Quality Provision in Two-Sided Markets: the Case of Managed Care

Mircea I. Marcu*

Department of Economics, University of Florida, 331 Matherly Hall P.O. Box 117140, Gainesville, FL 32601, (352) 392-0124, mircea@.ufl.edu

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ABSTRACT

I develop a two-sided market model to study the role played by indirect externalities in shaping the pricing and quality decisions of managed care organizations. I find that managed care quality, access to physicians, and physician reimbursements decrease when the cost of providing quality or the population health risk increases. These decreases are more pronounced when the externality exerted by doctors on patients is larger, when policyholders care more about quality, and when the supply of physicians is more elastic. The results may help to understand the increasing public dissatisfaction with managed care. They underscore the importance of implementing and monitoring the achievement of quality standards in the face of ongoing increases in health risks associated with an aging population.

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1. Introduction

Direct externalities, such as those exerted by insured individuals on other members of the same insurance pool through moral hazard and/or adverse selection, have long been analyzed in health care markets.[1] This article focuses on indirect externalities between doctors and patients and the two-sided nature of managed care markets.

Two-sided markets are markets in which there are two-way indirect externalities between the participants in a market that interact through an intermediary platform.[2] The number (and/or quality, or some other characteristic) of members on one side of an intermediary's network affects the utility of enrollees on the other side. A case in point is a managed care organization (MCO) that needs to attract both doctors and patients. Patients care about the availability of doctors. At the same time, doctors care about the number of patients enrolled in the managed care plan.

The presence of indirect externalities leads to important departures from fundamental results in traditional microeconomics. For example, departures from marginal cost pricing may not necessarily reflect market power. Such departures could also reflect the relative magnitude of the indirect externalities exerted by each side (Armstrong [2006]). The pricing structure in two-sided markets is often skewed in favor of one side, whose consumption is subsidized from the revenues collected on the other side. Most software platforms and internet portals operate in this manner. Recognizing the importance and consequences of the indirect externalities is essential to avoid the fallacies of applying the one-sided logic to two-sided markets (Wright [2004]).

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[1] See Arrow [1963], Zeckhauser [1970], Ellis and McGuire [1993], Ma and McGuire [1997], Ma and Riordan [2002].

[2] See Rochet and Tirole [2003] for a formal definition of two-sided markets.

Managed care differs from other two-sided markets because the MCO's expected cost is linked to the expected benefit derived by enrollees through their health risk. In contrast, the cost of producing a commodity like a video game is the same regardless how much gamers enjoy the game. Therefore, the health risk of policyholders is of particular importance in analyzing managed care. This health risk, in turn, depends on the quality of health care provided by the MCO, the size of its physician network, and the insurance premium it charges.

To date, two-sided market models have focused almost exclusively on the pricing decisions of intermediaries.[3] The study by Pezzino and Pignatarro [2007] is an exception. They study quality provision by competing hospitals in a horizontal differentiation model with regulated insurance premiums. To my knowledge, Bardey and Rochet's [2006] analysis of competition between managed care organizations is the only other two-sided approach to health care markets.

I extend the literature by analyzing an MCO's profit maximizing choice of quality of service. Quality here reflects the degree to which the MCO assists physicians in providing health care services. It can be thought of as a quality-increasing input supplied by the MCO, such as diagnostic tests or sophisticated health information processing, storage, and retrieval.

I find that the less elastic is the supply of doctors with respect to reimbursements, the more enrollees care about quality, and the lower the indirect externality exerted by doctors on patients, the more the MCO can reduce physician compensation when increasing quality and maintain the same premium and number of enrollees, particularly at low levels of quality relative to reimbursement.

[3] Evans and Schmalensee [2005], and Rochet and Tirole [2005] provide an overview of the two-sided markets literature.

In my model, an increase in the marginal cost of providing quality decreases the amount of the quality-increasing input provided by the MCO. This in turn decreases the effectiveness of physicians, which leads to lower physician reimbursements and fewer doctors in the preferred network. Individuals are unwilling to pay the same insurance premium as before, and the demand for managed care shifts downwards. The insurance premium decreases, but the decrease does not fully compensate for the decreases in quality and number of physicians. Thus, the MCO reduces the risk of its patients' mix to avoid providing the more costly quality.

For populations more at risk of becoming ill, as is the case of the aging population of the United States, the MCO provides services of lower quality, pays lower physician reimbursements, and includes fewer providers in its preferred network. The insurance premium decreases, but the MCO finds it profitable to enroll some higher risk individuals.

The decreases in managed care quality, access to doctors, and reimbursements to physicians due to increases in the population health risk and in the cost of quality provision are more pronounced when the externality exerted by doctors on patients is greater, when individuals care more about quality, and when the elasticity of the supply of doctors with respect to reimbursements is greater. Thus, although MCOs provide services of higher quality and access to more doctors in markets where individuals care more about quality and access to doctors, these markets are also the ones most affected when the cost of quality provision and population health risks increase.

The results illustrate the role played by indirect externalities in shaping the pricing and quality decisions of managed care organizations and may help to understand the growing public dissatisfaction with managed care. They underscore the importance of implementing and monitoring the achievement of quality standards in the face of ongoing increases in health risk

due to population aging, and that of adjusting the rates paid by government programs like Medicare and Medicaid according to the health risk of managed care enrollees.

The rest of the study is organized as follows. Section 2 presents the general model. Section 3 presents the results for iso-elastic distributions of patients' health risks and physicians' cost of treatment. Section 4 concludes.

2. The Model

There is a continuum of individuals of mass N_p that differ according to their probability of becoming ill, $\theta \in (0,1)$. The probability of becoming ill is distributed according to the cumulative distribution function $F(\theta)$, with twice continuously differentiable probability density function $f(\theta)$ that is positive everywhere in (0,1).

The expected utility of type θ individuals from joining the MCO depends on the number of physicians enrolled in the managed care network (n), the insurance premium (P) they have to pay, and the quality of service provided by the MCO (q): [4]

$$U(\theta) = \theta\left(q^{\alpha} n^{\beta}\right) - P, \quad \alpha, \beta \in (0,1). \tag{1}$$

For simplicity, I assume patients pay no out of pocket costs in the form of coinsurance, copayments, or deductibles. The model thus abstracts from moral hazard and adverse selection issues that have been extensively analyzed elsewhere in order to focus on network externality aspects of managed care.

The parameter β determines the magnitude of the externality exerted by doctors on patients. Doctors do not factor into their decision-making the effect their participation has on the utility of policyholders. The size of the provider network matters because the MCO does not pay

[4] As it is apparent from the utility specification, the model abstracts from income effects. This assumption is not incompatible with global risk aversion and health insurance demand from individuals if the probability of getting ill is small, the insurance premium is small, and the cost of treatment is large (Bardey and Rochet [2006]).

for any treatment that its enrollees might receive outside its network. The model thus captures the case of a health maintenance organization (HMO). A larger network of physicians may lower patients' transportation costs, or increase the likelihood that patients will find physicians matching their preferred style of treatment. There is reason to believe individuals feel strongly about the choice of providers MCOs offer. Concern about limited choice of providers has even prompted some states to enact laws ensuring that "willing" physicians are not excluded from managed care networks (the so-called "Any Willing Provider" laws), and patients have access to providers of their choice ("Freedom of Choice" laws).[5]

The utility of policyholders also depends on the quality of care provided by the MCO (q). The quality variable in this model captures the degree to which the MCO assists physicians in providing health care. It can be thought of as a quality-increasing input supplied by the MCO, such as diagnostic tests or sophisticated health information processing, storage, and retrieval. Managed care organizations use various means to control costs, from pre-authorization of procedures to utilization review. While these are meant to increase efficiency and decrease the cost of health care, they may also restrict the provision of health care by physicians and thus potentially affect the quality of treatment. Quality issues have not played a prominent role in two–sided market models to date, as studies have focused mainly on the credit card industry and electronic platforms.[6] However, quality of service is a particularly critical issue in health care, and hence merits careful attention.

There is a continuum of physicians of mass $N_d > 1$ that differ in their cost of treating patients $c(\mu) = \mu C$, where $C > 1$ is the highest cost incurred by any doctor for treating a single

[5] Some restrictions apply even under Any Willing Provider laws. Providers have to have a certain quality certification and have to be willing to accept the same contractual conditions (reimbursements, pre-authorization etc.) as enrolled physicians of the same quality.

[6] An exception is the study by Pezzino and Pignatarro [2007].

patient. The fraction $\mu \in [0,1]$ of this maximum cost incurred by type μ physicians is distributed according to the cumulative distribution function $G(\mu)$, with corresponding density function $g(\mu)$ which is assumed to be continuously differentiable and positive everywhere.[7]

To focus on the quality provided by the MCO, I assume that doctors do not differ in terms of quality. To this end, I also assume that doctors act in the best interest of their patients and do not engage in "skimping" (lowering the quality of service they provide) or "dumping" (the practice of refusing to treat high cost patients).

The utility that a physician derives from treating a patient is the difference between the reimbursement paid by the MCO in the form of a fee for service (r), and the cost of treatment ($c(\mu)$):[8]

$$v(\mu) = r - c(\mu) . \tag{2}$$

The MCO does not observe patient and physician types. However, the distributions of patient and doctor types are common knowledge.

