Price Tests for Entry into Markets in the Presence of Non-Convexities

Michael H. Rothkopf[a]

Richard P. O'Neill[b]

Benjamin J. Hobbs[c]

Paul M. Sotkiewicz[d]

William R. Stewart, Jr.[e]

March 27, 2004

Abstract

O'Neill, et al. (2004) showed how to find equilibrium-supporting prices in markets that have economies of scale and are modeled as Mixed Integer Programs. We review that result and describe the conditions under which a firm with a new technology can use these prices to determine whether market entry is profitable.

JEL Subject code: D4

Key words: Entry Prices, Nonconvex Markets

[a] Department of Management Science and Information Systems and RUTCOR, Rutgers University, 640 Bartholomew Road, Piscataway, NJ 08854-8003. 732 445-5632 rothkopf@rutcor.rutgers.edu

[b] Chief Economic Advisor, Federal Energy Regulatory Commission.

[c] Department of Geography and Environmental Engineering, The Johns Hopkins University.

[d] Public Utility Research Center, University of Florida.

[e] School of Business, College of William and Mary.

1. Introduction

Prices play an important role in markets. They signal what to produce, they signal investors about the value of proposed investments, and they indicate whether a firm with a new technology should enter a market. This works relatively well in the absence of non-convexities, such as economies of scale. However, prices to perform these functions when there are non-convexities in the market have not, until now, been available. Scarf (1990, 1994) summed up the dilemma caused by the inability to find such prices for the optimization and economic communities. He wrote, "[T]he essence of economies of scale in production is the presence of large and significant indivisibilities in production. … For a theorist, the major problem presented by indivisibilities in production is the failure of the pricing test for optimality or for welfare improvements. … [I]n the presence of indivisibilities in production, prices simply don't do the jobs that they were meant to do." Scarf gives an example which we use to illustrate our results.

When there are no non-convexities, a market can be modeled as a convex programming problem, and the dual variables (shadow prices) of this optimization problem give prices for commodities. The literature, dating back to Gomory and Baumol (1960), has noted the lack of equilibrium-supporting prices in non-convex models. However, as noted in O'Neill, et al. (2004), such prices can be found. The approach is to price *all* of the factors of production, the non-convexities, as well as the continuous variables. Thus, if it takes guns, butter, and a startup (a non-convexity) to make widgets, an equilibrium-supporting set of prices for widgets requires a price for the startup as well as prices for the guns and butter. This approach is of practical as well as of theoretical interest. Several daily electricity markets have evolved a pricing structure similar to the prices found by O'Neill, et al. (2004)

This paper starts with a review of the approach of O'Neill et al. (2004). That paper discusses prices that support an equilibrium in a nonconvex market. However, it does not discuss the role of prices for testing entry by new technologies in such markets. The paper then examine whether O'Neill et al. prices can be used to support such entry decisions. The answer is mixed. First, without non-convexities, their approach defaults to standard linear pricing. Second, when the existing technology has non-convexities but the new technology does not, their prices give a sufficient (but not necessary) test for profitable entry. Finally, when the new technology has non-convexities, they do not solve the problem in general, but they can sometimes help. The paper concludes with a reflection on the different thinking patterns of optimization theorists and economists and how this contributed to the delay of over 40 years in finding equilibrium supporting prices for markets with nonconvexites.

2. O'Neill et al. Prices

O'Neill et al. (2004) show that equilibrium-supporting prices can be found for any market with non-convexities that can be modeled as a Mixed Integer Programming Problem (MIP). First, they find the solution to the MIP.[1] Then, they constrain the integer variables to their optimal values and eliminate the integrality requirements. The resulting problem is a linear program with well-defined dual variables corresponding to the added constraints that give equilibrium-supporting prices for the discrete choices. These prices allow a decentralized implementation because, at them, firms have no incentive to deviate from the efficient solution.

Scarf's (1994) example illustrates the problem of deriving meaningful prices in the presence of non-convexities. Scarf postulates two types of plants, each with significant fixed costs (Table 1). The market-clearing problem can be formulated as a MIP to minimize the total cost of satisfying a fixed level of demand. The corresponding decentralized market problem is to satisfy market feasibility while each plant maximizes profits.

