxed in the next section.

Here s is the rate of growth in total equity from the sale of common stock. If $P_o = B_o$ (i.e., M/B = 1.0), then $g_2 = 0$. If M/B > 1.0, the larger the value of s, the greater will be g_2. If M/B < 1.0, then $g_2 < 0$, and the larger the value of s, the smaller will be g_2. At some combination of low M/B and high s, the negative g_2 could offset a positive $g_1 = br$, and total earnings growth could be negative even for a company that plows back some of its earnings.

The Modified M/B Equation. Equation (11), which is similar to (9) except that it is modified to show the effects of stock sales on the M/B ratio, is given below:[10]

$$M/B = \frac{r(1 - b)(1 + s) - s(1 + k - br)}{k - br - s}. \tag{11}$$

Equation (11) shows that the M/B ratio is dependent upon the allowed rate of return, the retention rate, the outside equity financing rate, the DCF cost of capital, and the asset growth rate.[11]

In evaluating equation (11), we are primarily interested in the relationship between M/B and r--this is the relationship upon which utility companies and their regulators focus--so we hold constant the values of the other variables and analyze M/B as r changes. However, as with the no-outside-equity-financing case, we still have the choice of holding G constant or letting G = br + s increase with increases in r. This is an important consideration, for M/B rises exponentially with r if G is permitted to vary but is approximately linear if G is constant.

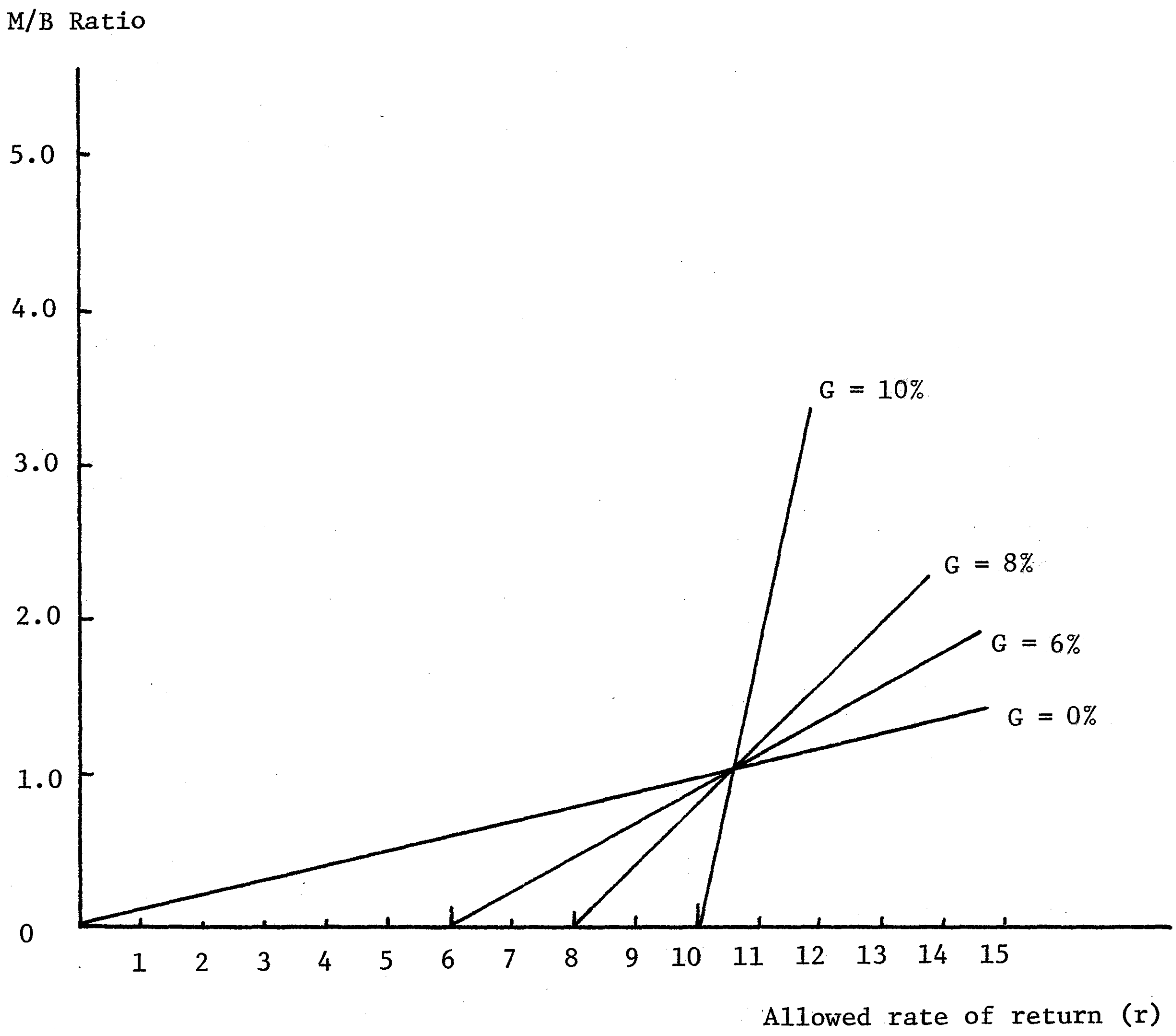
As noted above, it is probably more realistic to hold G constant than to let it vary, at least within a "reasonable" range of values for r. Under the assumption that G is constant, Figure 3 shows how M/B varies with r at different asset