2.1. Patients' Decision Making

If they choose to do so, individuals can secure health care services from an alternative provider, called plan A. Plan A provides services of quality q_A, and access to a network of physicians of size n_A, in exchange for a premium P_A. Plan A might be viewed as a traditional insurance plan (which gives enrollees access to all the doctors in the market) or Medicare fee for service (FFS), whose premium, quality, and number of physicians the MCO takes as given. I assume premiums are sufficiently high that patients choose not to join both plans, and the MCO finds it profitable to offer a lower quality of service and number of doctors in order to attract

[7] The cost $c(\mu)$ captures both monetary and non-monetary costs of treatment. For simplicity, I assume doctors have constant marginal costs of treating patients.

[8] The results of the model would not change if the MCOs paid physicians a fixed amount per year per enrollee regardless whether individuals seek treatment or not (capitation), rather than a fee for service.

individuals with lower probability of getting ill. To avoid confusion, in what follows I will refer to the MCO as plan *B* or MCO *B*, and index its characteristics accordingly.

Patients join MCO *B* if they derive a higher level of expected utility from joining it than from joining plan *A*:

$$U_B = \theta\left(q_B^{\alpha} n_B^{\beta}\right) - P_B \geq U_A = \theta q_A^{\alpha} n_A^{\beta} - P_A . \tag{3}$$

Given the alternative offered by plan *A*, the MCO chooses to provide a lower quality of service – network size combination such that it attracts individuals with lower probability of getting sick:

$$q_A^{\alpha} n_A^{\beta} \geq q_B^{\alpha} n_B^{\beta} . \tag{4}$$

MCO *B* charges a lower insurance premium than plan *A*, otherwise no patients will join MCO *B*. Therefore, there exists $\bar{\theta} \in (0,1)$ for which individuals are indifferent between joining MCO *B* and joining plan *A*:

$$\bar{\theta} = \bar{\theta}(P_A, P_B, q_A, q_B, n_A, n_B) = \frac{P_A - P_B}{q_A^{\alpha} n_A^{\beta} - q_B^{\alpha} n_B^{\beta}} . \tag{5}$$

Patients of lower health risk ($\theta \leq \bar{\theta}$) join MCO *B*, while patients of higher health risk ($\theta > \bar{\theta}$) join plan A. Empirical evidence suggests that managed care plans have been successful in attracting a mix of patients with lower health risk than traditional health insurers (Miller and Luft [1994], Glied [2000], Deb and Trivedi [2006], and Liu and Zimmer [2006]) and their insurance premiums are often substantially lower (Altman, Cutler, and Zeckhauser [2003]).

2.2. Physicians' Decision Making

I assume physicians face no capacity constraints. Consequently, they enroll in the network of MCO *B* if the utility they derive from treating patients is nonnegative:

$$v_B(\mu) = r_B - c(\mu) \geq 0 . \tag{6}$$

Only physicians for which the (monetary and non-monetary) cost of treating patients is lower than the reimbursement per patient ($\mu \leq \frac{r_B}{C}$) will join the MCO's preferred network. The number of physicians that join MCO *B* is thus:

$$n_B = G\left(\frac{r_B}{C}\right) \times N_d . \tag{7}$$

I assume MCO *B* does not observe the cost of treatment of each physician and cannot preferentially direct patients towards certain physicians (e.g., those that have the lowest cost of treatment). Moreover, patients are equally likely to go to any physician because treatment is free once the premium has been paid, and physicians are assumed to be of the same quality. Therefore, physicians enrolling in the preferred network of MCO *B* expect to equally share cases in the MCO with the other enrolled physicians. The total utility a physician of type μ expects to get from joining MCO *B* is the product of the expected utility per patient (v_B) and the expected case load (D_B):

$$V_B(\mu) = v_B(\mu) \times D_B = (r_B - \mu C)\frac{N_p}{G(r_B)}\int_0^{\bar{\theta}} \theta f(\theta) d\theta . \tag{8}$$

Physicians' utility thus depends on the number and type of enrollees. Individuals do not take this indirect externality into account when deciding whether to enroll in the MCO.

2.3. Profit Maximization

I assume MCO *B* is a for-profit MCO, and thus maximizes its profits given by:

$$\Pi_B = N_p \int_0^{\bar{\theta}} \left[P_B - \theta\left(r_B + k_B(q_B)\right)\right] f(\theta) d\theta , \tag{9}$$

where $k_B(q_B)$ is MCO *B*'s per patient cost of providing quality q_B, which is increasing and convex in q_B.

The first order conditions characterizing MCO B's profit maximizing choice of premium, reimbursement, and quality are given by:

$$\frac{\partial \Pi_B}{\partial P_B} = N_p \left[F(\bar{\theta}) + P_B \frac{\partial F(\bar{\theta})}{\partial \bar{\theta}} \frac{d\bar{\theta}}{dP_B} - \left(r_B + k_B(q_B)\right)\bar{\theta} f(\bar{\theta}) \frac{d\bar{\theta}}{dP_B} \right] = 0 \tag{10}$$

$$\frac{\partial \Pi_B}{\partial r_B} = N_p \left[P_B \frac{\partial F(\bar{\theta})}{\partial \bar{\theta}} \frac{d\bar{\theta}}{dr_B} - \int_0^{\bar{\theta}} \theta f(\theta) d\theta - \left(r_B + k_B(q_B)\right)\bar{\theta} f(\bar{\theta}) \frac{d\bar{\theta}}{dr_B} \right] = 0 \tag{11}$$

$$\frac{\partial \Pi_B}{\partial q_B} = N_p \left[P_B \frac{\partial F(\bar{\theta})}{\partial \bar{\theta}} \frac{d\bar{\theta}}{dq_B} - k_B^{'}(q_B) \int_0^{\bar{\theta}} \theta f(\theta) d\theta - \left(r_B + k_B(q_B)\right)\bar{\theta} f(\bar{\theta}) \frac{d\bar{\theta}}{dq_B} \right] = 0 \tag{12}$$

where

$$\frac{d\bar{\theta}}{dP_B} = \frac{\partial \bar{\theta}}{\partial P_B} = \frac{-1}{q_A^{\alpha} n_A^{\beta} - q_B^{\alpha} n_B^{\beta}} = -\frac{\bar{\theta}}{P_A - P_B} \tag{13}$$

$$\frac{d\bar{\theta}}{dr_B} = \frac{\partial \bar{\theta}}{\partial n_B} \frac{\partial n_B}{\partial r_B} = \frac{\left(P_A - P_B\right)\beta q_B^{\alpha} n_B^{\beta-1}}{\left(q_A^{\alpha} n_A^{\beta} - q_B^{\alpha} n_B^{\beta}\right)^2} \frac{\partial n_B}{\partial r_B} = \frac{\bar{\theta}^2}{P_A - P_B} \beta q_B^{\alpha} n_B^{\beta-1} \frac{\partial n_B}{\partial r_B} \tag{14}$$

$$\frac{d\bar{\theta}}{dq_B} = \frac{\partial \bar{\theta}}{\partial q_B} = \frac{\left(P_A - P_B\right)\alpha q_B^{\alpha-1} n_B^{\beta}}{\left(q_A^{\alpha} n_A^{\beta} - q_B^{\alpha} n_B^{\beta}\right)^2} = \frac{\bar{\theta}^2}{P_A - P_B} \alpha q_B^{\alpha-1} n_B^{\beta} \tag{15}$$

are the total derivatives of $\bar{\theta}$ with respect to P_B, r_B , and q_B. Sufficient conditions for profit maximization are provided in the appendix.

Equations 10 - 12 reflect the familiar microeconomic paradigm that the profit maximizing firm chooses the insurance premium, physician reimbursement, and quality such that the marginal benefit of each instrument equals its marginal cost. For example, increasing quality allows MCO B to attract more individuals $N_p \frac{\partial F(\bar{\theta})}{\partial \bar{\theta}} \frac{d\bar{\theta}}{dq_B}$ from which it collects premium P_B , a

fraction $\bar{\theta}$ of which will need treatment which costs $r_B + k_B(q_B)$, in addition to the increased costs of providing quality to all policyholders $k_B^{'}(q_B)N_p\int_0^{\bar{\theta}}\theta f(\theta)d\theta$ (Equation 12).

While these results are intuitive, they differ from the familiar microeconomic theory of one-sided markets through the presence of indirect externality effects. The profit maximizing premiums, reimbursements, and quality depend on the indirect externality physicians exert on policyholders, and the externality exerted by policyholders on doctors. When deciding whether to join MCO *B*, individuals do not take into account the effect of their decisions on the utility of doctors in the MCO's network. Similarly, physicians do not take into account the added benefit policyholders would enjoy from being able to choose among a larger network of providers.

The first order conditions can be rewritten in the familiar Ramsey form

$$\frac{P_B - \bar{\theta}\left(r_B + k_B(q_B)\right)}{P_B} = -\frac{1}{\eta_B}, \tag{16}$$

where

$$\eta_B = \frac{\partial F(\bar{\theta})}{\partial \bar{\theta}}\frac{\partial \bar{\theta}}{\partial P_B}\frac{P_B}{F(\bar{\theta})} \tag{17}$$

is the elasticity of the numbers of enrollees with respect to the insurance premium. The more elastic is the number of enrollees with respect to changes in the insurance premium the lower will be the markup charged by MCO *B*.

Physician reimbursement has taken the place that usually belongs to the marginal cost of treatment in one-sided market models. The utility of policyholders depends on the reimbursement paid to doctors by MCO *B* through its effect on the number of doctors in the preferred network

$$\frac{\partial n_B}{\partial r_B} = g\left(\frac{r_B}{C}\right) \times \frac{N_d}{C} \quad . \tag{18}$$

Similarly, the utility of doctors depends on the insurance premium provided by MCO *B* through its effect on the number and type of policyholders $N_p \frac{\partial F(\bar{\theta})}{\partial \bar{\theta}} \frac{d\bar{\theta}}{dP_B}$.

