Unbundling the Local Loop*

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Abstract

We study competition for high bandwidth services in the telecommunications industry by introducing the possibility of unbundling the local loop, where leased lines permit the entrant to provide services without building up its own infrastructure. We use a dynamic model of technology adoption and study the incentives of the entrant to lease loops and compete “service-based”, and/or to build up a new and more efficient infrastructure and compete “facility-based”, given the rental price.

We show that the incumbent sets too low a rental price for its loops, hence the entrant adopts the new technology too late, from a social welfare perspective. The distortion may not only appear on the timing of technology adoption, but also on the type (quality) of the new technology to be adopted. We also show that while regulating the rental price may suffice to achieve socially desirable outcomes, a sunset clause does not improve social welfare.

Keywords: Unbundling; Technology adoption; Regulation

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1 Introduction

In this paper we provide a dynamic framework within which the effect of unbundling on building alternative infrastructures to provide high bandwidth services can be analyzed. We use a dynamic model of technology adoption and study the incentives of the entrant to lease loops and compete “service-based”, and/or to build up a new and more efficient infrastructure and compete “facility-based”, given the rental price determined by the incumbent. We show that the incumbent sets too low a rental price for its loops, hence the entrant adopts the new technology too late, from a social welfare perspective. The distortion may not only appear in the timing of technology adoption, but also in the type (quality) of the new technology that it adopts. We also show that while regulating the rental price may suffice to achieve socially desirable outcomes, a sunset clause is redundant for improving social welfare.

Unbundling provides the entrants with access to the local loop of the incumbent operator so that they do not incur large fixed and sunk costs to build their own infrastructure. It is expected to improve service based competition, hence the variety of new services. Without unbundling, competitors have limited access to the essential facility which is reached through interconnection. On the other hand, for the same reason, in many policy discussions unbundling is claimed to undermine the incentives for building alternative networks. For instance, Kahn et al. (1999, p. 360) argue that in the United States, for the regulation of unbundled elements, the Federal Communications Commission (FCC) has “ignored or downplayed the likely discouraging effect on those competitors supplying their own needs and on risk-taking innovation by the ILECs themselves”. Sidak and Spulber (1998, p. 411) state that “pricing network services below economic cost is likely to discourage the building of competing facilities”.

Facility based competition in the telecommunications industry is perceived as a necessary condition for long term efficiency. For the full functioning of competition, it is necessary that each operator control its supply chain to the largest possible extent. The benefits from flexibility and innovation obtainable under this state of affairs exceed by far those achievable under facility sharing settlements.

“Build or by” decisions of the entrant depend on the supply conditions at which the unbundled loops are provided. In particular, rental prices of the loops assume predominance in the efforts to achieve desirable outcomes. The optimal price for the local loops reflects the trade off between short run benefits from service based competition and long run benefits of improved facility-based competition. A price too low deters (or delays) investment in alternative networks, and a price too high would discourage entrants from joining service based competition.1

Local loop unbundling in Europe has proven to be a complex and slow process as in the United States where it has been mandatory since 1996. Following the liberalization of European telecommunications market, possibility

1 See Kahn et al. (1999).
of local loop unbundling was considered only by a few number of European countries (e.g., Austria, Denmark, Finland and Germany). It is very recently (January 2001) that the European Commission put into force its decision on mandating local loop unbundling in all the E.U. member countries. That decision (EC/2887/2000) requires the incumbent operators to provide access to their copper lines on a ‘reasonable’ request. A reasonable rate should both ensure that the incumbent recovers its costs, and also should foster fair and sustainable competition in the local loop. In addition, the commission has also decided that the rental schemes should take into account the need for investment in alternative technologies. Although various policy studies discussed the possible effects of unbundling on building alternative technologies,\(^2\) to our knowledge, no formal analysis in this respect has been provided.\(^3\) This paper is an attempt at providing formal analysis of the issue.

The paper is organized as follows. We begin with setting up the basics of the model in Section 2. We devote Section 3 to the analysis of service based competition and facility based competition. In Section 4 we study the technology choice of the entrant both with and without unbundling. Proceeding backwards, in Section 5, we study the decision of the incumbent with respect to unbundling and the rental price. In Section 6 we provide a social welfare analysis whereby a comparison with the unregulated outcome is given. We also discuss the role of sunset clauses for improving socially desirable outcomes. Finally, we conclude.