[10]Equation (11) is derived in Appendix A.

[11]Equation (11) shows that M/B is dependent upon s. However, with a constant debt ratio, s = G - br, so for any given b and r, s varies with G.

Figure 3. M/B Ratios When Stock is Sold

k = 10.5%
b = 40%
G = br + s = constant for each line, with
s changing to maintain constancy



growth rates. The main features of the figure may be summarized as follows:

1. The relationship between M/B and r is approximately linear for relevant ranges of r when G is constant.

2. When $r = k$, $M/B = 1.0$ regardless of the asset growth rate, the retention rate, or the outside equity financing rate.

3. Whenever $r > k$, M/B is larger for higher values of G, and conversely if $r < k$.

4. The M/B ratio is most sensitive to changes in r if the asset growth rate is large; with $G = 10$ percent, a relatively small change in r produces a substantial change in M/B.

The third and fourth points above are particularly interesting. If a utility is in a rapidly growing service area, a value of r only slightly less than k will produce a very low M/B ratio; conversely, if r is even slightly greater than k, the M/B ratio will be relatively high. Thus, utilities in high growth areas are likely to exhibit a relatively high degree of price instability in periods when realized rates of return are varying.

Introducing Flotation Costs. Thus far we have implicitly assumed that flotation costs are zero, i.e., that new stock can be sold to net the company the current market price. Clearly, this is not a realistic assumption, so the model must be expanded to allow for flotation costs.

There are actually two types of flotation costs: (1) the specific costs associated with underwriting an issue, including whatever underpricing might be necessary to sell the issue, and (2) the more subtle impact of a continual increase in the supply of a given stock. The specific costs associated with a given flotation, as a percentage of the funds raised, is designated by the term F. Assume first that a stock is selling at $50 per share before a new financing is announced, that selling efforts enable the underwriters to market the stock at $50, and that underwriting costs are $2 per share, so the seller will net $48 per share. In this case, $F = \$2/\$50 = 4$ percent. However, if market pressure caused the stock to decline so that it was sold to the public at \$45 to net the seller \$43, then F = (\$50 - \$43)/\$50 = \$7/\$50 = 14 percent.

If the issue was a "one-time-shot," or at least if issues occurred only every four or five years, the market price would probably rebound to the original \$50 price. However, if the firm was forced to go to the market every year or two, then continuous pressure might hold the stock down indefinitely.

We have not made a detailed study of flotation costs, so we cannot specify precisely what level of flotation costs are likely to be encountered for a given issue. In our view, it is probably best to think of the permanent, long-run pressure of continuous large issues as affecting k, the cost of capital, and bring it into the model by adjusting k. In fact, for a firm that had a history of sizable stock issues, this long-run pressure would presumably be recognized in the estimate of k.