Two-sided market models so far have focused on the optimal pricing decisions of intermediaries. The present model also analyzes the optimal choice of quality of the intermediary MCO. The utility of physicians from joining MCO *B* depends indirectly on the quality of service provided by MCO *B* to patients through its effect on the number and type of policyholders $\frac{\partial F(\bar{\theta})}{\partial \bar{\theta}} \frac{d\bar{\theta}}{dq_B}$ (Equation 12).

Combining the first order conditions for profit maximization yields

$$\frac{\left(-\frac{d\bar{\theta}}{dP_B}\right)}{F(\bar{\theta})} = \frac{\frac{d\bar{\theta}}{dr_B}}{\int_0^{\bar{\theta}} \theta f(\theta) d\theta} = \frac{\frac{d\bar{\theta}}{dq_B}}{k_B^{'}(q_B) \int_0^{\bar{\theta}} \theta f(\theta) d\theta} . \tag{19}$$

This in turn can be rewritten to analyze the tradeoffs faced by MCO *B* when choosing the quality of service and physician reimbursement:

$$-k_B^{'}(q_B) = -\frac{\frac{\partial \bar{\theta}}{\partial q_B}}{\frac{\partial \bar{\theta}}{\partial n_B} \frac{\partial n_B}{\partial r_B}} = \left.\frac{\partial r_B}{\partial q_B}\right|_{\substack{\bar{\theta}=cst \\ P_B=cst}} . \tag{20}$$

Increasing the quality of service provided to patients allows the MCO to reduce the reimbursement per patient it pays doctors, charge the same insurance premium and attract the same number of patients. Similarly, increasing the reimbursement attracts more doctors which

provide policyholders additional utility, allowing the MCO to reduce the quality of service it offers and maintain the same number of patients and insurance premium.

Using Equations 14 and 15, Equation 20 can be rewritten as:

$$-k_B^{'}(q_B) = -\frac{\alpha r_B}{\beta q_B \frac{\partial n_B}{\partial r_B}\frac{r_B}{n_B}} = \left.\frac{\partial r_B}{\partial q_B}\right|_{\substack{\bar{\theta}=cst \\ P_B=cst}} . \tag{21}$$

The higher the marginal cost of providing quality $k_B^{'}(q_B)$ and the lower the elasticity of the number of doctors with respect to reimbursements $\sigma = \frac{\partial n_B}{\partial r_B}\frac{r_B}{n_B}$, the more the MCO can reduce payments to its doctors when it increases quality and maintain the same premium and number of enrollees. Also, the more enrollees care about quality (higher α) and the less they care about the number of doctors (lower β), the more the MCO can reduce reimbursements when increasing quality and keep the same premium and number of enrollees, particularly at low levels of quality relative to reimbursement.

3. Iso-Elastic Distributions of Health Risk and Cost

The model can be solved fully for the case in which the distributions of patient and physician types are iso-elastic and quality is produced with constant marginal cost, i.e.,

$$F(\theta) = \theta^{\varepsilon}\ ,\ G(\mu) = \mu^{\sigma}\ ,\ k_B^{'}(q_B) = \gamma_B . \tag{26}$$

These power distributions are a convenient way to summarize the concentration of high-risk patients and high-cost physicians in the market (Bardey and Rochet [2006]). Distributions with higher ε dominate those with lower ε in the sense of first order stochastic dominance. The higher is ε (the elasticity of the distribution of patient types) the higher is the expected number of illness cases. Similarly, the higher is σ, the elasticity of the distribution of physician types, the higher is the expected number of doctors with high cost of treating patients.

Solving the model yields the following solution (see Appendix for derivations):

$$r_B = \left[\frac{\varepsilon}{1+\varepsilon} \frac{\left(\frac{\alpha}{\beta\sigma\gamma_B}\right)^{-\alpha}}{\beta\sigma} \left(\frac{C^\sigma}{N_d}\right)^\beta \right]^{\frac{1}{\alpha+\beta\sigma-1}} \tag{27}$$

$$n_B = \left[\frac{\varepsilon}{1+\varepsilon} \frac{\left(\frac{\alpha}{\beta\sigma\gamma_B}\right)^{-\alpha}}{\beta\sigma} \right]^{\frac{\sigma}{\alpha+\beta\sigma-1}} \left(\frac{N_d}{C^\sigma}\right)^{1+\frac{\beta}{1-\alpha-\beta\sigma}} \tag{28}$$

$$q_B = \frac{\alpha}{\beta\sigma\gamma_B} \left[\frac{\varepsilon}{1+\varepsilon} \frac{\left(\frac{\alpha}{\beta\sigma\gamma_B}\right)^{-\alpha}}{\beta\sigma} \left(\frac{C^\sigma}{N_d}\right)^\beta \right]^{\frac{1}{\alpha+\beta\sigma-1}} \tag{29}$$

$$P_B = \frac{P_A}{1+\varepsilon} \frac{\beta(1+\varepsilon)\sigma q_A^\alpha n_A^\beta + \varepsilon[(\alpha+\beta\sigma)(1+\varepsilon)-1] \left[\frac{\varepsilon}{1+\varepsilon} \frac{\left(\frac{\alpha}{\beta\sigma\gamma_B}\right)^{-\alpha}}{\beta\sigma} \left(\frac{C^\sigma}{N_d}\right)^\beta \right]^{\frac{1}{\alpha+\beta\sigma-1}}}{\beta(1+\varepsilon)\sigma q_A^\alpha n_A^\beta + \varepsilon(\alpha+\beta\sigma-1) \left[\frac{\varepsilon}{1+\varepsilon} \frac{\left(\frac{\alpha}{\beta\sigma\gamma_B}\right)^{-\alpha}}{\beta\sigma} \left(\frac{C^\sigma}{N_d}\right)^\beta \right]^{\frac{1}{\alpha+\beta\sigma-1}}} \tag{30}$$

$$\bar{\theta} = \frac{\beta\varepsilon\sigma P_A}{\beta(1+\varepsilon)\sigma q_A^\alpha n_A^\beta + \varepsilon(\alpha+\beta\sigma-1) \left[\frac{\varepsilon}{1+\varepsilon} \frac{\left(\frac{\alpha}{\beta\sigma\gamma_B}\right)^{-\alpha}}{\beta\sigma} \left(\frac{C^\sigma}{N_d}\right)^\beta \right]^{\frac{1}{\alpha+\beta\sigma-1}}} \tag{31}$$

The following propositions describe the impact of changes in parameters of interest on MCO *B*'s reimbursement, quality, number of doctors, marginal health risk, and insurance premium. They are summarized in Table 1. The proofs of these propositions are sketched in the appendix. The results were derived using Mathematica 5. A separate file detailing the derivations is available from the author upon request.

Proposition 1. When individuals care more about quality (higher α) or access to doctors (higher β) the MCO provides more of the quality-increasing input, pays higher reimbursements, and attracts more doctors. The marginal health risk insured by the MCO decreases as more individuals choose the alternative plan *A*.

As individuals become more concerned about the quality of the health care they receive, the MCO provides more of the quality increasing input. This increases the marginal utility of doctors, so MCO *B* pays higher reimbursements and attracts more doctors. Similarly, the indirect externality exerted by doctors on patients does not affect solely the number of doctors in MCO *B*'s network. When policyholders care more about the number of doctors, the MCO finds it profitable to provide doctors with more of the inputs they need to better treat patients. However, if individuals care more about quality or access to doctors, more of them will choose the alternative plan *A* that provides higher quality of service and access to more physicians.

Proposition 2. The MCO reduces the risk of its mix of patients when the elasticity of the supply of physicians with respect to reimbursements (σ) increases.

Just as firms charge lower prices when demand for their products is more elastic, the MCO pays higher reimbursements to attract doctors when the supply of doctors is more elastic. In the model, σ captures both the elasticity of the supply of physicians and the concentration of doctors with high treatment costs. More doctors have high treatment costs when σ is greater. Because it is more costly to attract physicians the MCO insures fewer individuals and reduces the health risk of its patient mix.

The effect of an increase in σ on reimbursement, the number of doctors, and the quality provided by the MCO is ambiguous. Decreasing reimbursements or keeping them constant may be profit maximizing if the number of doctors and thus the willingness to pay of policyholders doesn't decrease too much. If a decrease in reimbursement would lead to a large decrease in the number of doctors and in the policyholders' utility and willingness to pay, then the MCO may find it profitable to increase reimbursement, even if the net effect is still a reduction in the marginal health risk insured by the MCO.

Proposition 3. MCO *B* provides less of the quality-increasing input, pays lower reimbursements, attracts fewer doctors, charges a lower insurance premium, and insures individuals with lower health risk when its marginal cost of providing quality (γ_B) increases.

The MCO decreases reimbursements and attracts fewer doctors because doctors are less efficient when they receive less of the quality-increasing input, and the marginal utility derived by policyholders from access to doctors decreases. The demand for managed care shifts downwards, as individuals' willingness to pay for managed care decreases. The insurance premium decreases as a result, but the decrease does not fully compensate for the decrease in policyholder's utility due to lower quality and fewer physicians. Thus, the MCO reduces the risk of its patients' mix and insures fewer people in order to avoid providing the more costly quality.

