USAGE-BASED PRICING AND BROADBAND USERS DIFFERENTIATION

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ABSTRACT

Fixed line broadband service providers in the United States are trying to propose different broadband pricing strategies centered on usage. In our research, we wish to explore these issues and model them in an analytical framework. The results should be of interest to broadband service providers as they consider the merits and demerits of the different strategies, and researchers as well as policymakers as they debate the appropriate regulatory responses.

*Keywords*: Internet access pricing; congestion pricing; public policy; market regulation
1. Introduction

In 2007, it was independently verified that the broadband Internet service provider\(^1\) Comcast was slowing down network traffic within its servers that originated from the popular peer-to-peer (P2P) networks (McCullagh 2007). After initially denying any such behavior, Comcast defended its actions by claiming that the traffic from the P2P networks, which was dominated by just a small fraction of the total number of users, was slowing down the network traffic for the rest of the users. The United States Federal Communications Commission (FCC) later declared Comcast’s actions to be illegal.

In this article, our focus is to analyze the economic rationale for and against the proposals put forth by several broadband service providers who intend to differentiate between different classes of users. For example, the cable broadband service provider Time Warner Cable has recently started an experiment in certain markets where they plan to charge Internet customers based on how much Web data they consume. The experiment started in a single market (Beaumont, TX) in the summer of 2008, and the company plans to introduce tiered pricing in several other markets in the near future. By charging a premium to the heaviest broadband users, much the same way cell phone providers collect fees from subscribers who exceed their allotted minutes, Time Warner would upend a longstanding uniform pricing strategy among (fixed-line) Internet service providers in the United States, whereby phone and cable companies have charged flat fees for unlimited access to the Web. AT&T has started a similar experiment with its own customers, also in Beaumont, TX.

As expected, such experiments have ignited fierce debate. Consumer advocates and online content providers have opined that a tiered Web-use pricing would limit customer choice and could stifle innovation by crimping demand for high-bandwidth services such as

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\(^1\) Here, and in the rest of the article, we have uses the terms broadband internet service provider, broadband service provider (or BSP for short) and internet service provider (ISP) interchangeably.
online video and music (Al-Chalabi 2008). However, cable and phone companies have countered by saying that they need the flexibility in setting prices for use of large, expensive, heavily used broadband networks, so as to effectively serve the majority of their customers and encourage greater efficiency in the way customers use capacity (Tweney 2008).

As consumers spend more time online, and also use the Internet to consume various types of data-intensive content (like music and video – a high-definition movie typically consumes around 8 GB of traffic), the decision to charge data consumption by volume can be expected to have profound implications in the way online content is consumed in future. In such scenarios, heavy users can expect to spend much more than what they currently spend on the erstwhile “all you can eat” plans. However, Time Warner has countered that most people are actually not downloading that much data. The company’s trial in Beaumont, TX, lasted several months: of the 10,000 broadband customers enrolled – which represented about 25% of the company’s total number of consumers in Beaumont – about 14% exceeded their cap and had to pay additional fees that averaged about $19 a month. Time Warner Cable also discovered that the top 25% of users consumed 100 times more data than the bottom 25% of users, suggesting an enormous gap in usage patterns.

Broadband service providers have often mentioned that as more and more people download TV shows and movies, particularly those in high-definition, the broadband network infrastructure faces enormous strain. Time Warner Cable has said its strategy is intended to alleviate some of that strain, with users self-regulating themselves under the new plan. But critics have expressed concerns that the pricing scheme will discourage broadband use and impede new online media businesses before they even have a chance to flourish.

The entire debate has raised a number of unanswered questions that are of interest to researchers and practitioners alike, not to mention the regulatory agencies. While legal scholars might debate whether such pricing plans (as those that Time Warner and AT&T are experimenting with) are fair on the consumers, it is an entirely separate issue whether there are
economic incentives for the BSPs to pursue such strategies. In other words, facing a highly dynamic and differentiated data usage patterns from different classes of users, would a BSP gain (as compared to the status quo) by employing different pricing strategies? The focus of our analysis is to explore this issue.