2 The Model

In this model, we assume that the incumbent, who owns and operates the local loop, to be making all the decisions regarding unbundling. Hence, if it decides to unbundle its local loop, it sets the rental rate for it. Later, we introduce a social welfare maximizing regulator in order to compare the unregulated outcome with the socially efficient one. We also discuss regulatory tools to achieve desirable outcomes when the local loop is unbundled.

We distinguish between two types of entry to the high bandwidth services market. Service-based entry takes place when the local loop is unbundled and the entrant decides to lease a local loop. In this type of entry, we assume that the incumbent and the entrant are restricted to providing horizontally differentiated products, as it is less likely that firms using the same infrastructure achieve different quality of services. On the other hand, in case of facility-based entry, i.e., when the entrant invests in an alternative technology (e.g., wireless local loop, cable networks or fiber optic networks), we assume that the entrant obtains a quality advantage over the services provided with the traditional local loops. The quality advantage the entrant may gain depends on the technology it chooses to adopt. A proper interpretation of quality in this context is the size of the bandwidth.


\(^3\)Kim et al. (2000) examine economic effects of local loop unbundling, however, they do not address the incentives to invest in alternative facilities.
Service-based entry occurs only if the local loop is unbundled, whereas facility-based entry can take place whether the local loop is unbundled or not. Furthermore, if the local loop is unbundled, the entrant may lease loops and compete on the basis of services, prior to its adoption of a new technology. We assume that the incumbent uses some Digital Subscriber Line (DSL) technology, and cannot invest in alternative technologies.\footnote{This may be either because of its previous sunk investments in the copper loop, or because of a regulatory ban.}

\section{Firms}

The incumbent \((I)\) has a constant marginal cost of providing high bandwidth services, which is normalized to zero. If the incumbent unbundles the local loop, it sets the rental rate \(r\), and receives a marginal revenue of \(r\) per line if the entrant decides to lease lines.\footnote{Here, we don’t consider any credibility issues. We assume that the incumbent is able to commit to a rental price with long-term contracts. This assumption is a strong one, but enables us to focus on the effects of a fixed rental price on the adoption of alternative technologies.} As we show later, value of \(r\) determines whether service based competition yields competitive equilibrium, corner equilibrium, or quasi-monopolistic equilibrium.

There is a sunk cost of entry, \(f\), when the entrant leases loops, which is due to co-location and order handling. The quality of service that is provided with the existing local loop is normalized to zero. On the other hand, the new technology brings a superior quality of service. In order to simplify the analysis for three-dimensional competition (price, quality, and variety), we restrict the choice set of the entrant for the new technology in terms of quality. There are two different technologies, technology \(L(ow)\) and technology \(H(igh)\), which are available to the entrant \(\(E)\) for adoption. They respectively yield qualities \(q_L\) and \(q_H\) and enable the entrant to compete with a superior quality of service.\footnote{In reality, incumbents can make upgrading investments which increase the bandwidth (here the quality) of their local loop. What is essential to our analysis is that the new technology brings a sufficiently superior quality to the maximum of what can be achieved through the copper lines. For instance, it seems unlikely that in the future DSL copper lines will achieve the same bandwidth as fiber optic loops.}

The choice for the technology to be adopted reflects the flexibility of the entrant for market targeting.

Adoption cost for technology \(\tau = L, H\) is

\[
A_\tau(\Delta_\tau) = \frac{a_\tau}{2} \Delta_\tau^2,
\]

where \(\Delta_\tau \in [0, 1)\) is the discount factor determined by the adoption date. Here we use the same notation and interpretation of Riordan (1992): \(\Delta = \exp(-\delta t)\), where \(\delta\) is the discount rate, and \(t\) denotes time. We normalize \(\delta\) to 1.\footnote{Note that a very low discount rate may change some of our findings. See end of Section 5 for a discussion on low discount rates.} Throughout the paper we will refer to \(\Delta\) as the adoption date. Note that higher \(\Delta\) corresponds to an earlier adoption date. We have \(A'_\tau(\Delta_\tau) \geq 0\), and \(A''_\tau(\Delta_\tau) > 0\).
We assume that at any time it is more costly to adopt technology $H$, i.e.,

$$a_H > a_L \Leftrightarrow A_H(\Delta) > A_L(\Delta).$$

### 2.2 Consumers

Consumers are uniformly distributed on the unit square $[0, 1] \times [0, 1]$. A consumer of type $(x, \theta)$ has a taste $x$ for variety (its location on the horizontal segment) and a valuation $\theta$ for quality (location on the vertical segment), with $x, \theta \in [0, 1]$.