The specific flotation cost associated with an individual issue is incorporated into equation (12), which is derived in Appendix B:

$$M/B = \frac{r(1 - b)(1 + s)(1 - F) - s(1 + k - br)}{(k - br - s)(1 - F)}. \tag{12}$$

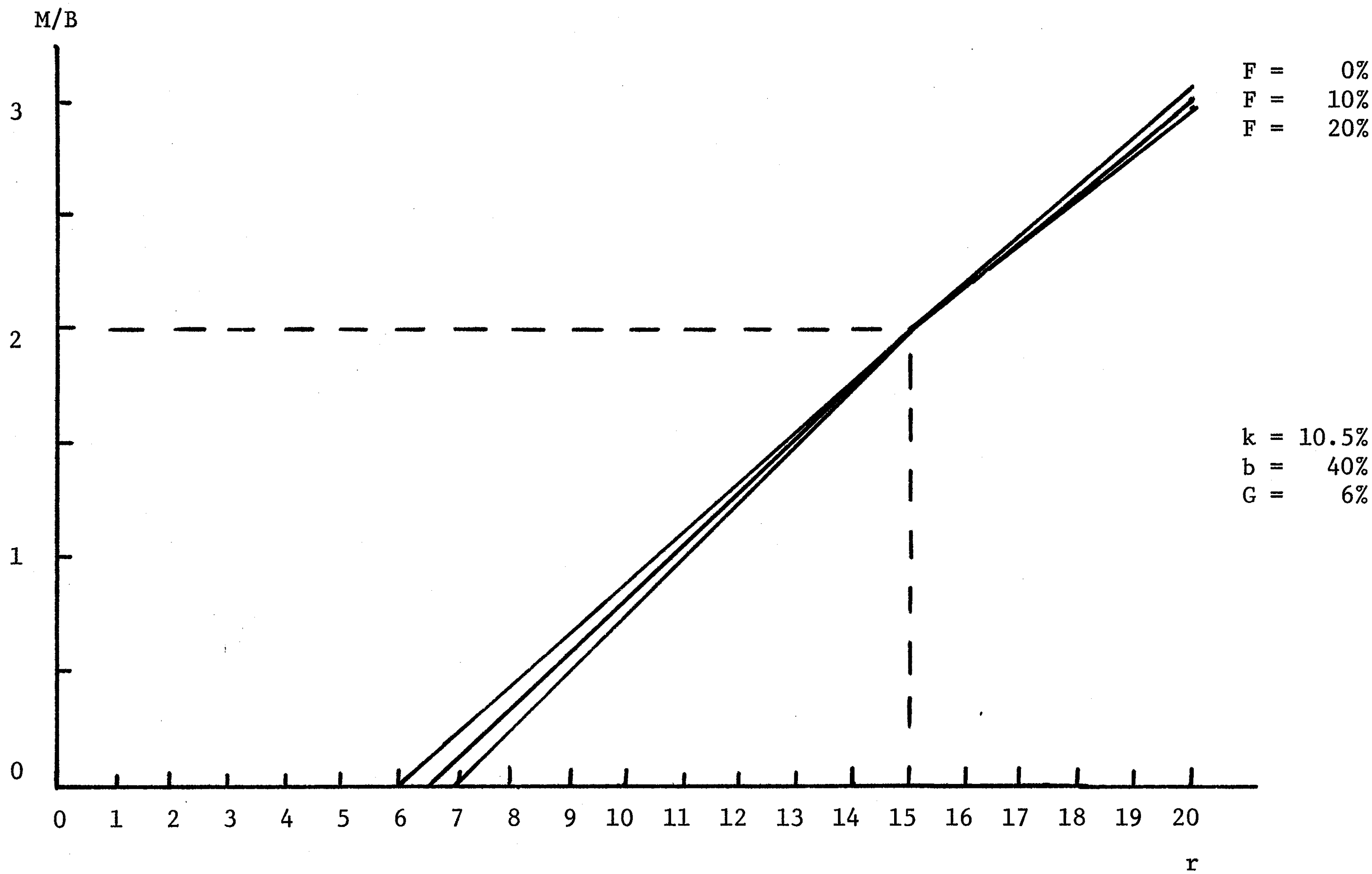
If $F = 0$, then equation (12) reduces to (11); but if $F > 0$, M/B is lower than it would otherwise be.

Figure 4 shows how the M/B ratio varies with r assuming different values for F. Some interesting features of the figure include the following:

1. If $r = k$, $b > 0$, and $G > 0$, then $M/B = 1$ only if $F = 0$. In our example, $b = .4$ and $G = 6$ percent. Thus, only on the $F = 0$ line is $M/B = 1.0$ where $k = r = 10.5$ percent. In the other cases, $M/B < 1.0$ where $r = 10.5$ percent.[12]

[12]Where $F > 0$, the cost of capital obtained by selling new stock is greater than k, the DCF required rate of return on the common stock. This point, which is discussed in detail in [5], pp. 306-307, explains why the M/B ratio is less than 1.0 if r is set equal to 10.5. The true DCF cost of capital is greater than 10.5 percent if the firm incurs flotation costs, so setting $r = 10.5$ implies $r < k$, which leads to $M/B < 1.0$.