While the BSP might prefer certain pricing policies under certain circumstances, such a move might be detrimental to the consumers or the society as a whole. Thus, from a social planner’s perspective, the issue is somewhat different: how would such actions affect consumer surplus or social welfare? Depending on that answer, the social planner might wish to regulate on the issue.

In this paper, we explore these issues and model them in an analytical framework and examine the economic impacts of user discrimination from the perspectives of both the BSP and the social planner. We characterize the dynamic and differentiated data demand of the end consumers by their valuations for data consumption and their usage patterns. Specifically, we consider a stylized model that segments the consumers into two types, H (for heavy) and L (light), with the H-type consumers constituting a (relatively small) fraction of the entire consumer base (for example, as we point out later, AT&T characterized their heavy users constituting about 5% of the total consumer base). These two types of consumers are characterized by their valuations for data consumption ($V_H$ and $V_L$, where $V_H > V_L$) and their usage patterns ($\lambda_H$ and $\lambda_L$ where $\lambda_H > \lambda_L$). We will discuss these user characteristics in greater detail in next section.

Under the current scenario (which can be thought of as a uniform fixed fee pricing strategy), all users are charged the same fixed price for accessing broadband content. Both types of users face similar delays while accessing their desired content – the delay arises from the fact that the users’ packets are serviced by the broadband service provider who has a fixed capacity. Facing this heterogeneous user data demand, the potential instruments for user discrimination for broadband
service providers is through price discrimination. We explore three different strategies that a BSP might employ to differentially charge its users:

1. Broadband user traffic from different user types face the same delay, and all users are charged the same fixed fee (i.e., the status quo).

2. Broadband user traffic from different user types face the same delay, and different types of users are charged different fixed fees.

3. Broadband user traffic from different user types face the same delay, and different types of users are charged a two-part tariff.

These three different options help us model a broad swath of strategies that a BSP might employ. Depending on the characteristics of users’ valuations for content and their usage patterns, different types of pricing regimes yield different profits for the BSP. However the optimal choice for the BSP might not coincide with that of a policymaker who intends to maximize the total social surplus. The results of the analysis should therefore be useful both for the broadband service providers as they mull over the introduction of the different pricing strategies in an age where consumers increasingly get their information and entertainment online, and for policymakers who might wish to regulate the BSPs’ practice of user discrimination in order to maximize social surplus.

We find that with price-based differentiation, the BSP would prefer to charge a two-part tariff for Internet access that generates higher profit than the traditional all-you-can-eat for a flat rate. We show that the total social welfare remains the same for all three pricing options.

The remainder of the paper is organized as follows: In next section, we propose a stylized model of the BSP’s pricing mechanisms. We then analyze the BSP’s pricing options with price-only differentiation in Section 3. This enables us to examine the impact of pricing mechanisms from the perspectives of the BSP and the social planner respectively. We then conclude with a summary of our findings and some directions for future research.
2. The Model

We assume a monopolist BSP who provides Internet access to the end consumers. While the monopoly assumption is a simplification in some geographies, it is to be noted that unlike many other countries, the extent of competition in the local broadband services market is very limited in the United States, so much so that in many places, a single broadband service provider is often a de facto monopolist (Hausman et al. 2001; Economides 2008). Some of the factors leading to this scenario are the high switching costs induced by long-term service contracts and by incompatible broadband technologies between cable and phone companies. Further, many customers are not qualified for DSL broadband services from phone companies because they exceed the maximal distance limit from the phone company’s nearest switching office, making the cable operators the only feasible broadband service providers in several local markets (Turner 2007). Thus, in addition to providing the benefit of making the analysis tractable, the assumption closely reflects the reality of local broadband services in the U.S. market.

