Modeling Electric Congestion Charges in a Composed Error Framework

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ABSTRACT

Congestion charges, the opportunity cost of transmission in a regional electric market, present a unique modeling challenge. Their properties render many popular methods used to model energy prices ineffective. In this paper, we present a model utilizing the composed error framework to capture many of the unique properties of congestion charges. We then demonstrate how this model can be used to assess the risk associated with a financial instrument derived from congestion charges and compare our approach with an assessment that utilizes the bootstrap method.

1. INTRODUCTION

The electricity market in the United States continues to move toward a collection of regional markets each administered by an Independent System Operator (ISO). When the Southwest Power Pool begins operation in 2009,[1] only the western (excluding California) and southeastern portions of the continental United States will retain the traditional electric market structure built around vertically integrated utilities.[2] Whereas traditional utilities once generated the electricity and then transmitted and distributed it to customers at rates periodically reviewed by government regulators, in today's markets, the role of the utility is changing. Utilities in markets administered by ISOs submit daily bids and offers for electricity, and regional electric market models use supply and demand, as well as the physics of electric power flows, to calculate market prices for hundreds of locations within the region.

These market prices are called the locational marginal prices for electricity. Electric customers have traditionally paid a so-called "bundled" rate, where the costs associated with the production, transmission, and distribution of electric power are aggregated into a rate that stayed constant for months, or even years. Locational marginal prices are often expressed as hourly aggregates, but the individual cost components of delivering electricity to a particular point, or node, are transparent. The specific terms may vary between regional markets, but conceptually, the three components of locational marginal prices are the same.

The first component is the energy portion. The energy charge is the marginal cost of generating electricity sufficient to meet the demand, and does not vary across the

[1] http://www.spp.org/.
[2] http://www.ferc.gov/market-oversight/mkt-electric/overview/elec-ovr-rto-map.pdf.

region. The energy charge is determined by comparing the hourly regional supply and demand curves. The second component is the congestion charge. The congestion charge is a monetization of the opportunity cost of transmitting the power on the regional electric grid, and is determined by performing complex simulations of physical power flows within the electric grid. The third component is the loss component. The loss component is the cost of any electricity that might be lost during the process of transmitting the electricity. These losses can be associated with energy losses on the transmission lines themselves, or energy losses that result from changes in voltage as the electricity travels along the transmission grid. The loss component is typically a percentage of the sum of the energy and congestion charges at a particular node. Of the three types of charges (energy, congestion, and loss), the congestion charge is the most complex, both to calculate and to analyze. Moreover, the properties of congestion charges make their analysis with traditional energy models difficult, if not impossible.

With a surfeit of electricity price models to choose from, such as the single- and multi-factor mean reverting models of Pindyck (1999) and Schwartz (1997), the mean reverting jump diffusion models of Clewlow and Strickland (2000) and Clewlow, Strickland, and Kaminski (2001), the price spike models of Kholodnyi (2004), and the regime switching models derived from Hamilton (1994), a robust model of congestion charges might seem superfluous. Nonetheless, a dedicated model of congestion charges fills a void in the quantitative analyst's toolbox. First, the factors that affect the behavior of congestion charges may vary from the factors that affect the energy charges or loss charges, so an explicit model of congestion charges is necessary for any bottom up model of electricity prices in an ISO market. Second, the difference in the congestion charges

between two pricing nodes forms the value for financial instruments known as financial transmission rights (FTRs), and any analysis that attempts to quantify the uncertainty surrounding the value of FTRs must begin with a model of congestion charges.

In this article, we will explain why the congestion charge is so difficult to analyze and why a composed error model lends itself best for analyzing congestion charges. We will first examine the properties of congestion charges and discuss why traditional energy pricing models fail to capture them. We will then discuss the composed error framework, and demonstrate that this framework is applicable to the properties of congestion charges. Although we present pricing nodes within the Midwest Independent System Operator (MISO) to provide an example of how this framework might be best understood, this paper can be easily extended to other ISOs, such as Pennsylvania-New Jersey-Maryland (PJM). Finally, we will use the framework to construct a price distribution for a financial instrument derived from congestion charges and find that this distribution compares favorably with one generated from a bootstrap methodology.