Figure 4. M/B Ratios When Stock is Sold With Flotation Costs



2. The higher the value of F, the larger the value of r needed to attain a specified M/B ratio.

3. At the point where r = 15 percent, the firm can finance its 6 percent growth rate through internal equity only, and here the M/B ratio is independent of flotation costs.

4. The lines bend down past r = 15 because, if G, b, and the debt ratio are all to remain constant, then the firm must repurchase and retire stock, and F becomes a brokerage rather than a flotation cost.

Equation (12) is probably the best, or most realistic, of our models relating M/B ratios to allowed rates of return--if we were called upon to recommend to a utility commission the allowed rate of return needed to attain a specific M/B ratio, we would use equation (12) as the basis for our recommendation. However, as noted below, the model may be subject to faults which would cause the actual M/B ratio to deviate from the one predicted by equation (12).

III. SOME QUESTIONS ABOUT THE M/B MODELS

Some Potential Shortcomings of Equation (12)

The various M/B models developed above have several potential shortcomings; they are discussed briefly in this section.

The Constant Growth Assumption. Our M/B equation is based on the constant dividend growth model, so it has all the shortcomings of that model. In the first place, the model assumes that growth is expected to be constant at some specified level for an indefinite period. This is always a worrisome assumption, but it is made throughout most of the financial literature, and it is probably not too inconsistent with reality: investors do make subjective estimates of growth, and these estimates may generally be stated in terms of a constant growth rate.

The constant growth assumption may, however, be a more serious limitation in the present use of the model than in the case of unregulated firms. Especially if $r < k$, the calculated growth rate from retention ($g_1 = br$) may understate the long-run expected growth rate--investors may reason that, eventually, regulators will raise r to the level of k, thereby causing g_1 = br to increase.[13]

Also, as we noted in the preceding section, our model--or at least the way we evaluate it--assumes an inelastic demand function. For relatively small changes in r, hence prices, this is probably not a bad assumption, but for large changes in r the model might give seriously misleading results.

The Dividend Hypothesis. We are, in equation (12), implicitly following the Miller-Modigliani dividend hypothesis; i.e., we implicitly assume that investors are indifferent between returns in the form of dividends or capital gains. This follows from the fact that we have k independent of b. If we knew that a relationship exists between k and b, and if we knew the nature of this relationship, we could incorporate it into the model. Based on the available evidence, the independence assumption is probably not bad, at least for relatively small changes in the retention rate, but the assumption could cause the model to be questioned for comparisons among firms with radically different dividend policies. Fortunately, most utility companies have payout ratios in the range of 50-75 percent, so the dividend controversy is probably not a serious issue for our M/B model.

Debt Policy. We implicitly assume a constant debt ratio--this follows from the fact that we set G = br + s, and if the debt ratio could vary, G could be different from br + s. The model could be expanded to permit a variable debt ratio, and we may do so in the future. However, this will require us to specify the relationship between k and leverage. Unlike the dividend indifference assumption, we feel that k is related to leverage, so we anticipate problems incorporating debt policy into the M/B model.

Marginal versus Average r. We are implicitly assuming that the rate of re-

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[13]The g_1 based on r > k may, on the other hand, be projected into the future if it produces a M/B ratio that regulators do not consider unreasonably high.

turn on new investments is identical to the rate of return on existing plant; i.e., that marginal returns are equal to average returns. This is probably not a bad assumption under "normal" conditions, but in inflationary periods and in the presence of regulatory lag, it is a bad assumption. To see why, take the case of AT&T's operating unit in Florida. The average investment per telephone in service is about \$1,200, and the price of telephone service is set to earn a rate of return r on that amount. However, the marginal capital cost per new telephone in 1973 is over \$2,400. Accordingly, the average cost of plant in service is rising, and the larger is G, the faster the rise in average plant cost. It is clear that if prices are based on average plant costs, and if prices are raised only after a lag, then the realized r will tend to fall below the target r, and the shortfall will be larger the higher the value of G.