To model the demand for broadband Internet access service, we consider a unit mass of end consumers. As mentioned earlier, we assume that there are two types of users: a fraction $\alpha$ of H-type consumers and $1 - \alpha$ fraction of L-type consumers. High-type users request more content (the requested rate of data packets by the two user types are given by $\lambda_H$ and $\lambda_L$ respectively, where $\lambda_H > \lambda_L$) and have higher valuation for that content ($V_H > V_L$) than the Low-type users. Considering the consumers’ heterogeneous demand patterns, the BSP may charge a uniform fixed fee ($F$) per unit time to all consumers; different fixed fees ($F_H$ and $F_L$) per unit time to different types of consumers; or a usage-based fee ($p$) per packet to consumers for Internet access, a pricing strategy that has been already employed in some Scandinavian countries (Economist 2003; Bandyopadhyay and Cheng 2006). Since the consumers are serviced by the BSP which has a fixed network infrastructure capacity, the former encounter a disutility while they wait for the packets to arrive. The consumers’ utility function thus takes the following form:
Consumers request data from various websites and the requested data packets are transmitted through the BSP’s network. We model the congestion in the network after (Mendelson 1985; Bandyopadhyay and Cheng 2006), and accordingly, consumers’ request for data packets follows a Poisson process with arrival rate $\lambda_H$ and $\lambda_L$ for H-type and L-type consumers respectively. The gross valuations the two types of consumers receive are denoted by $V_H$ and $V_L$. Consumers face a delay cost due to network congestion during the data transmission process. The BSP’s capacity is fixed and denoted by $\mu$. As noted in the afore-mentioned literature, we assume an M/M/1 queue to model the data transmission service provided by the BSP. Then the time that a data packet spent in the system is $\frac{1}{\mu - \lambda}$ and the corresponding delay cost is $\frac{d}{\mu - \lambda}$ where $d$ is the delay parameter that captures the unit cost of delay for consumers waiting for the content to arrive from the websites. We summarize all the notations in Table 1.

---Insert Table 1 about here---

The delay cost for the consumers is:

$$w_i = \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L}, \quad i = H \text{ or } L$$

3. Pricing Strategies for the BSP

In this section, we analyze three potential pricing structures for the BSP – uniform fixed fee, differential fixed fees and charging a two-part tariff.
Option 1: Uniform fixed fee

The most simple and common pricing mechanism for the BSP is to charge a uniform fixed fee for all consumers. The BSP’s profit maximization problem is formulated as follows:

$$\max_{F_i} \pi_i = F_i$$

s.t. $$V_H - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - F_i \geq 0 \quad (i)$$

$$V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - F_i \geq 0 \quad (ii)$$

Constraint (i) is the participation constraint for H-type consumers and constraint (ii) is the participation constraint for L-type consumers. Since $$V_H > V_L$$, the BSP will charge a fixed access fee that is high enough to just keep the L-type consumers to participate, i.e.,

$$F^*_L = V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L}$$, and the BSP then receives a corresponding profit of

$$\pi^*_L = F^*_L = V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L}.$$  

The corresponding consumer surplus, defined as the sum of the utility of all consumers, is given by

$$CS_i = \alpha \left( V_H - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - F_i \right) + (1 - \alpha) \left( V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - F_i \right)$$

$$= \alpha (V_H - V_L),$$ and the social welfare, defined as the sum of both the BSP’s profit and consumer surplus, is

$$SW_i = \pi^*_i + CS_i = \alpha V_H + (1 - \alpha) V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L}.$$  

Option 2: Differential fixed fees

It is easy to see that this option reduces to the Option 1 above. This is because the BSP has just one service offering at its disposal, and therefore will not be able to differentiate between the two classes of users by using different prices (if the two user types are offered two different price
points, the H-type users will always choose the lower price, as would the L-type users). The
formal statement of the BSP’s profit maximization problem is as follows:

\[
\max_{F_{2,H}, F_{2,L}} \pi_2 = \alpha F_{2,H} + (1 - \alpha) F_{2,L}
\]

s.t. \( V_H = \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - F_{2,H} \geq 0 \) \hspace{1cm} (i)

\( V_L = \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - F_{2,L} \geq 0 \) \hspace{1cm} (ii)