Of particular importance is the way in which the distribution of health risks in the population affects the insurance premium, reimbursement, quality provided, as well as the number and mix of patients served by the MCO. More people are at risk of becoming ill as a result of the aging of the US population. In the model this corresponds to a shift in the distribution of health risks towards higher health risks, captured by a higher value of ε. The distribution of health risks for higher ε statistically dominates those with lower ε in the sense of first order stochastic dominance. Comparative statics with respect to ε can illustrate the effects of

population ageing, or allow for comparisons between markets with different distributions of health risks.

Proposition 4. When the health risk of the population ε increases the MCO provides lower quality, pays lower reimbursements, attracts fewer physicians, charges a lower insurance premium, and finds it profitable to attract some individuals with higher health risk.

When it faces a higher number of illness cases, the MCO tries to control costs by reducing quality, paying lower reimbursements, and providing access to fewer doctors. These predictions of the model are broadly consistent with the growing public dissatisfaction with the quality of health care provided by managed care organizations, and their attempts to control costs through a variety of mechanisms such as mandatory referrals and pre-authorization procedures. Because the policyholders' willingness to pay decreases when quality and access to physicians are reduced, the MCO charges a lower insurance premium. Because there are fewer low-risk and more high-risk individuals when ε increases, the MCO insures some individuals with higher health risk. Proposition 5 describes the factors that determine the magnitude of the changes in quality, reimbursement, and number of doctors.

Proposition 5. The percentage decreases in MCO quality, reimbursement, and number of doctors due to increases in population health risk or in the marginal cost of quality provision are greater when people care more about quality (higher α), when the externality exerted by doctors on patients is greater (higher β), and when the concentration of doctors with high treatment costs is higher (higher σ).[9]

Proposition 1 showed that the MCO provides higher quality and access to more physicians in markets in which individuals care more about quality and access to physicians. Proposition 5 shows that these are also the markets in which the decreases in quality and access to physicians

[9] The externality exerted by the enrollment of an additional doctor in MCO B's network, the marginal utility of policyholders with respect to the number of doctors, increases when β increases if the number of doctors in the network is greater than one (Assumption 63 in the Appendix). Similarly, Assumption 64 in the Appendix ensures that the marginal utility of quality increases when α increases.

are more pronounced, as the MCO tries to contain costs when the population health risk increases for reasons such as population aging.

Proposition 6. The MCO pays higher reimbursements, attracts more doctors, provides more of the quality increasing input, insures more policyholders, and charges a higher insurance premium when the number of doctors in the market N_d increases.

The MCO pays higher reimbursements when the number of doctors in the market increases because the same increase in reimbursements attracts more doctors in its network. This in turn increases the utility of policyholders, more of whom enroll in managed care. The insurance premium increases as a result of the increase in the policyholders' willingness to pay. This is a somewhat counterintuitive result because the increased supply of an input results in a higher price for the "good" produced. The key difference from the usual setting is that the number of doctors also affects the marginal utility and thus the willingness to pay of policyholders.

Proposition 7. The MCO pays lower reimbursements, attracts fewer doctors, provides less of the quality increasing input, insures fewer policyholders, and charges a lower insurance premium when the maximum cost of treatment (C) increases.

While the decrease in the insurance premium charged by the MCO is somewhat surprising, it is due to the absence of income effects in the model, and to the fact that the MCO takes the alternative plan's quality, network of doctors, and insurance premium as given. This limits the MCO's ability to increase its insurance premium. The MCO then resorts to quality reductions and reduced access to physicians to curtail costs. This framework is appropriate for analyzing the behavior of managed care organizations given the choices made by the Center for Medicare and Medicaid Service regarding the traditional Medicare fee for service plan. Future research should model the strategic interaction between health insurers to study also the alternative plan's choice of quality, number of doctors, and insurance premium.

4. Conclusion

This paper has developed a two-sided market model of managed care to analyze the behavior of a profit maximizing MCO in the presence of indirect network externalities between doctors and patients. Previous two-sided market studies have focused on the pricing decisions of intermediaries. Given the crucial role played by quality in determining the health risk of policyholders, I analyze both the MCO's pricing and its quality decisions.

The MCO's choice of quality, premiums, and reimbursements depends on the distributions of patient health risks and physician costs. It also depends on the marginal cost of providing quality, the utility derived by patients from the quality of health care services, the magnitude of the indirect externality exerted by doctors on patients, and the elasticity of the supply of physicians with respect to reimbursements.

In the case of iso-elastic distributions of patient health risk and physician cost of treatment, an increase in the cost of providing quality decreases the quality provided by the MCO, which reduces the effectiveness of physicians, the marginal utility derived by policyholders, and thus the reimbursement and the number of doctors in the preferred network. The demand for managed care services shifts downward, and the insurance premium also decreases. However, the decrease in premium is smaller than the decrease in patient utility due to lower quality and number of physicians in the network. Thus, the MCO reduces the health risk of its patient mix, and avoids providing, at least in part, the more costly quality.

For populations more at risk of becoming ill, as is the case of the ageing population of the United States, the MCO provides services of lower quality, pays lower physician reimbursements, includes fewer providers in its preferred network, yet finds it profitable to insure some higher risk individuals.

Although MCOs provide services of higher quality and access to more doctors in markets where individuals care more about quality and access to doctors, these markets are also the ones in which the MCO reduces quality, reimbursements, and access to physicians more aggressively to curtail costs when the cost of quality provision or population health risks increase.

These results may help to understand the increasing public dissatisfaction with managed care. They underscore the importance of implementing and monitoring the achievement of quality standards, as well as adjusting the capitation rates paid by government programs like Medicare and Medicaid to account for health risk.

While most of the research on two-sided markets has so far been theoretical, there is a growing empirical two-sided market literature (Rysman [2004], Rysman [2007], Chandra & Collard-Wexler [2008]). Yet to my knowledge there is no empirical study analyzing the two-sided market aspects of managed care. Given the potential important implications of indirect externalities, empirical studies would be useful to gauge the relative magnitude and the role played by indirect externalities in health care markets.

APPENDIX

Iso-elastic Distributions of Health Risk and Treatment Cost

Derivation of the Profit Maximizing Solution

To better understand the results, the model is solved assuming iso-elastic distributions of patient and physician types $F(\overline{\theta}) = \overline{\theta}^{\varepsilon}$ and $G(c) = c^{\sigma}$, and constant marginal costs of providing quality $k_B^{'}(q_B) = \gamma_B$.

It is easiest to express the insurance premium charged by the MCO in terms of quality, reimbursement, and the cutoff probability $\overline{\theta}$ from Equation 5 and solve the model with respect to quality, reimbursement, and the cutoff probability of illness. We have

$$P_B = P_A - \bar{\theta}(q_A^{\alpha} n_A^{\beta} - q_B^{\alpha} n_B^{\beta}) \tag{32}$$

and the MCO B's profit is

$$\Pi_B = N_p \int_0^{\bar{\theta}} \left[P_A - \bar{\theta}(q_A^{\alpha} n_A^{\beta} - q_B^{\alpha} n_B^{\beta}) - \theta\left(r_B + k_B(q_B)\right) \right] f(\theta) d\theta . \tag{33}$$

Replacing $f(\theta) = \varepsilon\theta^{\varepsilon-1}$ in Equation 33 and integrating yields:

$$\Pi_B = N_p \left[P_A \bar{\theta}^{\varepsilon} - (q_A^{\alpha} n_A^{\beta} - q_B^{\alpha} n_B^{\beta}) \bar{\theta}^{\varepsilon+1} - \left(r_B + k_B(q_B)\right) \frac{\varepsilon}{\varepsilon+1} \bar{\theta}^{\varepsilon+1} \right]. \tag{34}$$

The first order conditions with respect to $\bar{\theta}$, q_B, r_B are:

$$\frac{\partial \Pi_B}{\partial \bar{\theta}} = N_p \left[P_A \varepsilon \bar{\theta}^{\varepsilon-1} - (q_A^{\alpha} n_A^{\beta} - q_B^{\alpha} n_B^{\beta})(\varepsilon+1)\bar{\theta}^{\varepsilon} - \left(r_B + k_B(q_B)\right)\varepsilon\bar{\theta}^{\varepsilon} \right] = 0 . \tag{35}$$

$$\frac{\partial \Pi_B}{\partial r_B} = N_p \left[\frac{\beta\sigma}{r_B} q_B^{\alpha} n_B^{\beta} \bar{\theta}^{\varepsilon+1} - \frac{\varepsilon}{\varepsilon+1} \bar{\theta}^{\varepsilon+1} \right] = 0 . \tag{36}$$

$$\frac{\partial \Pi_B}{\partial q_B} = N_p \left[\alpha q_B^{\alpha-1} n_B^{\beta} \bar{\theta}^{\varepsilon+1} - k'(q_B) \frac{\varepsilon}{\varepsilon+1} \bar{\theta}^{\varepsilon+1} \right] = 0 . \tag{37}$$

Simplifying, we get:

$$P_A \varepsilon - (q_A^{\alpha} n_A^{\beta} - q_B^{\alpha} n_B^{\beta})\bar{\theta}(\varepsilon+1) - \left(r_B + k_B(q_B)\right)\varepsilon\bar{\theta} = 0 . \tag{38}$$

$$\frac{\beta\sigma}{r_B} q_B^{\alpha} n_B^{\beta} - \frac{\varepsilon}{\varepsilon+1} = 0 . \tag{39}$$

$$\alpha q_B^{\alpha-1} n_B^{\beta} - k'(q_B) \frac{\varepsilon}{\varepsilon+1} = 0 . \tag{40}$$