The indirect utility function of the consumer of type $(x, \theta)$, who purchases a unit of service (an access line) from firm $i$ is

$$U = v + \theta q_i - (x - y_i)^2 - p_i,$$

where $v > 3$ is the fixed utility derived by using high bandwidth services. The horizontal location of firm $i$ on the unit segment is denoted by $y_i$, whereas its price is denoted by $p_i$, with $i = I, E$. The quality of the service provided by the incumbent is $q_I$, and $q_L = 0$. The quality of service provided by the entrant is $q_E$, and depends on the technology it uses. If it leases loops and competes on the basis of services it has the same quality as the incumbent, $q_E = 0$. If it adopts technology ($L$), its quality is $q_L$, and is given in the interval $q_L \in (0, 3/2)$. If it adopts technology ($H$), its quality, $q_H$, is given and belongs to the interval $q_H \in (2, 3)$.

Competition with technology $H$ brings forth “vertical dominance” whereas technology $L$ leads to “horizontal dominance”. Vertical dominance occurs when the quality differentiation dominates horizontal differentiation, and horizontal dominance occurs when it is the other way around.\footnote{See Neven and Thisse (1989).}

The assumption on the lower bound of $v$ ensures market coverage for all possible cases: i) when incumbent operates alone, ii) when firms compete on the basis of services, iii) when firms compete on the basis of facilities.

Finally, we make the following assumptions on $a_H$, $a_L$, and $f$.

A1. $a_H \in (8q_H / 9 - 1/2, v - 3/4 + q_H / 3)$.

A2. $a_L \in (6 + q_L)^2 / 36 - 1/2, v - 3/4 + q_L / 3)$.

A3. $f \in F$, with $F = \{f, \overline{f}\}$, where

$$f \equiv (v - 1) / (4v - 3),$$

and

$$\overline{f} = \min \left\{ \frac{1}{2} + \frac{1}{8a_L} - \frac{(6 + q_L)^2}{144a_L}, \frac{1}{2} + \frac{1}{8a_H} - \frac{2q_H}{9a_H} \right\}.$$

These assumptions rule out the uninteresting cases for our analysis, i.e., they rule out the case under which the entrant never leases loops (even when the rental price is set at zero), and the case in which the incumbent never unbundles...
its loops. Hence, the lower bounds on \( a_r \) together with the upper bound on \( f \) ensure that if the rental price is set at marginal cost (zero), then the entrant is willing to lease lines. On the other hand, the upper bounds on \( a_r \) exclude the case in which the incumbent never unbundles its loops.\(^9\) Finally, given the lower bound on \( f \), the entrant obtains non-positive profits for relatively high rental prices, and hence, we restrict our attention to the competitive and corner equilibria.\(^{10}\)

2.3 The Timing

The timing of the game is as follows.

1. The incumbent decides whether to unbundle or not.

2. The incumbent commits to the rental rate of the loop, \( r \), if it unbundles the local loop.

3. At any time \( t \), the entrant decides whether to rent loops if the local loop is unbundled, and compete on service-base (S), and/or decide to adopt a new technology. In case it decides to adopt a new technology, it also decides on the type of technology to adopt \((L)\) or \((H)\). The entrant can compete on the basis of services by leasing lines before it decides to introduce any new technology. The incumbent and the entrant obtain profits flows during the phase of service based competition \( \pi_I^S(r) \), \( \pi_E^S(r) \), and during the phase of facility based competition \( \pi_I^F \), \( \pi_E^F \), respectively.

We have assumed that the entrant never adopts the technology at time zero, and that the adoption cost has no fixed component and declines over time. Furthermore, as it will be seen in the next section, competing on the basis of facilities brings a higher profit flow to the entrant than when it competes on the basis of services, thus it always end up by adopting a new technology. As a consequence, we find ourselves in one of the following situation: either entry takes place with an alternative technology only, or before adopting any new technology, the entrant leases local loops and competes on the basis of services. Once it adopts the new technology, it stops leasing lines.