\( V_H = \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - F_{2,H} \geq V_H - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - F_{2,L} \) \hspace{1cm} (iii)

\( V_L = \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - F_{2,L} \geq V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - F_{2,H} \) \hspace{1cm} (iv)

Constraints (i) and (ii) are participation constraints for H-type and L-type consumers respectively.
Constraints (iii) and (iv) are incentive compatibility constraints for H-type and L-type consumers respectively. Constraint (iii) can be reduced to \( F_{NN2_H} \leq F_{NN2_L} \) and Constraint (iv) can be reduced to \( F_{NN2_H} \geq F_{NN2_L} \). So \( F_{NN2_H} = F_{NN2_L} \). As a result, Option 2 can be reduced to Option 1 with \( \pi^*_2 = F^*_2 = V_L = \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} \).

The corresponding consumer surplus will still be \( CS_2 = \alpha (V_H - V_L) \), and the social welfare will be given by \( SW_2 = \alpha V_H + (1 - \alpha) V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} \).

**Option 3: Two-part tariff**

Under this option, the BSP charges a two-part tariff for Internet access – a lump-sum fee \( F \) and a per-unit charge \( p \). The BSP’s profit maximization problem is:
\[
\max_{F_3, p_3} \pi_3 = F_3 + \left[ \alpha \lambda_H + (1 - \alpha) \lambda_L \right] p_3
\]

s.t. \[
V_H - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - F_3 - \lambda_H p_3 \geq 0 \quad (i)
\]
\[
V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - F_3 - \lambda_L p_3 \geq 0 \quad (ii)
\]

Constraint (i) is the participation constraint for H-type consumers and constraint (ii) is the participation constraint for L-type consumers. By solving the BSP’s problem (see Appendix A for derivation details), we find when the two types of consumers’ valuations for data consumption are comparable (we denote this as Case 3_1, with the exact criterion being)

\[
V_H \leq \frac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} - \frac{d\left(\lambda_H - \lambda_L\right)}{\lambda_L \left[\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L\right]},
\]

the BSP will charge a positive lump-sum fee \( F_{3,1}^* = \frac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} - \frac{d\left(\lambda_H - \lambda_L\right)}{\lambda_L \left[\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L\right]} \) and a positive usage-based fee \( p_{3,1}^* = \frac{V_H - V_L}{\lambda_H - \lambda_L} \);

however, if the two types of consumers differ significantly in their valuations for their requested content (or more precisely, if \( V_H > \frac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} - \frac{d\left(\lambda_H - \lambda_L\right)}{\lambda_L \left[\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L\right]} \), which we denote as Case 3_2), the BSP will charge a zero lump-sum fee and rely only on usage-based fee: \( F_{3,2}^* = 0 \) and \( p_{3,2}^* = \frac{1}{\lambda_L} \left[ V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} \right] \).

The corresponding consumer surpluses are

\[
CS_{3,1} = 0 \text{ and } CS_{3,2} = \alpha \left\{ \left[ \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} \right] \left( \frac{\lambda_H - \lambda_L}{\lambda_L} \right) - \left( \frac{\lambda_H V_L - \lambda_L V_H}{\lambda_L} \right) \right\} .
\]

The resulting social welfare is \( SW_{3,1} = SW_{3,2} = \alpha V_H + (1 - \alpha) V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} \). Note that under Case 3_1, the entire consumer surplus is extracted away completely by the BSP.
4. The Preferred Choice for the BSP and the Social Planner

In this section we study the effects of pricing structure on the BSP’s profit.

The BSP’s preference for pricing structure

Comparing the BSP’s three pricing options yields \( \pi_{3,1}^* > \pi_1^* = \pi_2^* \) and \( \pi_{3,2}^* > \pi_1^* = \pi_2^* \). This result is summarized in the following proposition.

**Proposition 1**: (BSP’s preferred pricing structure)

The BSP prefers a two-part tariff.

**Proof**: See Appendix B.

The social planner’s preference for pricing structure

In this subsection, we examine the social planner’s preference for different pricing structures by comparing the social welfare levels when the BSP adopts the three pricing structures. The following proposition summarizes the analysis.