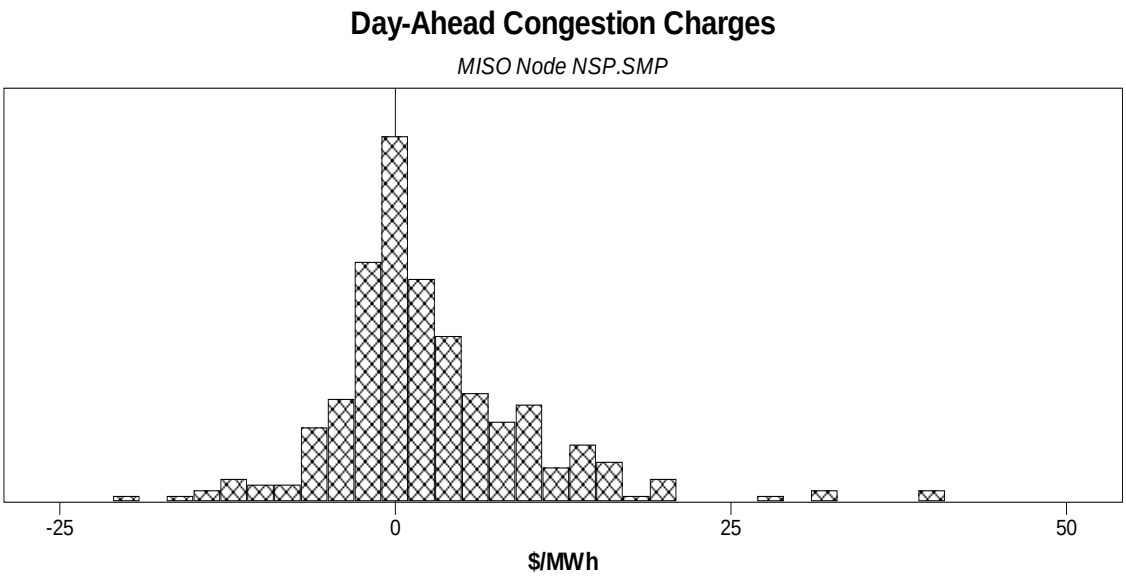
2. BACKGROUND

Properties of Congestion Charges

Congestion charges are the monetization of the opportunity costs of transmission and are used to price transmission and provide incentives in the marketplace. A positive congestion charge, for example, encourages generation and discourages consumption at a particular node within a regional system. A negative congestion charge has the opposite effect.

The first property, then, of congestion charges is that they can be both positive and negative, as shown by the distribution of daily congestion charges from July 1, 2007 through April 27, 2008 at the NSP.SMP node[3] in the MISO in Figure 1.

Figure 1. Sample Distribution of Congestion Charges



Many existing asset pricing models break down in the face of negative prices. The random walk pricing model of Samuelson (1965) utilized in the seminal asset pricing model of Black and Scholes (1973) modeled the price of an asset as a function of its periodic returns. For an asset that can have positive and negative value, the concept of returns is meaningless because returns are ambiguous as prices move from positive to negative and negative to positive. The mean reverting price framework of Pindyck (1999) and Schwartz (1997) modeled asset prices in terms of their logarithms. The logarithm of a negative price is undefined. Further, congestion charges have no theoretical bounds in either direction, so truncated distributions, which assume the existence of such bounds,

[3] NSP.SMP is one of the approximately 1,750 pricing nodes in the MISO.

cannot be used to model the charges. Finally, the distribution of congestion charges is typically skewed in one direction, so a single symmetric distribution may not be suitable for modeling purposes.

Financial Transmission Rights

The application of locational marginal pricing has also spawned a new class of energy derivative. Financial transmission rights (FTRs) have emerged as an instrument used to hedge against volatile congestion charges in the day-ahead markets, a forward market where the price of the commodity is set on the day before physical delivery of the electricity. The value of a particular FTR is determined by the hourly differences between the congestion charges at two points on the regional electric grid, aggregated over an entire calendar month. FTRs are also utilized by speculators willing to assume the risk of congestion charge volatility. The concept of a financial instrument to hedge against volatile congestion charges exists in every regional transmission organization in the United States, though sometimes under a different name (e.g., Congestion Revenue Rights in California). Accurately modeling the characteristics of these congestion charges is essential for either pricing, or assessing the risk associated with, a given FTR. For example, on December 20, 2007, PJM notified its members that Power Edge LLC, a subsidiary of Tower Research Capital, had defaulted on $80 million of its FTR obligations in the PJM market.[4] Power Edge contended that the credit requirements of PJM were inadequate for the magnitude of its positions. The case is being investigated by

[4] http://www.pjm.com/Media/about-pjm/newsroom/2007-releases/20071226-credit-default-news-release.pdf.

the Federal Energy Regulatory Commission,[5] but the risk associated with the FTR positions held by Power Edge was almost certainly underestimated by at least one of the parties involved.