Combining Equations 39 and 40 we get:

$$\frac{\alpha r_B}{\beta\sigma q_B} = k_B^{'}(q_B) . \tag{41}$$

Taking into account that $k_B^{'}(q_B) = \gamma_B$, we can express quality in terms of reimbursement:

$$q_B = \frac{\alpha}{\beta\sigma\gamma_B} r_B . \tag{42}$$

Replacing Equations 42 and 7 into Equation 39 and solving for reimbursement yields:

$$r_B = \left[\frac{\varepsilon}{1+\varepsilon} \frac{\left(\frac{\alpha}{\beta\sigma\gamma_B}\right)^{-\alpha}}{\beta\sigma} \left(\frac{C^\sigma}{N_d}\right)^\beta \right]^{\frac{1}{\alpha+\beta\sigma-1}} . \tag{43}$$

Because $n_B = \left(\frac{r_B}{C}\right)^\sigma N_d$ we can compute the number of doctors in the preferred network:

$$n_B = \left\{ \frac{1}{C} \left[\frac{\varepsilon}{1+\varepsilon} \frac{\left(\frac{\alpha}{\beta\sigma\gamma_B}\right)^{-\alpha}}{\beta\sigma} \left(\frac{C^\sigma}{N_d}\right)^\beta \right]^{\frac{1}{\alpha+\beta\sigma-1}} \right\}^\sigma N_d . \tag{44}$$

Replacing the expression for the reimbursement from Equation 43 into Equation 42 yields the following expression for the quality provided by MCO *B*:

$$q_B = \frac{\alpha}{\beta\sigma\gamma_B} \left[\frac{\varepsilon}{1+\varepsilon} \frac{\left(\frac{\alpha}{\beta\sigma\gamma_B}\right)^{-\alpha}}{\beta\sigma} \left(\frac{C^\sigma}{N_d}\right)^\beta \right]^{\frac{1}{\alpha+\beta\sigma-1}} . \tag{45}$$

Substituting Equations 43, 44, and 45 into Equation 38 and solving for the marginal health risk of MCO *B*'s patients $\bar{\theta}$ yields

$$\bar{\theta} = \frac{\beta\varepsilon\sigma P_A}{\beta(1+\varepsilon)\sigma q_A^{\alpha} n_A^{\beta} + \varepsilon(\alpha+\beta\sigma-1)\left[\frac{\varepsilon}{1+\varepsilon}\frac{\left(\frac{\alpha}{\beta\sigma\gamma_B}\right)^{-\alpha}}{\beta\sigma}\left(\frac{C^{\sigma}}{N_d}\right)^{\beta}\right]^{\frac{1}{\alpha+\beta\sigma-1}}} \tag{46}$$

Finally, substituting in Equation 32 yields the solution for the insurance premium

$$P_B = \frac{P_A}{1+\varepsilon}\frac{\beta(1+\varepsilon)\sigma q_A^{\alpha} n_A^{\beta} + \varepsilon[(\alpha+\beta\sigma)(1+\varepsilon)-1]\left[\frac{\varepsilon}{1+\varepsilon}\frac{\left(\frac{\alpha}{\beta\sigma\gamma_B}\right)^{-\alpha}}{\beta\sigma}\left(\frac{C^{\sigma}}{N_d}\right)^{\beta}\right]^{\frac{1}{\alpha+\beta\sigma-1}}}{\beta(1+\varepsilon)\sigma q_A^{\alpha} n_A^{\beta} + \varepsilon(\alpha+\beta\sigma-1)\left[\frac{\varepsilon}{1+\varepsilon}\frac{\left(\frac{\alpha}{\beta\sigma\gamma_B}\right)^{-\alpha}}{\beta\sigma}\left(\frac{C^{\sigma}}{N_d}\right)^{\beta}\right]^{\frac{1}{\alpha+\beta\sigma-1}}} \tag{47}$$

Sufficient Conditions for Profit Maximization

Note that the denominator in Equations 46 and 47 can be written as

$$D = \beta(1+\varepsilon)\sigma q_A^{\alpha} n_A^{\beta} - \varepsilon(1-\alpha-\beta\sigma) r_B = \beta\sigma(1+\varepsilon)\left[q_A^{\alpha} n_A^{\beta} - (1-\alpha-\beta\sigma) q_B^{\alpha} n_B^{\beta}\right] > 0\,, \tag{48}$$

where the second equality comes from Equation 39. The denominator is positive since MCO B offers lower quality and number of doctors than the alternative plan A, and so are $\bar{\theta}$ and P_B as a result. We also need

$$P_A < \frac{1+\varepsilon}{\varepsilon} q_A^{\alpha} n_A^{\beta} + \frac{\alpha+\beta\sigma-1}{\beta\sigma}\left[\frac{\varepsilon}{1+\varepsilon}\frac{\left(\frac{\alpha}{\beta\sigma\gamma_B}\right)^{-\alpha}}{\beta\sigma}\right]^{\frac{1}{\alpha+\beta\sigma-1}} \tag{49}$$

to guarantee that $\bar{\theta} < 1$.

The second order derivatives of MCO B's profit function with respect to $\overline{\theta}$, q_B, and r_B are:

$$\frac{\partial^2 \Pi_B}{\partial \overline{\theta}^2} = N_p \left[P_A(\varepsilon - 1) - (q_A^{\alpha} n_A^{\beta} - q_B^{\alpha} r_B^{\beta\sigma})(\varepsilon + 1)\overline{\theta} - \left(r_B + k_B(q_B)\right)\varepsilon\overline{\theta} \right] \varepsilon \overline{\theta}^{\varepsilon - 2}. \tag{50}$$

$$\frac{\partial^2 \Pi_B}{\partial r_B^2} = N_p \frac{\beta\sigma(\beta\sigma - 1)}{r_B^2} q_B^{\alpha} n_B^{\beta} \overline{\theta}^{\varepsilon + 1}. \tag{51}$$

$$\frac{\partial \Pi_B}{\partial q_B^2} = N_p \left[\alpha(\alpha - 1) q_B^{\alpha - 2} n_B^{\beta} \overline{\theta}^{\varepsilon + 1} - k''(q_B) \frac{\varepsilon}{\varepsilon + 1} \overline{\theta}^{\varepsilon + 1} \right]. \tag{52}$$

$$\frac{\partial^2 \Pi_B}{\partial r_B \partial \overline{\theta}} = N_p \left(\frac{\beta\sigma}{r_B} q_B^{\alpha} n_B^{\beta} - \frac{\varepsilon}{\varepsilon + 1} \right) (\varepsilon + 1) \overline{\theta}^{\varepsilon} = 0. \tag{53}$$

$$\frac{\partial^2 \Pi_B}{\partial q_B \partial \overline{\theta}} = N_p \left(\alpha q_B^{\alpha - 1} n_B^{\beta} - k'(q_B) \frac{\varepsilon}{\varepsilon + 1} \right) (\varepsilon + 1) \overline{\theta}^{\varepsilon} = 0. \tag{54}$$

$$\frac{\partial^2 \Pi_B}{\partial r_B \partial q_B} = N_p \alpha \frac{\beta\sigma}{r_B} q_B^{\alpha - 1} n_B^{\beta} \overline{\theta}^{\varepsilon + 1}. \tag{55}$$

Taking Equation 38 into account, Equation 50 can be written:

$$\frac{\partial^2 \Pi_B}{\partial \overline{\theta}^2} = -N_p P_A \varepsilon \overline{\theta}^{\varepsilon - 2} < 0. \tag{56}$$

The Hessian matrix of the MCO's profit function is:

$$H = \begin{pmatrix} -N_p P_A \varepsilon \overline{\theta}^{\varepsilon - 2} & 0 & 0 \\ 0 & N_p \frac{\beta\sigma}{r_B^2} (\beta\sigma - 1) q_B^{\alpha} n_B^{\beta} \overline{\theta}^{\varepsilon + 1} & N_p \alpha \frac{\beta\sigma}{r_B} q_B^{\alpha - 1} n_B^{\beta} \overline{\theta}^{\varepsilon + 1} \\ 0 & N_p \alpha \frac{\beta\sigma}{r_B} q_B^{\alpha - 1} n_B^{\beta} \overline{\theta}^{\varepsilon + 1} & N_p \left(\alpha(\alpha - 1) q_B^{\alpha - 2} n_B^{\beta} - k''(q_B) \frac{\varepsilon}{\varepsilon + 1} \right) \overline{\theta}^{\varepsilon + 1} \end{pmatrix}. \tag{57}$$

The diagonal elements of the Hessian matrix are negative if

$$k''(q_B) \geq 0,\ 0 < \beta < 1,\ 0 < \sigma < 1, \text{ and } 0 < \alpha < 1, \tag{58}$$

that is, if the cost of providing quality is convex, and the utility of enrollees exhibits diminishing returns to quality and reimbursement. Furthermore, all the 2×2 determinant minors are positive

under these assumptions if the utility of policyholders also exhibits decreasing returns to scale, that is if

$$1-\alpha-\beta\sigma>0. \tag{59}$$

The fact that the 2×2 determinant minors are positive is obvious with the exception of the one corresponding to the second derivatives with respect to quality and reimbursement, which can be written as:

$$Det(H_2)=\begin{vmatrix} N_p\dfrac{\beta\sigma}{r_B^2}(\beta\sigma-1)q_B^{\alpha}n_B^{\beta}\overline{\theta}^{\varepsilon+1} & N_p\alpha\dfrac{\beta\sigma}{r_B}q_B^{\alpha-1}n_B^{\beta}\overline{\theta}^{\varepsilon+1} \\ N_p\alpha\dfrac{\beta\sigma}{r_B}q_B^{\alpha-1}n_B^{\beta}\overline{\theta}^{\varepsilon+1} & N_p\left(\alpha(\alpha-1)q_B^{\alpha-2}n_B^{\beta}-k''(q_B)\dfrac{\varepsilon}{\varepsilon+1}\right)\overline{\theta}^{\varepsilon+1} \end{vmatrix}= \tag{60}$$

$$N_p^2\overline{\theta}^{2\varepsilon+2}\left[\left(\alpha\frac{\beta\sigma}{r_B^2}q_B^{2\alpha-2}n_B^{2\beta}\right)[1-\alpha-\beta\sigma]-k''(q_B)\frac{\varepsilon}{\varepsilon+1}\frac{\beta\sigma}{r_B^2}(\beta\sigma-1)q_B^{\alpha}n_B^{\beta}\right]>0$$

Under the conditions in Inequalities 58 and 59 the Hessian matrix in Equation 57 is negative definite. These conditions together with the first order conditions in Equations 35, 36, and 37 are sufficient conditions for profit maximization.