3 Service-based competition vs. Facility-based competition

In this section we compute the payoff flows for both service-based and facility-based competition. Service-based competition is modeled à-la-Hotelling. On

\(^9\) Later we show that if the adoption cost of a new technology is sufficiently high, which implies that there is no threat of facility based entry, the incumbent is better off by not unbundling and maintaining its monopoly profits. However, we include the possibility of unbundling to be a favorable strategy of the incumbent.

\(^{10}\) See Appendix A for the determination of \( f \). The value of \( T \) is determined in the proof of lemma 6.
the other hand, facility-based competition introduces a quality dimension to the former.

Since immediate technology adoption is assumed away, prior to the adoption (or prior to the lease of loops, if available) the incumbent is the monopolistic provider of high bandwidth services, and it locates at 1/2. Corresponding prices and profits are denoted by $p^M_i$ and $\pi^M_i$, with

$$p^M_i = \pi^M_i = v - \frac{1}{4}.$$ 

### 3.1 Service-based competition

As we have already stated, firms compete on the basis of services when they use the same infrastructure, i.e., the traditional local loops. In this case, quality of service provided by both firms is zero. We assume that firms choose maximum horizontal differentiation, and hence locate at the two extremes of the unit line. Therefore, the indirect utility function of the consumer of type $(x, \theta)$, who purchases a unit of service from firm $i$ is

$$U = v - (x - y_i)^2 - p_i.$$ 

Profit flows of the incumbent and the entrant are

$$\pi^S_I(r) = p^S_I x + (1 - \overline{\theta}) r$$

and

$$\pi^S_E(r) = (p^S_E - r)(1 - \overline{\theta}),$$

where $\overline{\theta}$ is the marginal customer who is indifferent for purchasing the access from the incumbent and the entrant. Equilibrium prices and profits depend on the rental rate $r$. For $r < v - 5/4$, we have a competitive equilibrium. Letting $y_I = 0$ and $y_E = 1$, in the phase of service based competition equilibrium prices and profit flows for the competitive range of $r$ are $p^S_I(r) = p^S_E(r) = 1 + r$, $\pi^S_E = 1/2$ and $\pi^S_I(r) = 1/2 + r$.$^{11}$

### 3.2 Facility-based competition

In this section, we employ a simplified version of Neven and Thisse (1989) three dimensional competition. The entrant has a technology choice between $H$ and $L$. We show that the entrant obtains a higher profit flow with technology $H$ than with technology $L$. While computing the payoffs in this section, we assume that $y_I \geq y_E$ without any loss of generality.

**Technology $H$ (Vertical Dominance)**

Vertical dominance is defined by

$$\left| \frac{\partial \overline{\theta}(x)}{\partial x} \right| < 1$$

$^{11}$ See Appendix B for the definition of prices and profits for all $r$. 

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where \( \overline{\vartheta}(x) \) is the marginal consumer with
\[
\overline{\vartheta}(x) = \frac{(p_E - p_I) + (y_E^2 - y_I^2) - 2(y_E - y_I)x}{q_H}.
\]
Vertical differentiation dominates horizontal differentiation for all \( q_E = q_H \in (2,3) \). For vertical dominance it is necessary to have \( q_E > 2 \). On the other hand, the upper bound for ensures the existence of the price equilibrium on the linear part of the demand curve. Unless \( q_E = 2 \) (there is neither vertical nor horizontal dominance), the demand curve is composed of three parts. In addition to a linear part in the middle, the demand curve has a convex and a concave part (for very high and very low prices, respectively). From here on, we will focus on the linear part of the demand curve for computing equilibrium profit flows. Considering these non-linear parts would complicate the exposition without substantively enhancing our analysis.

We solve the game backwards and hence start with equilibrium prices for given \( q_H \). Then, we determine the equilibrium horizontal differentiation.

**Lemma 1** During the phase of facility based competition with technology \( H \), the equilibrium is characterized by minimum horizontal differentiation \( (y_I = y_E) \), and equilibrium prices and profit flows are \( p_H^I = q_H/3 \), \( p_H^E = 2q_H/3 \), \( \pi_H^I = q_H/9 \), and \( \pi_H^E = 4q_H/9 \).