[1] In general, MIPs are NP-hard problems. Hence, computation in the worst case grows exponentially with the size of the problem. However, commercial codes for solving MIPs are now practical for substantial problems (Hobbs et al. 2001). In practice, the worst-case bounds are extremely loose.

To satisfy a fixed demand of 61 units, the optimal solution is to build three Smokestack plants and two High Tech plants, running the High Tech plants at capacity and the Smokestack plants at less than their combined capacity. Is there a price that can support a competitive equilibrium in the decentralized market problem? One candidate price is 3, the marginal cost of Smokestack plants. Since they are producing, but not at capacity, equilibrium is impossible at any other price. However, at that price both types of plans lose money. Hence, that price cannot be an equilibrium, and no classical equilibrium price exists.

Table 1. Production Characteristics: Smokestack versus High Tech (from Scarf, 1994)

Characteristic	**Smokestack (Type 1 Plant)**	**High Tech (Type 2 Plant)**
Capacity	16	7
Construction Cost	53	30
Marginal Cost	3	2
Average Cost at Capacity	6.3125	6.2857
Total Cost at Capacity	101	44

Now, consider the construction (start-up) for each type as a separate commodity; there are now three commodities needing prices: the final output, construction of the Smokestack type, and construction of the High Tech type. Let 3, the marginal cost of the higher cost type, be the price for the final output. Let 53, the Smokestack construction cost, be the price for building the Smokestack type. Finally, let 23 be the price for building High Tech types. A price of 3 on the final output makes sense, since at that price all Smokestack units receive 3 per unit for the output that they can produce at a marginal cost of 3. Each Smokestack unit also receives a price of 53 for construction, leaving it with zero profits. The High Tech units each receive a price of 3 for the final output they can produce at a marginal cost of 2, leaving each with a margin of 1 per unit of output. At the construction price of 23 (7 below construction cost), each High Tech unit gets precisely zero profit. Thus, if startup decisions are viewed as commodities, competitive-equilibrium-supporting prices exist.

3. O'Neill et al. Prices and Entry Decisions

Scarf (1994) points out that prices, in addition to supporting equilibrium activities by market participants, provide signals to potential entrants. O'Neill et al. (2004) derive prices that serve the first of these roles. They don't discuss the use of their prices as signals for entry as we now do. (Note that our focus is on the role of prices, not the optimal decision. An infallible procedure for evaluating entry by a potential new technology is to add it to the MIP and resolve.) Do the O'Neill at al. prices alone allow potential new entrants to make correct entry decisions?

It is useful to distinguish four separate cases:

1. Neither the existing technologies nor the new one have non-convexities,
2. Existing technologies have non-convexities, but new technologies do not,
3. Existing technologies do not have non-convexities, but new ones do, and
4. Both the existing technologies and the new ones have non-convexities.

For the purposes of discussion, we focus on economies of scale. Case 1 is classical economics. The market provides prices for commodities; the new technology can test entry profitability by comparing the prices for the commodities with its marginal cost of production.

In case 2, the O'Neill et al. prices provide prices for both non-convexities and continuous commodities. Since the new technology is convex, it can use the O'Neill et al. prices for the continuous commodities just as in case 1 to decide if it is profitable for it to enter at some scale. (If there is degeneracy, this scale might be zero, but that is also true with linear programming models of convex markets.) Thus, it can test its entry decision at the margin. This is a new capability for prices. In Scarf's example, a new technology without economies of scale can enter if it can produce at cost of 3 or less. This test is sufficient but not necessary. Entry could be profitable at higher marginal costs, but checking this depends upon inframarginal information.

In case 3, the prices for the existing technologies are just commodity prices. However, the new technology is non-convex. If the non-convexities are small enough, the commodity prices can be used to evaluate the potential of entry. However, if the non-convexities are large, there is a risk that there is no economic scale for entry. For example, in a market with a commodity price of 1, a new technology with a fixed cost of 100, a capacity of 200, and a variable cost of 0.49 *seems* slightly profitable. It will be if its entry does not change the price appreciably. However, if the commodity price would fall below 0.99 in the presence of another 100 units of supply, the new technology would be unable to enter profitably. Thus, in this case the classical marginal prices are insufficient to guarantee the advisability of entry. An informed decision needs information about infra-marginal demand