**Proposition 2**: (Social planner’s preferred pricing structure)

The social welfare is the same for one-level fixed fee, two-level fixed fee, and two-part tariff, i.e., \( SW_{NN1} = SW_{NN2} = SW_{NN3} \). Hence, the social planner is indifferent in the pricing strategy adopted by the BSP.

5. Conclusion

In the early days of dial-up access for Internet, consumers were charged for the actual dial-up time they used Internet services. This metered charge pricing later gave way to one fixed fee for all consumers in always-on broadband Internet services. Initially, the broadband service providers (BSPs) implemented the flat rate broadband services at a time when most consumers used the service for e-mails and browsing Web pages that are mainly static (i.e., text and graphics with no videos) in nature. The BSPs’ attempt to introduce usage-based pricing for Internet access amid a surge in Internet traffic as consumers nowadays use the Internet to download digital songs,
videos, and high-def movies with usage-based experiments conducted by AT&T in Beaumont, Texas and metered-based pricing by Time Warner for cities of Austin, Texas and Rochester, New York. These usage-based pricing initiatives by the BSPs have met stiff resistance from consumers. Consumer advocates argue that the traditional fixed fee Internet access has been critical to the growth of Internet usage and the formation of online start-ups.

To complicate the matter for BSPs, the FCC announced in September 2009 its proposal to strengthen the existing principles on network neutrality. Should the net neutrality be enforced, the only option for the BSPs is to consider usage-based pricing option.

We develop an analytical model to analyze the economic impact of the proposals put forth by several broadband service providers who intend to differentiate between different classes of users. Specifically, we examine three potential pricing options for the BSPs – same fixed fee for both heavy users and regular users, differential fixed fees, and two-part tariff. We analyze the implications of these three pricing strategies on BSP’s profits and total social welfare. We find that two-part tariff generates the highest profit for the BSP. When the two types of consumers’ valuation of Internet access are comparable, the BSP will charge a positive fixed fee component of the two-part tariff pricing strategy. However, if the two types of consumers differ significantly in their valuations, the BSP will charge a pure usage-based pricing policy. Interestingly, the total social welfare remains the same irrespective of the pricing policy adopted by the monopolistic BSP. Hence, the social planner or the policy maker should be indifferent to the pricing plan of the BSP.
Table 1: List of Notations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>Percentage of H-type consumers</td>
</tr>
<tr>
<td>$\lambda_H, \lambda_L$</td>
<td>Rate of content requested from H-type and L-type consumers in packets per unit of time</td>
</tr>
<tr>
<td>$V_H, V_L$</td>
<td>The gross value function of retrieving content for H-type and L-type consumers respectively</td>
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<tr>
<td>$F$</td>
<td>A uniform fixed fee per unit of time charged by the BSP to end consumers</td>
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<tr>
<td>$F_H, F_L$</td>
<td>Fixed fees charged to H-type and L-type consumers respectively</td>
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<tr>
<td>$p$</td>
<td>Unit price per packet for data packet transmission</td>
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<tr>
<td>$w_H, w_L$</td>
<td>Consumers’ delay cost (congestion cost) for H-type and L-type consumers respectively</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Capacity of the BSP in packets per unit of time</td>
</tr>
<tr>
<td>$d$</td>
<td>Consumers’ delay parameter that converts the delay for consumers waiting for the content to arrive from the websites to the unit cost of delay per unit of time</td>
</tr>
<tr>
<td>$u_H, u_L$</td>
<td>The utility function for H-type and L-type consumers respectively</td>
</tr>
<tr>
<td>$\pi_i$</td>
<td>The BSP’s profit, $i = 1, 2, 3$</td>
</tr>
<tr>
<td>$CS_i$</td>
<td>Consumer surplus, $i = 1, 2, 3$</td>
</tr>
<tr>
<td>$SW_i$</td>
<td>Social Welfare, $i = 1, 2, 3$</td>
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<tr>
<td>Pricing Options</td>
<td>Same fixed fee for all consumers</td>
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<td>( \pi_1^* = F_1^* = V_L - \frac{d}{\mu - \alpha \lambda_H - (1-\alpha) \lambda_L} )</td>
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<tr>
<td></td>
<td>( CS_1 = \alpha (V_H - V_L) )</td>
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<td></td>
<td>( SW_1 = \alpha V_H + (1-\alpha) V_L - \frac{d}{\mu - \alpha \lambda_H - (1-\alpha) \lambda_L} )</td>
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Appendices