The following assumptions on the parameters are made in addition to the sufficient conditions for maximization:

$$C>1,\ N_d>1 \tag{61}$$

$$C^{1-\alpha}>\frac{1+\varepsilon}{\varepsilon}\left(\frac{\alpha}{\gamma_B}\right)^{\alpha}(\beta\sigma)^{1-\alpha}N_d^{\beta} \tag{62}$$

$$C^{1-\alpha}<\frac{1+\varepsilon}{\varepsilon}\left(\frac{\alpha}{\gamma_B}\right)^{\alpha}(\beta\sigma)^{1-\alpha}N_d^{\frac{1-\alpha}{\sigma}} \tag{63}$$

$$C^{\beta\sigma}<\frac{1+\varepsilon}{\varepsilon}(\beta\sigma)^{\beta\sigma}\left(\frac{\alpha}{\gamma_B}\right)^{1-\beta\sigma}N_d^{\beta}. \tag{64}$$

Assumption 62 is needed to guarantee that $0 < G\left(\frac{r_B}{C}\right) = \left(\frac{r_B}{C}\right)^{\sigma} < 1$. Assumption 63 ensures that $n_B > 1$ and the utility of policyholders is higher for higher values of β. This allows us to interpret an increase in β as an increase in the externality exerted by doctors on patients. Similarly, Assumption 64 guarantees that $q_B > 1$ and policyholders derive a higher utility from quality when α increases. Inequalities 62, 63, and 64 can be simultaneously satisfied if $1 - \alpha - \beta\sigma > 0$ and the marginal cost of the quality increasing input is not too large

$$\gamma_B < \frac{1+\varepsilon}{\varepsilon} \alpha N_d^{\beta} . \tag{65}$$

Under all these assumptions, the impact of changes in parameters of interest on the MCO's reimbursement, quality, number of doctors, marginal health risk, and insurance premium is summarized in Table 1. The following section sketches the proofs of the main propositions. The results were derived using Mathematica 5. For brevity, only the main results of the derivations are presented. A separate file detailing the derivations is available from the author upon request.

Proof of Proposition 1

Differentiating Equations 43 through 47 with respect to α it can be shown that

$$\frac{\partial r_B}{\partial \alpha} \frac{1}{r_B} = \frac{1 + \ln q_B}{1 - \alpha - \beta\sigma} > 0 . \tag{66}$$

$$\frac{\partial n_B}{\partial \alpha} \frac{1}{n_B} = \sigma \frac{\partial r_B}{\partial \alpha} \frac{1}{r_B} = \sigma \frac{1 + \ln q_B}{1 - \alpha - \beta\sigma} > 0 . \tag{67}$$

$$\frac{\partial q_B}{\partial \alpha} \frac{1}{q_B} = \frac{1 - \beta\sigma + \alpha \ln q_B}{1 - \alpha - \beta\sigma} > 0 . \tag{68}$$

$$\frac{\partial \bar{\theta}}{\partial \alpha} \frac{1}{\bar{\theta}} = -\frac{q_A^{\alpha} n_A^{\beta} \ln q_A - q_B^{\alpha} n_B^{\beta} \ln q_B}{q_A^{\alpha} n_A^{\beta} - (1 - \alpha - \beta\sigma) q_B^{\alpha} n_B^{\beta}} < 0 . \tag{69}$$

$$\frac{\partial P_B}{\partial \alpha} = \frac{P_A \varepsilon^2 r_B \beta\sigma\left\{\left[1+(\alpha+\beta\sigma)\left(\ln q_B - (1-\alpha-\beta\sigma)\ln q_A\right)\right] q_A^{\alpha} n_A^{\beta} - (1-\alpha-\beta\sigma) q_B^{\alpha} n_B^{\beta}\right\}}{(1-\alpha-\beta\sigma)\left(\beta\sigma(1+\varepsilon) q_A^{\alpha} n_A^{\beta} - (1-\alpha-\beta\sigma)\varepsilon r_B\right)^2}. \tag{70}$$

One would expect the MCO's insurance premium to increase when policyholders care more about quality (higher α) because the MCO's quality and number of doctors increase and the marginal health risk decreases. The change in MCO B's insurance premium is ambiguous because the utility of the alternative plan also increases when patients care more about quality. A small enough decrease in reimbursement coupled with small increases in quality and number of doctors can lead to a decrease in the marginal health risk when the alternative plan's quality and number of doctors are large enough.

Differentiating Equations 43 through 48 with respect to β to study the effects of an increase in the externality exerted by doctors on patients yields

$$\frac{\partial r_B}{\partial \beta}\frac{1}{r_B} = \frac{1-\alpha+\beta\ln n_B}{\beta(1-\alpha-\beta\sigma)} > 0 \tag{71}$$

$$\frac{\partial q_B}{\partial \beta}\frac{1}{q_B} = \frac{\sigma+\ln n_B}{1-\alpha-\beta\sigma} > 0 \tag{72}$$

$$\frac{\partial n_B}{\partial \beta}\frac{1}{n_B} = \sigma\frac{\partial r_B}{\partial \beta}\frac{1}{r_B} = \sigma\frac{1-\alpha+\beta\ln n_B}{\beta(1-\alpha-\beta\sigma)} > 0 \tag{73}$$

$$\frac{\partial \bar{\theta}}{\partial \beta}\frac{1}{\bar{\theta}} = -\frac{q_A^{\alpha} n_A^{\beta}\ln n_A - q_B^{\alpha} n_B^{\beta}\ln n_B}{q_A^{\alpha} n_A^{\beta} - (1-\alpha-\beta\sigma) q_B^{\alpha} n_B^{\beta}} < 0 \tag{74}$$

$$\frac{\partial P_B}{\partial \beta} = \frac{P_A \varepsilon^2 r_B \beta\sigma\left\{\left[\sigma+(\alpha+\beta\sigma)\left(\ln n_B - (1-\alpha-\beta\sigma)\ln n_A\right)\right] q_A^{\alpha} n_A^{\beta} - \sigma(1-\alpha-\beta\sigma) q_B^{\alpha} n_B^{\beta}\right\}}{(1-\alpha-\beta\sigma)\left(\beta\sigma(1+\varepsilon) q_A^{\alpha} n_A^{\beta} - (1-\alpha-\beta\sigma)\varepsilon r_B\right)^2}. \tag{75}$$

The change in MCO B's insurance premium when policyholders care more about access to doctors (higher β) is ambiguous for reasons similar to those for which the change in MCO B's insurance premium is ambiguous when policyholders care more about quality (higher α). The utility from joining the alternative plan A also increases.

Proof of Proposition 2

Differentiating Equations 43 through 47 with respect to σ yields

$$\frac{\partial r_B}{\partial \sigma}\frac{1}{r_B} = \frac{1-\alpha+\beta\sigma(\ln r_B - \ln C)}{\sigma(1-\alpha-\beta\sigma)} \tag{76}$$

$$\frac{\partial n_B}{\partial \sigma}\frac{1}{n_B} = \ln\frac{r_B}{C} + \sigma\frac{\partial r_B}{\partial \sigma}\frac{1}{r_B} = \frac{(1-\alpha)(1+\ln r_B - \ln C)}{1-\alpha-\beta\sigma} \tag{77}$$

$$\frac{\partial q_B}{\partial \sigma}\frac{1}{q_B} = \beta\frac{1+\ln r_B - \ln C}{1-\alpha-\beta\sigma} \tag{78}$$

$$\frac{\partial \bar{\theta}}{\partial \sigma}\frac{1}{\bar{\theta}} = -\frac{\varepsilon r_B(\ln C - \ln r_B)}{\sigma(1+\varepsilon)\left(q_A^{\alpha} n_A^{\beta} - (1-\alpha-\beta\sigma) q_B^{\alpha} n_B^{\beta}\right)} < 0 \tag{79}$$

$$\frac{\partial P_B}{\partial \sigma} = \frac{P_A \varepsilon^2 r_B \beta^2 \sigma\left(\left[1-(\alpha+\beta\sigma)(\ln C - \ln r_B)\right] q_A^{\alpha} n_A^{\beta} - (1-\alpha-\beta\sigma)\varepsilon r_B\right)}{(1-\alpha-\beta\sigma)\left(\beta\sigma(1+\varepsilon) q_A^{\alpha} n_A^{\beta} - (1-\alpha-\beta\sigma)\varepsilon r_B\right)^2}. \tag{80}$$

The sign of the derivatives depends on the magnitude of ($\ln C - \ln r_B$), with the exception of the cutoff health risk. The MCO reduces the marginal health risk it insures when the concentration of high treatment cost doctors increases.