Similarly, in case 4, O'Neill et al. prices will not always guarantee the advisability of entry decisions. However, in both cases 3 and 4, there is some chance that they will. Dominance arguments may be sufficient. Suppose, for example, that in Scarf's example, a third technology becomes possible, with capacity 16, marginal cost 3, but construction cost 50 (less than that of a Smokestack plant). Clearly, due to its lower construction cost, it dominates Smokestack plants; it can accept profitably the prices they get. Similarly, if the new technology is inferior to an existing technology--say a plant with capacity 16, marginal cost 3, but construction cost 55--it is uneconomic. Dominance arguments go beyond one-for-one comparisons. A plant with capacity 23, construction cost 83, and variable cost of 2.1 would dominate the combination of one Smokestack plant and one High Tech plant and be profitable at the O'Neill et al. prices.

4. Discussion

Since Gomory and Baumol (1960), it has been believed that markets with non-convexities might have no equilibrium-supporting prices. O'Neill et al. showed that there are such prices if, in addition to the "traditional" commodities themselves, the non-convexity "commodities" are priced. They left open, however, the question of whether the prices they calculate are a sufficient guide for *new* technologies considering entering the market. We point out that if the new technology has no non-convexities, the O'Neill et al. commodity prices can be sufficient. If the new technology is itself non-convex, these prices are not generally sufficient, but may in some cases provide definitive guidance.

Why it has taken over 40 years to find the O'Neill et al. prices? One reason is that those interested in mathematical programming have thought of dual variables as computational tools for finding optimality, not primarily for pricing. Often, they viewed prices on non-convexities as "artificial" (e.g., Gomory and Baumol 1960). Also, such prices are "non-anonymous" and, therefore, not as useful as pure commodity prices. While Wolsey [1981] gives a theory of duality in integer programming, it has not, to our knowledge, been used to develop useful prices for non-convexities in real-world markets. Meanwhile, economists have tended to try to work around non-convexities and to de-emphasize their importance since they are inconvenient and inconsistent with many theorems in welfare economics (even though, as Scarf [1994] notes, they are necessary for the existence of firms). Economics has tended to start by searching for equilibrium prices and then show these support optimal outcomes. O'Neill et al. start from optimality and look for equilibrium prices to support it by expanding the commodity space to include non-convexities.

The optimization of O'Neill et al. involves a centralized decision algorithm. The existence of significant non-convexities requires this for optimality. However, even with a centralized algorithm, equilibrium prices exist that allow decentralized decision making as in the new electricity markets. These prices may support decentralized entry decisions for technologies without significant economies of scale, but they are insufficient in general.

Acknowledgments. The Federal Energy Regulatory Commission supported this research. NSF Grant ECS 0080577 provided Hobbs with partial support. The authors acknowledge the helpful comments and suggestions of Karla Hoffman, Herbert Scarf, and William Hogan. Any errors and opinions are solely the responsibility of the authors.

References

Gomory, R. E. and Baumol, W. J., 1960, "Integer Programming and Pricing," *Econometrica*, Vol. 28, No. 3, pp. 521-550.

Hobbs, B. F., Stewart, W. R. Jr., Bixby, R. E., Rothkopf, M. H., O'Neill, R. P., and Chao, H-P., 2001, "Why This Book?: New Capabilities and New Needs for Unit Commitment Modeling," in Hobbs, B. F., Rothkopf, M. H., O'Neill, R. P., and Chao, H-P. (eds.), *The Next Generation of Electric Power Unit Commitment Models*, Kluwer Academic Press, pp. 1-14.

O'Neill, R. P., Sotkiewicz, P. M., Hobbs, B.F., Rothkopf, M. H., and Stewart, W. R. Jr., 2004, "Efficient Market-Clearing Prices in Markets with Non-convexities," forthcoming in the *European Journal of Operations Research*.

Scarf, H. E., 1990, "Mathematical Programming and Economic Theory," *Operations Research* **38**, pp. 377-385.

Scarf, H. E., 1994, "The Allocation of Resources in the Presence of Indivisibilities," *Journal of Economic Perspectives*, Vol. 8, Number 4, pp. 111-128.

Wolsey, L. A., "Integer Programming Duality: Price Functions and Sensitivity Analysis," *Mathematical Programming*, **20**, 173-195, 1981.