3. MODEL

Composed Error Models

The stochastic frontier production function models (more generally known as composed error models) of Aigner, Lovell, and Schmidt (1977), Meeusen and van den Broeck (1977), and Battese and Corra (1977) arose as an attempt to quantify technical inefficiency in production. These models all decompose the error term in the estimation of a production possibility frontier into two components: one symmetric and one asymmetric. This functional form suits our application for congestion charges as well. Congestion charges are most often influenced by factors that upset, or have the potential to upset, the balance between load and generation. Regional factors, like weather events or the change in status of an important regional generating resource, can have a positive or a negative effect on congestion, and would be best modeled with a symmetric term. Local factors, like the underlying balance between load and generating resources at a particular node, can have an effect as well. A node that is well-balanced will likely have small congestion charges associated with it. A node that has much more generation than load will likely have negative congestion charges associated with it, while a node that has more load than generation will likely have positive congestion charges associated with it. Because the balance between demand and supply changes slowly over time, if at all, it would be best modeled with an asymmetric term.

[5] FERC Docket EL08-44.

We can, then, assume that the error term in a model of congestion charges can be expressed as:

$$\varepsilon_t = v_t + u_t \tag{1}$$

where $v_t \sim N(0, \sigma_v^2)$, $u_t \sim N(0, \sigma_u^2)$ and u_t non-negative.

Per Aigner, Lovell, and Schmidt, the mean and variance of this error term are given by:

$$E(\varepsilon) = E(u) = \sqrt{\frac{2}{\Pi}}\sigma_u \tag{2}$$

$$V(\varepsilon) = \left(\frac{\Pi - 2}{\Pi}\right)\sigma_u^2 + \sigma_v^2 \tag{3}$$

Because of the half normal distribution of u_t, the mean of the error term is no longer zero, and the variance is less than the variance of the sum of two independent, normally distributed, variables. Note also that v and u are considered, by construction, to be independent.

Model of Congestion Charges

The model for the congestion charge itself will depend on the granularity required in the application. A common application for the simulation of congestion charges is the quantification of the risks associated with FTRs. A model with daily resolution is sufficient for this purpose, and because congestion charges exhibit mean reverting

tendencies, the mean reverting framework of Pindyck (1999) and Schwartz (1997) can be used. In the mean reverting framework, prices follow:

$$p_t = p_{t-1} + \alpha(\mu - p_{t-1}) + \varepsilon_t \tag{4}$$

where p_t is the price at time t, μ is the long-term equilibrium price, and α is the rate at which prices revert to μ.

From Aigner, Lovell, and Schmidt, the log likelihood function for estimating this model, with constants omitted, is,

$$\ln L = -N \ln \sigma + \sum_{i=1}^{N} \ln\left[\Phi\left(\frac{\varepsilon_i \lambda}{\sigma}\right)\right] - \frac{1}{2\sigma^2}\sum_{i=1}^{N} \varepsilon_i^2 \tag{5}$$

where

$$\begin{aligned} \lambda &= \frac{\sigma_u}{\sigma_v} \\ \sigma^2 &= \sigma_u^2 + \sigma_v^2 \end{aligned} \tag{6}$$

This functional form, as well as the interpretation of these models, is particularly suited to the question of congestion charges.

We estimated these parameters using observations of congestion charges at 12 nodes in the MISO. The dataset consists of daily observations from the period July 1,

2007 to April 27, 2008. The estimated values of σ_v and σ_u for these nodes are listed in Table 1.