Appendix A: Solution of Option 3

In Formulation (5), from (i), we get \( F_3 \leq V_H - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - \lambda_H p_3 \). From (ii), we get

\[
F_3 \leq V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - \lambda_L p_3.
\]

We consider two cases:

Case 1: \( p_3 \geq \frac{V_H - V_L}{\lambda_H - \lambda_L} \). Then

\[
V_H - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - \lambda_H p_3 \leq V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - \lambda_L p_3. \]

So constraint (i) is binding, i.e., \( F_3 = V_H - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - \lambda_H p_3 \). Substituting into the objective function gives

\[
V_H - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - (1 - \alpha)(\lambda_H - \lambda_L) p_3. \]

The optimal solution is

\[
F_3 = \frac{\lambda_L V_H + \lambda_H V_L}{\lambda_H - \lambda_L} - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L}, \quad p_3 = \frac{V_H - V_L}{\lambda_H - \lambda_L}, \quad \text{and}
\]

\[
\pi^*_3 = \alpha V_H + (1 - \alpha) V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L}.
\]

Case 2: \( p_3 \leq \frac{V_H - V_L}{\lambda_H - \lambda_L} \). Then

\[
V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - \lambda_L p_3 \leq V_H - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - \lambda_H p_3. \]

So constraint (ii) is binding, i.e., \( F_3 = V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - \lambda_L p_3 \). Substituting into the objective function gives

\[
V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} + \alpha (\lambda_H - \lambda_L) p_3.
\]
Case 21: \( \frac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} \geq \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} \). The optimal solution is

\[
F_3^* = \frac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L}, \quad p_3^* = \frac{V_H - V_L}{\lambda_H - \lambda_L},
\]

and

\[
\pi_3^* = \alpha V_H + (1 - \alpha) V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L}.
\]

Case 22: \( \frac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} \leq \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} \). The optimal solution is \( F_3^* = 0 \),

\[
p_3^* = \frac{1}{\lambda_L} \left[ V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} \right]
\]

(Since \( \frac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} \leq \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} \),

\[
p_3^* = \frac{1}{\lambda_L} \left[ V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} \right] \leq \frac{1}{\lambda_L} \left( V_L - \frac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} \right) = \frac{V_H - V_L}{\lambda_H - \lambda_L}
\]

\[
\pi_3^* = \frac{\lambda H V_L - \lambda L V_H}{\lambda H - \lambda L} \left[ V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} \right].
\]

The above cases can be summarized as:

Case 3_1: If \( \frac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} \geq \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} \), i.e.,

\[
V_H \leq \frac{\lambda_H V_L - \lambda_L V_H}{\lambda_L} - \frac{d (\lambda_H - \lambda_L)}{\lambda_L \left[ \mu - \alpha \lambda_H - (1 - \alpha) \lambda_L \right]}, \quad F_{3_1}^* = \frac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L},
\]

\[
p_{3_1}^* = \frac{V_H - V_L}{\lambda_H - \lambda_L}, \quad \pi_{3_1}^* = \alpha V_H + (1 - \alpha) V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L}
\]

The corresponding consumer surplus is
\[ CS_{3,1} = \alpha \left( V_H - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - F_{NN3,1} - \lambda_H p_{NN3,1} \right) \]
\[ + (1 - \alpha) \left( V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - F_{NN3,1} - \lambda_L p_{NN3,1} \right) = 0 \]