Proof of Proposition 3

Differentiating Equation 45 with respect to γ_B, we see that the MCO provides less of the quality increasing input when the marginal cost of quality provision increases.

$$\frac{\partial q_B}{\partial \gamma_B}\frac{1}{q_B} = -\frac{1-\beta\sigma}{\gamma_B(1-\alpha-\beta\sigma)} < 0. \tag{81}$$

The reimbursement paid to physicians also decreases:

$$\frac{\partial r_B}{\partial \gamma_B}\frac{1}{r_B} = -\frac{\alpha}{\gamma_B(1-\alpha-\beta\sigma)} < 0. \tag{82}$$

The reason is that the reduction in the amount of the quality increasing input provided by the MCO also diminishes the marginal utility of physicians. Therefore, the number of physicians in the MCO's network also decreases:

$$\frac{\partial n_B}{\partial \gamma_B}\frac{1}{n_B} = -\frac{\alpha\sigma}{\gamma_B(1-\alpha-\beta\sigma)} < 0. \tag{83}$$

With lower quality and access to fewer doctors the marginal willingness to pay of policyholders decreases, and so does the insurance premium charged by the MCO:

$$\frac{\partial P_B}{\partial \gamma_B} = -\frac{P_A \varepsilon^2 r_B \alpha\beta\sigma(\alpha+\beta\sigma) q_A^{\alpha} n_A^{\beta}}{\gamma_B(1-\alpha-\beta\sigma)\left[\beta\sigma(1+\varepsilon) q_A^{\alpha} n_A^{\beta} - (1-\alpha-\beta\sigma)\varepsilon r_B\right]^2} < 0. \tag{84}$$

However, the decrease is such that the MCO insures a mix of patients with lower health risk, as can be seen by differentiating Equation 46 with respect to γ_B and simplifying:

$$\frac{\partial \bar{\theta}}{\partial \gamma_B}\frac{1}{\bar{\theta}} = -\frac{\alpha q_B^{\alpha} n_B^{\beta}}{\gamma_B\left\{q_A^{\alpha} n_A^{\beta} - (1-\alpha-\beta\sigma) q_B^{\alpha} n_B^{\beta}\right\}^2} < 0 \tag{85}$$

Finally, the MCO attracts a smaller number of individuals:

$$\frac{\partial N_p F(\bar{\theta})}{\partial \gamma_B} = N_p \frac{\partial \bar{\theta}^{\varepsilon}}{\partial \gamma_B} = N_p \varepsilon \bar{\theta}^{\varepsilon-1}\frac{\partial \bar{\theta}}{\partial \gamma_B} < 0. \tag{86}$$

Proof of Proposition 4

Differentiating Equations 43, 44 , and 45 with respect to ε it can be shown that

$$\frac{\frac{\partial r_B}{\partial \varepsilon}}{r_B} = \frac{\frac{\partial q_B}{\partial \varepsilon}}{q_B} = \frac{1}{\sigma}\frac{\frac{\partial n_B}{\partial \varepsilon}}{n_B} = -\frac{1}{1-(\alpha+\beta\sigma)}\frac{1}{\varepsilon(1+\varepsilon)} < 0. \tag{87}$$

When the concentration of high-risk patients increases the MCO reduces quality and includes fewer physicians in its network by paying lower reimbursements to control costs.

To see how the mix of patients insured by the MCO changes when the distribution of heath risks changes, differentiate Equation 46 with respect to ε. Taking also the expression for reimbursement from Equation 43 into account we get

$$\frac{\partial\bar{\theta}}{\partial\varepsilon}=\frac{P_A\beta\sigma\left[\beta\sigma(1+\varepsilon)q_A^{\alpha}n_A^{\beta}-\varepsilon r_B\right]}{(1+\varepsilon)\left[\beta\sigma(1+\varepsilon)q_A^{\alpha}n_A^{\beta}-(1-\alpha-\beta\sigma)\varepsilon r_B\right]^2}. \tag{88}$$

From Equation 39 we have:

$$\varepsilon r_B=(1+\varepsilon)\beta\sigma q_B^{\alpha}r_B^{\beta\sigma}. \tag{89}$$

Substituting Equation 89 in Equation 88 yields

$$\frac{\partial\bar{\theta}}{\partial\varepsilon}=\frac{P_A\beta^2\sigma^2\left[q_A^{\alpha}n_A^{\beta}-q_B^{\alpha}r_B^{\beta\sigma}\right]}{\left[\beta\sigma(1+\varepsilon)q_A^{\alpha}n_A^{\beta}-(1-\alpha-\beta\sigma)\varepsilon r_B\right]^2}>0. \tag{90}$$

The derivative in Equation 90 is positive because the quality and number of doctors provided by MCO *B* is lower than that provided by the alternative plan *A*. The MCO finds it profitable to attract some higher risk individuals because fewer individuals with low risk remain.

To determine how the insurance premium charged by the MCO changes, rewrite Equation 5 as

$$P_A-P_B=\bar{\theta}(q_A^{\alpha}n_A^{\beta}-q_B^{\alpha}n_B^{\beta}). \tag{91}$$

Totally differentiating Equation 91 with respect to ε we get

$$(q_A^{\alpha}n_A^{\beta}-q_B^{\alpha}n_B^{\beta})\frac{\partial\bar{\theta}}{\partial\varepsilon}-\bar{\theta}\frac{\partial(q_B^{\alpha}n_B^{\beta})}{\partial\varepsilon}=-\frac{\partial P_B}{\partial\varepsilon}. \tag{92}$$

Because $\frac{\partial\bar{\theta}}{\partial\varepsilon}>0$, $\frac{\partial q_B}{\partial\varepsilon}<0$, and $\frac{\partial n_B}{\partial\varepsilon}<0$ it must be that $\frac{\partial P_B}{\partial\varepsilon}<0$. Hence, the insurance premium also decreases.

Proof of Proposition 5

To show that the percentage decreases in quality, reimbursement, and number of doctors in MCO *B*'s network due to increases in population health risk are greater when the externality is greater, differentiate the absolute value of the decreases in Equation 87 with respect to β

$$\frac{\partial\left|\frac{\partial r_B}{\partial\varepsilon}\frac{1}{r_B}\right|}{\partial\beta}=\frac{\partial\left|\frac{\partial q_B}{\partial\varepsilon}\frac{1}{q_B}\right|}{\partial\beta}=\frac{\partial\left|\frac{1}{\sigma}\frac{\partial n_B}{\partial\varepsilon}\frac{1}{n_B}\right|}{\partial\beta}=\frac{\sigma}{\left[1-(\alpha+\beta\sigma)\right]^2}\frac{1}{\varepsilon(1+\varepsilon)}>0\,. \tag{93}$$

Similarly, one can show that the percentage decreases are greater when

- individuals care more about the quality of health care service

$$\frac{\partial\left|\frac{\partial r_B}{\partial\varepsilon}\frac{1}{r_B}\right|}{\partial\alpha}=\frac{\partial\left|\frac{\partial q_B}{\partial\varepsilon}\frac{1}{q_B}\right|}{\partial\alpha}=\frac{\partial\left|\frac{1}{\sigma}\frac{\partial n_B}{\partial\varepsilon}\frac{1}{n_B}\right|}{\partial\alpha}=\frac{1}{\left[1-(\alpha+\beta\sigma)\right]^2}\frac{1}{\varepsilon(1+\varepsilon)}>0 \tag{94}$$

- the elasticity of the supply of doctors with respect to reimbursements is higher

$$\frac{\partial\left|\frac{\partial r_B}{\partial\varepsilon}\frac{1}{r_B}\right|}{\partial\sigma}=\frac{\partial\left|\frac{\partial q_B}{\partial\varepsilon}\frac{1}{q_B}\right|}{\partial\sigma}=\frac{\beta}{\left[1-(\alpha+\beta\sigma)\right]^2}\frac{1}{\varepsilon(1+\varepsilon)}>0 \tag{95}$$

$$\frac{\partial\left|\frac{\partial n_B}{\partial\varepsilon}\frac{1}{n_B}\right|}{\partial\sigma}=\frac{1-\alpha}{\left[1-(\alpha+\beta\sigma)\right]^2}\frac{1}{\varepsilon(1+\varepsilon)}>0 \tag{96}$$

- the concentration of high health risks in the population is smaller to start with

$$\frac{\partial\left|\frac{\partial r_B}{\partial\varepsilon}\frac{1}{r_B}\right|}{\partial\varepsilon}=\frac{\partial\left|\frac{\partial q_B}{\partial\varepsilon}\frac{1}{q_B}\right|}{\partial\varepsilon}=\frac{\partial\left|\frac{1}{\sigma}\frac{\partial n_B}{\partial\varepsilon}\frac{1}{n_B}\right|}{\partial\varepsilon}=\frac{1}{1-(\alpha+\beta\sigma)}\frac{1}{\varepsilon(1+\varepsilon)}<0\,. \tag{97}$$