Table 1. Parameter Estimates for Selected MISO Nodes

MISO Node	Day Ahead		Real Time	
	σ_v	σ_υ	σ_v	σ_υ
ALTE.ROCKGEN1	2.273	3.290	4.162	7.649
ALTW.SMP	3.954	6.388	6.391	14.263
ALTW.SPRVAUN1	2.194	8.495	7.098	10.433
ALTW.WELLS1	2.748	8.522	5.668	14.462
NSP.NEW.PNEW	4.900	5.289	7.635	9.224
NSP.SMP	4.308	6.386	7.458	9.413
NSP.WPPI	3.053	7.300	6.870	11.518
SMP.BLOOMING	2.243	9.108	6.900	11.435
SMP.OWANASM7	4.289	7.414	7.404	9.942
SMP.PRESTON	2.208	8.371	7.253	10.403
SMP.SMP	2.645	10.366	7.510	11.054
SMP.ST_PEST1	4.263	11.210	7.584	9.619

The volatility parameters for the real-time markets are higher than for the day-ahead markets. This is consistent with our understanding of the energy markets as forward prices are typically less volatile than spot prices. We can also see that the asymmetric effects appear to be stronger than the symmetric effects, and the symmetric effects seem to be more homogeneous. This is also consistent with our hypothesis that the symmetric terms reflect regional effects, and the asymmetric terms reflect local effects.

Cross Correlations

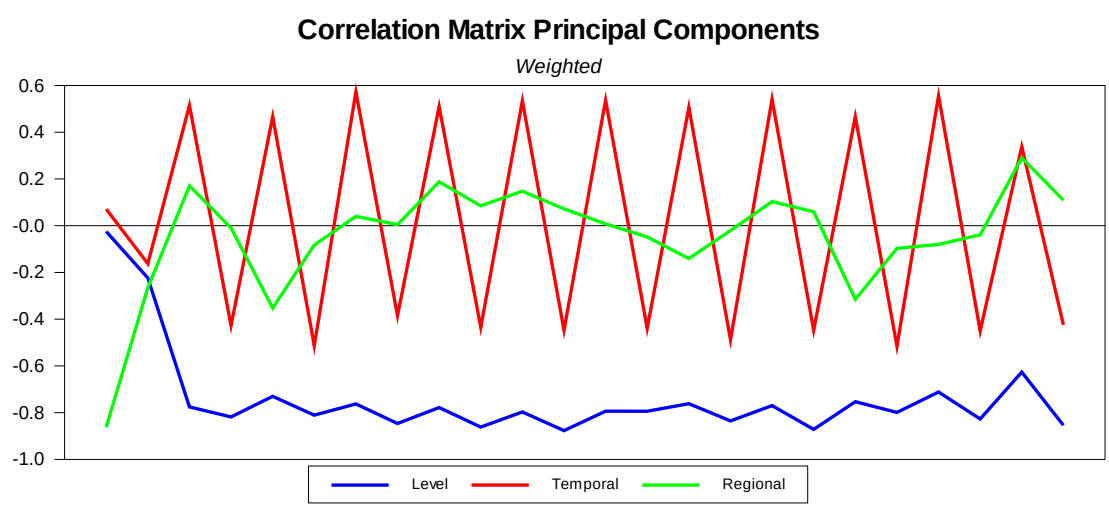
These initial composed error models were concerned with estimating the variance of the v and u components of the residual distributions. Jondrow, Lovell, Materov, and Schmidt (1982) studied the v and u parameters associated with the residual of each observation, and derived the equation:

$$E\left(u_i \mid \varepsilon_i\right) = \left(\frac{\sigma_u \sigma_v}{\sigma}\right)\left[\frac{\phi\left(\frac{\varepsilon_i \lambda}{\sigma}\right)}{\Phi\left(\frac{\varepsilon_i \lambda}{\sigma}\right)} + \frac{\varepsilon_i \lambda}{\sigma}\right] \tag{7}$$

to extract the individual u terms from the maximum likelihood residuals ε.[6] We can use this technique to decompose the daily residuals of each pricing node to calculate a correlation matrix from the components.

Eigen decomposition of this correlation matrix can help us to identify the interrelationships between the various pricing points. We have constructed the correlation matrix by alternating the residuals of the estimation equations for first the day-ahead and then the real-time prices. We repeat this pattern for all 12 nodes in Table 1 to construct the x-axis in Figure 2. The three principal components of this correlation matrix – level, temporal, and regional – are shown in Figure 2.

Figure 2. Weighted Eigenvectors of the Correlation Matrix



[6] As seen in, for example, Mester (1996).

The level component explains 58% of the variation in the correlation matrix. This component measures the tendency for the congestion charges at different points and in different markets to move up or down together. The temporal component explains 21% of the variation. Note that all of the eigenvectors associated with the day-ahead prices are positive and eigenvectors associated with the real-time prices are negative. This component measures the tendency for congestion charges in the day-ahead and real-time markets to move in opposite directions. Finally, the third component appears to be a regional component, measuring the tendency for nodes in different regions to move in different directions. This component explains only 5% of the variation.