Therefore the social welfare is
\[ SW_{3,1} = \pi_{3,1}^* + CS_{3,1} = \alpha V_H + (1 - \alpha) V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} . \]

Case 3_2: If \( \frac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} < \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} \), i.e.,

\[ V_H > \frac{\lambda_H V_L}{\lambda_L} - \frac{d}{\lambda_L \left[ \mu - \alpha \lambda_H - (1 - \alpha) \lambda_L \right]} \cdot \frac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} , F_{3,2}^* = 0 , \]
\[ p_{3,2} = \frac{1}{\lambda_L} \left[ V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} \right] \cdot \frac{\alpha \lambda_H + (1 - \alpha) \lambda_L}{\lambda_L} . \]

The corresponding consumer surplus is
\[ CS_{3,2} = \alpha \left( V_H - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - F_{NN3,2} - \lambda_H p_{NN3,2} \right) \]
\[ + (1 - \alpha) \left( V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} - F_{NN3,2} - \lambda_L p_{NN3,2} \right) \]
\[ = \alpha V_H + (1 - \alpha) V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} \cdot \frac{\alpha \lambda_H + (1 - \alpha) \lambda_L}{\lambda_L} \left[ V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} \right] \]
\[ = \alpha \left\{ \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} \left( \frac{\lambda_H V_L - \lambda_L V_H}{\lambda_L} + \frac{\lambda_H V_L - \lambda_L V_H}{\lambda_L} \right) \right\} \]

Therefore the social welfare is
\[ SW_{3,2} = \pi_{3,2}^* + CS_{3,2} = \alpha V_H + (1 - \alpha) V_L - \frac{d}{\mu - \alpha \lambda_H - (1 - \alpha) \lambda_L} . \]
Appendix B: Proof of Proposition 1

Comparing the BSP’s three pricing options, one has \( \pi_1^* = V_L - \frac{d}{\mu - \alpha \lambda_H - (1-\alpha) \lambda_L} = \pi_{NN2}^* \).

Then we compare \( \pi_1^* \) and \( \pi_2^* \) to \( \pi_3^* \).

If \( \frac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} \geq \frac{d}{\mu - \alpha \lambda_H - (1-\alpha) \lambda_L} \), i.e., \( V_H \leq \frac{\lambda_H V_L}{\lambda_L} - \frac{d (\lambda_H - \lambda_L)}{\lambda_L [\mu - \alpha \lambda_H - (1-\alpha) \lambda_L]} \),

then \( \pi_3^* = \pi_{3,1}^* = \alpha V_H + (1-\alpha) V_L - \frac{d}{\mu - \alpha \lambda_H - (1-\alpha) \lambda_L} \).

Since \( \alpha V_H + (1-\alpha) V_L - \frac{d}{\mu - \alpha \lambda_H - (1-\alpha) \lambda_L} > V_L - \frac{d}{\mu - \alpha \lambda_H - (1-\alpha) \lambda_L} \), \( \pi_{3,1}^* > \pi_1^* = \pi_2^* \).

If \( \frac{\lambda_H V_L - \lambda_L V_H}{\lambda_H - \lambda_L} < \frac{d}{\mu - \alpha \lambda_H - (1-\alpha) \lambda_L} \), i.e., \( V_H \geq \frac{\lambda_H V_L}{\lambda_L} - \frac{d (\lambda_H - \lambda_L)}{\lambda_L [\mu - \alpha \lambda_H - (1-\alpha) \lambda_L]} \),

then \( \pi_3^* = \pi_{3,2}^* = \left[ \frac{\alpha \lambda_H + (1-\alpha) \lambda_L}{\lambda_L} \right] \left[ V_L - \frac{d}{\mu - \alpha \lambda_H - (1-\alpha) \lambda_L} \right] \).

Then \( V_L - \frac{d}{\mu - \alpha \lambda_H - (1-\alpha) \lambda_L} = \pi_1^* = \pi_2^* \) since \( \frac{\alpha \lambda_H + (1-\alpha) \lambda_L}{\lambda_L} > 1 \).
References