To show that the percentage decreases in quality, reimbursement, and number of doctors in MCO *B*'s network due to increases in the marginal cost of providing quality are greater when the

externality is greater, differentiate the absolute value of the decreases in Equations 81, 82, and 83 with respect to β

$$\frac{\partial\left|\frac{\partial r_B}{\partial \gamma_B}\frac{1}{r_B}\right|}{\partial\beta}=\frac{\partial\left|\frac{\partial q_B}{\partial \gamma_B}\frac{1}{q_B}\right|}{\partial\beta}=\frac{\partial\left|\frac{1}{\sigma}\frac{\partial n_B}{\partial \gamma_B}\frac{1}{n_B}\right|}{\partial\beta}=\frac{\alpha\sigma}{\gamma_B\left[1-(\alpha+\beta\sigma)\right]^2}>0\,. \tag{98}$$

Similarly, one can show that the percentage decreases are greater when

- individuals care more about the quality of health care service

$$\frac{\partial\left|\frac{\partial r_B}{\partial \gamma_B}\frac{1}{r_B}\right|}{\partial\alpha}=\frac{\partial\left|\frac{\partial q_B}{\partial \gamma_B}\frac{1}{q_B}\right|}{\partial\alpha}=\frac{\partial\left|\frac{1}{\sigma}\frac{\partial n_B}{\partial \gamma_B}\frac{1}{n_B}\right|}{\partial\alpha}=\frac{1-\beta\sigma}{\gamma_B\left[1-(\alpha+\beta\sigma)\right]^2}>0 \tag{99}$$

- the elasticity of the supply of doctors with respect to reimbursements is higher

$$\frac{\partial\left|\frac{\partial r_B}{\partial \gamma_B}\frac{1}{r_B}\right|}{\partial\sigma}=\frac{\partial\left|\frac{\partial q_B}{\partial \gamma_B}\frac{1}{q_B}\right|}{\partial\sigma}=\frac{\alpha\beta}{\gamma_B\left[1-(\alpha+\beta\sigma)\right]^2}>0 \tag{100}$$

$$\frac{\partial\left|\frac{\partial n_B}{\partial \gamma_B}\frac{1}{n_B}\right|}{\partial\sigma}=\frac{\alpha(1-\alpha)}{\gamma_B\left[1-(\alpha+\beta\sigma)\right]^2}>0\,. \tag{101}$$

Proof of Proposition 6

Differentiating Equations 43 through 47 with respect to N_d yields

$$\frac{\partial r_B}{\partial N_d}\frac{1}{r_B}=\frac{\partial q_B}{\partial N_d}\frac{1}{q_B}=\frac{\beta}{N_d(1-\alpha-\beta\sigma)}>0 \tag{102}$$

$$\frac{\partial n_B}{\partial N_d}\frac{1}{n_B}=\frac{1-\alpha}{N_d(1-\alpha-\beta\sigma)}>0 \tag{103}$$

$$\frac{\partial\overline{\theta}}{\partial N_d}\frac{1}{\overline{\theta}}=\frac{\beta q_B^{\alpha}n_B^{\beta}}{N_d\left(q_A^{\alpha}n_A^{\beta}-(1-\alpha-\beta\sigma)q_B^{\alpha}n_B^{\beta}\right)}>0 \tag{104}$$

$$\frac{\partial P_B}{\partial N_d} = \frac{P_A \varepsilon^2 r_B \beta^2 \sigma(\alpha + \beta\sigma) q_A^\alpha n_A^\beta}{N_d (1-\alpha-\beta\sigma)\left(\beta\sigma(1+\varepsilon) q_A^\alpha n_A^\beta - (1-\alpha-\beta\sigma)\varepsilon r_B\right)^2} > 0 . \tag{105}$$

Proof of proposition 7

Differentiating Equations 43 through 47 with respect to C yields

$$\frac{\partial r_B}{\partial C}\frac{1}{r_B} = \frac{\partial q_B}{\partial C}\frac{1}{q_B} = -\frac{\beta\sigma}{C(1-\alpha-\beta\sigma)} < 0 \tag{106}$$

$$\frac{\partial n_B}{\partial C}\frac{1}{n_B} = -\frac{(1-\alpha)\sigma}{C(1-\alpha-\beta\sigma)} < 0 \tag{107}$$

$$\frac{\partial \bar{\theta}}{\partial C}\frac{1}{\bar{\theta}} = -\frac{\beta\sigma q_B^\alpha n_B^\beta}{C\left(q_A^\alpha n_A^\beta - (1-\alpha-\beta\sigma) q_B^\alpha n_B^\beta\right)} < 0 \tag{108}$$

$$\frac{\partial P_B}{\partial C} = -\frac{P_A \varepsilon^2 r_B \beta^2 \sigma^2 (\alpha + \beta\sigma) q_A^\alpha n_A^\beta}{C(1-\alpha-\beta\sigma)\left(\beta\sigma(1+\varepsilon) q_A^\alpha n_A^\beta - (1-\alpha-\beta\sigma)\varepsilon r_B\right)^2} < 0 . \tag{109}$$

The MCO prefers to reduce quality and the number of doctors in its preferred network to restrain costs, even though this means it has to charge a lower premium.

Table 1. Effect of increase in parameters on the MCO's reimbursement, quality, number of doctors, marginal health risk, and insurance premium

Variables of interest Parameters	Reimbursement r_B	Quality q_B	Number of doctors n_B	Cutoff probability $\bar{\theta}$	Insurance premium P_B
Concentration of health risks in the population ε	-	-	-	+	-
Importance of quality for policyholders α	+	+	+	-	+/-
Indirect externality parameter (Importance of number of doctors for policyholders) β	+	+	+	-	+/-
Elasticity of supply of doctors with respect to reimbursement σ	+/-	+/-	+/-	-	+/-
Marginal cost of quality γ_B	-	-	-	-	-
Doctors' maximum treatment cost per patient C	-	-	-	-	-
Number of doctors in the market N_d	+	+	+	+	+

REFERENCES

Altman, D., Cutler, D. M., and R. J. Zeckhauser, 2003. Enrollee Mix, Treatment Intensity, and Cost in Competing Indemnity and HMO Plans. Journal of Health Economics 22, 23-45.

Armstrong, M., 2006. Competition in Two-Sided Markets. Rand Journal of Economics 37(3), 668-691.

Arrow, K. J., 1963. The Welfare Economics of Medical Care. American Economic Review 53, 941-973.

Bardey, D. and Rochet, J-C., 2006. Competition Among Health Plans: a Two-Sided Market Approach. Available online at http://economia.uniandes.edu.co/es/content/download/9220/45219/file/Bardey.pdf.

Chandra, A., and A. Collard-Wexler, 2008. Mergers in Two-Sided Markets: An Application to the Canadian Newspaper Industry. Available online at http://pages.stern.nyu.edu/~acollard/Mergers%20in%20Two-Sided%20Markets.pdf.

Deb, P., and P. K. Trivedi, 2006. Specification and Simulated Likelihood Estimation of a Non-Normal Treatment-Outcome Model with Selection: Application to Health Care utilization. Econometrics Journal 9, 307-331.

Ellis, R. P. and T. G. McGuire, 1993. Supply-Side and Demand-Side Cost Sharing in Health Care. Journal of Economic Perspectives 7, 135-152.

Evans, D.S. and R. Schmalensee, 2005. The Industrial Organization of Markets with Two-Sided Platforms. NBER Working Paper No. 11603.

Glied, S., 2000. Managed Care. Handbook of Health Economics, Ed. by A. J. Culyer and J. P. Newhouse, Vol. 1A.

Liu, F. and D. M. Zimmer, 2006. The Effect of Switching Private Insurance Plans on Health Care Utilization. Contributions to Economic Analysis and Policy Vol. 5(1), Article 22.

Ma, C. A. and T. G. McGuire, 1997. Optimal Health Insurance and provider Payment. American economic Review 87, 685-704.

Ma, C. A. and M. H. Riordan, 2002. Health Insurance, Moral Hazard, and Managed Care. Journal of Economics and Management Strategy 11(1), 81-107.

Miller, R. H. and L. S. Luft, 1994. Managed Care Plan Performance Since 1980. Journal of the American Medical Association 271, 1512-1519.

Pezzino, M. and G. Pignatarro, 2007. Competition in the Health Care market: a "Two-Sided" Approach. Annals of the 19th Conference of the Italian Society for Public Economics (SIEP). Electronic document at http://www-1.unipv.it/websiep/wp/200771.pdf.

Rochet, J-C. and J. Tirole, 2003. Platform Competition in Two-Sided Markets. IDEI Working Papers 152, Institut d'Économie Industrielle, Toulouse.

Rochet, J-C. and J. Tirole, 2005. Two-Sided Markets: A Progress Report. IDEI Working Papers 275, Institut d'Économie Industrielle, Toulouse. Electronic document at http://idei.fr/doc/wp/2005/2sided_markets.pdf.

Rysman, M., 2004. Competition Between Networks: A Study of the Market for Yellow Pages. Review of Economic Studies 71(2), 483-512.

Rysman, M., 2007. The Empirics of Antitrust in Two-Sided Markets. Competition Policy International 3(1), 197-209.

Wolfram Research. Mathematica 5.

Wright, J., 2004. One-Sided Logic in Two-Sided Markets. Review of Network Economics 3(1), 44-64.

Zeckhauser, R., 1970. Medical Insurance: A case Study of the Tradeoff between Risk Spreading and Appropriate Incentives. Journal of Economic Theory 2, 10-26.