Thus, if marginal $r <$ average r, then a high asset growth rate will tend to depress the value of r observed in the company's accounting statements and used in equation (12). This particular factor should not cause a problem in using the model during a given period with specified rates of inflation and regulatory lags; inflation and lags will lower the observed value of r from what it otherwise would have been, but it will not affect the working of the model. However, a problem does arise in attempts to verify the model and to compare the empirical results over time. What happens is that in noninflationary, short-lag periods, G tends to exert a positive influence on M/B ratios, while the converse holds in inflationary, long-lag periods.

IV. SUMMARY: USING THE M/B EQUATIONS IN UTILITY RATE PROCEEDINGS

In rate cases, witnesses frequently testify that the rate of return on book equity should be set so as to attain a specified target M/B ratio. The question immediately arises: Given a target M/B ratio, what rate of return on book equity should the company be allowed to earn? For example, if a given firm's M/B ratio is

1.0, but the target M/B ratio is 1.3, what rate of return on equity should be allowed? As we have seen, the answer depends upon a number of factors--k, s, b, G, and F. Given estimates of these factors, we can determine the appropriate value of r through the use of equation (12). For example, if k = 10.5 percent, b = 40 percent, G = 8 percent[14], and F = 5 percent, then the value of r needed to bring the M/B ratio up to the 1.3 target is 11.4 percent.

As we have indicated, the models do have potential weaknesses--they assume that certain conditions exist, and if these assumptions are incorrect, then the M/B ratio that results from a specific r will be different from the predicted M/B ratio. We plan to undertake an empirical study in the near future to determine just how well the model works. Preliminary tests suggest that the model predicts at an accuracy rate of about 75 percent; i.e., the model explains about 75 percent of the variation in M/B ratios among utility companies. With this level of accuracy, we believe that the model can be applied in rate cases; that is, the value of r as determined in equation (12), if allowed by a commission, will produce an M/B ratio reasonably close to the target ratio.[15]

Also, as we noted at the outset, we have underway research dealing with reasonable target levels, or ranges, for M/B ratios. Assuming that a valid target range can indeed be determined, and that our M/B models prove reliable in our further empirical tests, then we will have another useful input for rate case decisions.

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[14]With G = 8 percent and b = 40 percent, then s = .08 - .4 r.

[15]We have seen rate case testimony that suggests the value of r needed to produce a 1.3 M/B ratio under these conditions is 13.65 percent, found by increasing the old r of 10.5 percent (r must have been 10.5 percent if M/B = 1.0) by 30 percent, the desired increase in the M/B ratio. The equation approach shows that such a proportional increase greatly overstates the proper value of r.

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Appendix A

Derivation of M/B Equation When Stock is Sold

The basic dividend growth model is

$$P_o = \frac{D_1}{k - g} = \frac{D_1}{k - g_1 - g_2} \tag{1a}$$

when growth occurs both from earnings retention and from sale of stock.

Substituting for D_1 and dividing through by B_o, we obtain

$$P_o = \frac{r(1 - b)B_o + r(1 - b)B_o g_2}{k - g_1 - g_2}$$

$$= \frac{r(1 - b)B_o(1 + g_2)}{k - g_1 - g_2}$$

$$\frac{P_o}{B_o} = \frac{r(1 - b)(1 + g_2)}{k - g_1 - g_2}. \tag{2a}$$

Let $x = r(1 - b)$ and $y = k - g_1 = k - br$, and substitute into (2a):

$$\frac{P_o}{B_o} = \frac{x(1 + g_2)}{y - g_2}. \tag{3a}$$

Now substitute equation (10) from the text into (3a) for g_2 and cancel terms:

$$\frac{P_o}{B_o} = \frac{x\left[1 + \frac{P_o(1 + s)}{P_o + sB_o} - 1\right]}{y - \left[\frac{P_o(1 + s)}{P_o + sB_o} - 1\right]}$$

$$= \frac{x\, P_o(1 + s)}{y(P_o + sB_o) - P_o(1 + s) + P_o + sB_o}$$

$$= \frac{xP_o\ (1 + s)}{yP_o + ysB_o - P_o s + sB_o} \quad . \tag{4a}$$

Now rearrange terms, then divide through by P_oB_o:

$$yP_o^2 + ysB_oP_o - sP_o^2 + sB_oP_o = xB_oP_o + xB_oP_os$$

$$y\frac{P_o}{B_o} + ys - s\frac{P_o}{B_o} + s = x + xs$$

$$\frac{P_o}{B_o}(y - s) + s(y + 1) = x(1 + s)$$

$$\frac{P_o}{B_o} = \frac{x(1 + s) - s(1 + y)}{y - s} \quad . \tag{5a}$$

Finally, substitute for x and y to complete the derivation of text equation (11):

$$\frac{P_o}{B_o} = \frac{r(1 - b)(1 + s) - s(1 + k - br)}{k - br - s} . \tag{11}$$