When we compare the eigenvalues for congestion charges to the eigenvalues of natural gas and crude oil on the New York Mercantile Exchange (NYMEX), there is a stark contrast. We see that the correlation matrix for these congestion charges decay slowly relative to these other commodities. This is an indication that the correlation matrix for congestion charges is more complex than the correlation matrices for natural gas and crude oil forward contracts. The level component of the NYMEX crude oil forward curve explains about 99% of the variation in the correlation matrix for the first 12 futures contracts. The level component of the NYMEX natural gas curve explains about 92% of the variation in the correlation matrix of the first 24 futures contracts, and the temporal component explains about 5% of the variation (Kury 2008). So the first principal component of the crude oil futures curve alone explains 99% of the variation in the correlation matrix. The first two principal components of the natural gas forward curve explain 97% of the variation in the correlation matrix. But the first three principal

components of the correlation matrix for congestion charges explain only 84% of the variation. While the scope of our sample of congestion charges is not comprehensive, this seems to indicate that the interrelationships in the market for congestion charges are much more complex than those for liquid fuels markets where most of the variation can be explained by the level component.

4. RESULTS

Quantification of the risk of FTRs

We can illustrate the utility of a parametric model for modeling congestion charges by measuring the risk associated with the purchase of a FTR. Without a parametric approach, we are limited to estimating the distribution of the FTR's value with the bootstrap method. Under the bootstrap methodology, we can resample contiguous historical blocks of congestion charges at two different points and compute the sum of their differences. Resampling of contiguous blocks is necessary because congestion charges are correlated from one day to the next, as well as across nodes. Through multiple iterations of this process, we can build a distribution of value for this asset. A significant limitation of the bootstrap method, however, is that it makes the implicit assumption that the past perfectly mirrors the future. By extension, the only risk in the future will be risk that has already existed in the past.

With our parametric model, we can use the parameters in Table 1 and the interrelationships illustrated in Figure 2 to build a parametric simulation of the value of a FTR from the MISO nodes NSP.SMP to SMP.SMP. The results of these simulations, under each estimation technique, with 10,000 iterations are given in Table 2.

Table 2. Summary Statistics for Bootstrap and Parametric Distributions of FTR Value

	Bootstrap	Parametric
Mean	-35.98	-64.64
Minimum	-172.33	-278.18
Maximum	37.75	149.90
5th Percentile	-156.12	-156.99
95th Percentile	28.84	26.60

The difference in mean values between the two simulations is due to the bootstrap methodology utilizing historical averages as the expected case, while the parametric method attempts to extract an (unobserved) long-run equilibrium price from the data. This long-run equilibrium price is utilized as the expected case in the parametric simulation. The difference in the extremes of the value distribution is due to the assumption, under the bootstrap methodology, that the past perfectly reflects the future. Under the parametric approach, we assume that history is a subset, but not a comprehensive view, of future possibility. Despite these differences in the distributions, the 5th and 95th percentiles under the two methodologies are nearly identical.

5. FUTURE RESEARCH

The composition of a normal and half-normal term to model the error distribution of a particular congestion charge may still be insufficient, but a strength of the composed error framework is its flexibility. In their original paper, Aigner, Lovell, and Schmidt also derived a normal-exponential composition for modeling the distribution of error terms. Greene (1990) incorporated a gamma distribution into the composed error framework.

The extent to which these models, or further compositions, apply to congestion charges is an interesting question.

6. CONCLUSION

The properties of congestion charges distinguish them from many other assets in the energy sector. Most traditional models used to model energy assets fail when confronted with the unique properties of congestion charges. Nonetheless, a distinct model that captures the unique properties of congestion charges is critical for assessing the risk of prices in an ISO framework, or the risk associated with financial products like FTRs. In this paper, we have proposed a framework utilizing the composed error model to capture these properties. We have shown that the residuals from individual observations can be broken down into their symmetric and asymmetric components, and that their correlation matrix can be analyzed through eigen decomposition, to extract important interrelationships between the charges. Finally, we simulated a parametric distribution of the value of a sample FTR and found that it compared favorably with a bootstrap estimation of the FTR value.

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