Appendix B

Derivation of the M/B Equation when Flotation Costs are Incurred

Step 1: Definitions

B_o	=	Original book value per share (before sale of stock)
B_{oa}	=	Book value after sale of stock
S	=	Total number of original shares outstanding
SB_o	=	Total book equity
s	=	Growth rate of total book equity from sale of stock
sSB_o	=	Total dollar increase in book equity from sale of stock
	=	Amount of funds available after flotation costs
F	=	Flotation costs expressed as a percentage of the total funds raised by sale of stock
$\frac{sSB_o}{1 - F}$	=	Amount of funds that must be raised through sale of stock in order to leave sSB_o available after flotation costs; i.e., gross funds raised before flotation costs.
ΔS	=	Number of shares that must be sold to raise funds in the amount of $sSB_o/(1 - F)$
	=	$\frac{sSB_o}{1 - F} \div P_o = \frac{sSB_o}{P_o(1 - F)}$
g_2	=	Growth in book equity per share resulting from sale of stock. This g_2 is, of course, different from the g_2 that would result in the absence of flotation costs.

Step 2: Determining the Value of g_2

$$B_{oa} = \frac{SB_o + \Delta SP_o(1 - F)}{S + \Delta S}.$$

Substituting for ΔS, we obtain:

$$B_{oa} = \frac{SB_o + \left[\frac{sSB_o}{P_o(1 - F)}\right]\left[P_o(1 - F)\right]}{S + \frac{sSB_o}{P_o(1 - F)}}$$

$$= \frac{SB_o + sSB_o}{\frac{SP_o(1 - F) + sSB_o}{P_o(1 - F)}}$$

$$= \frac{B_oP_o(1 + s)(1 - F)}{P_o(1 - F) + sB_o}.$$

But g_2 is defined as follows:

$g_2 = B_{oa}/B_o - 1.$

Substituting for B_{oa}, we obtain:

$$g_2 = \frac{\left[\frac{B_o P_o (1 + s)(1 - F)}{P_o(1 - F) + sB_o}\right]}{B_o} - 1$$

$$= \frac{P_o(1 + s)(1 - F) - P_o(1 - F) - sB_o}{P_o(1 - F) + sB_o}$$

$$= \frac{sP_o(1 - F) - sB_o}{P_o(1 - F) + sB_o} .$$

Step 3: Developing the M/B Equation

Substituting the new equation for g_2 into equation (3a) from Appendix A:

$$\frac{P_o}{B_o} = \frac{x(1 + g_2)}{y - g_2} \tag{3a}$$

$$= \frac{x\left[1 + \frac{sP_o(1 - F) - sB_o}{P_o(1 - F) + sB_o}\right]}{y - \frac{sP_o(1 - F) - sB_o}{P_o(1 - F) + sB_o}}$$

$$= \frac{x\left[\frac{P_o(1 - F) + sB_o + sP_o(1 - F) - sB_o}{P_o(1 - F) + sB_o}\right]}{\frac{yP_o(1 - F) + ysB_o - sP_o(1 - F) + sB_o}{P_o(1 - F) + sB_o}}$$

$$= \frac{xP_o(1 - F)(1 + s)}{P_o(1 - F)(y - s) + sB_o(1 + y)} .$$

Now rearrange terms, obtaining:

$$P_o^2(1 - F)(y - s) + sB_oP_o(1 + y) = xB_oP_o(1 - F)(1 + s).$$

Divide through by B_oP_o, obtaining:

$$\frac{P_o}{B_o}(1 - F)(y - s) + s(1 + y) = x(1 - F)(1 + s)$$

$$\frac{P_o}{B_o} = \frac{x(1 - F)(1 + s) - s(1 + y)}{(1 - F)(y - s)}.$$

Now substitute $x = r(1 - b)$ and $y = k - g_1 = k - br$ and rearrange:

$$\frac{P_o}{B_o} = \frac{r(1 - b)(1 + s)(1 - F) - s(1 + k - br)}{(k - br - s)(1 - F)} \tag{12}$$