**Proof.** See Appendix C.

**Technology L (Horizontal Dominance)** For all \( q_E = q_L \in (0,3/2) \), we have horizontal dominance, which is defined by
\[
\left| \frac{\partial \overline{\vartheta}(x)}{\partial x} \right| > 1.
\]

The upper bound for \( q_L \) ensures the existence of a price equilibrium at the linear part of the demand curve.

**Lemma 2** During the phase of facility based competition with technology \( L \), the equilibrium is characterized by maximum horizontal differentiation \( (y_I = 0, y_E = 1) \), and equilibrium prices and profit flows are \( p_L^I = 1 - q_L/6 \), \( p_L^E = 1 + q_L/6 \), \( \pi_L^I = (6 - q_L)^2 / 72 \), and \( \pi_L^E = (6 + q_L)^2 / 72 \).

**Proof.** See Appendix C.

It is easy to verify that the entrant obtains a higher profit flow with technology \( H \) than with technology \( L \). This immediately follows from comparing \( \pi_H^E \) and \( \pi_L^E \) determined in Lemma 1 and Lemma 2, respectively. For all \( q_H \) and \( q_L \), we have \( \pi_H^E > \pi_L^E \). However, the entrant may sometimes end up adopting technology \( L \) as the adoption cost of technology \( H \) is higher than technology \( L \) (i.e., \( a_H > a_L \)).
Lemma 3 The incumbent obtains higher profit flows when it competes on the basis of services than when it competes on the basis of facilities.

Proof. See Appendix D.

However, for the ultimate preference of the incumbent for unbundling, we can not make a conclusive statement yet. Intuitively, the incumbent obtains a higher profit with service-based competition as the competing services have the same quality and the incumbent obtains additional revenues from the leased lines that are non-existent in facility based competition.

Up to this point, we have looked for the equilibrium profit flows for given technology choice of the entrant. In the next section we study the optimal date of adoption for each technology, both for cases of with and without unbundling.

4 Adoption of New Technology

In our setting, regardless of whether the local loop is unbundled or not, the entrant eventually builds its own facility. However, the date of adoption and the technology type to be adopted may change with the rental price. Indeed, we show that when the entrant leases local loops prior to its technology adoption, the adoption date is retarded compared to the case in which local loops are not available for lease. This is not surprising, and similar types of ‘replacement effect’ have already been mentioned in several studies.\(^\text{12}\) Furthermore, we show that when the entrant decides to lease loops, the optimal choice of technology type may differ compared to the case when there is no unbundling. In particular, we show that the entrant who chooses to adopt technology \(L\) when there is no unbundling, may choose to adopt technology \(H\) if it leases lines prior to technology adoption. This is because the cost of adopting both technologies approaches zero as time approaches infinity. We know that technology \(H\) yields a higher profit flow to the entrant compared to technology \(L\). Hence, if unbundling sufficiently retards technology adoption, it makes technology \(H\) more attractive for adoption.

4.1 No Unbundling of the Local Loop

When the local loop is not unbundled or if the entrant chooses not to lease local loops, the entrant maximizes its discounted profits net of the adoption cost, and the problem can be defined as

\[
\max_{\Delta r} \left\{ \Delta r \pi^E_r - \frac{a_r}{2} \Delta_r^2 \right\}.
\]

\(^{12}\)The ‘replacement effect’ has been introduced in the R&D literature by Arrow (1962). It states that an incumbent firm has less incentives to invest in R&D, because by increasing its R&D investment, it hastens its own replacement (see Tirole, 1988). The ‘replacement effect’ in our model is very similar to the ‘replacement effect’ considered in the licensing literature. Indeed, by licensing its technology, an incumbent reduce the entrants’ incentives to innovate (see Gallini, 1984).
Note that, given our assumption on the lower bound of \( a_\tau \), adoption does not occur at \( \Delta_\tau = 1 \). The optimal date of adoption is defined by the first order condition,

\[
\Delta_\tau^* = \frac{\pi_E^*}{a_\tau}.
\]

The discounted net profits of the entrant are

\[
\Pi_E^* = \frac{(\pi_E^*)^2}{2a_\tau},
\]

and the entrant obtains profits equal to

\[
\Pi_E^* = \max \left\{ \frac{(\pi_E^*)^2}{2a_L}, \frac{(\pi_H^*)^2}{2a_H} \right\}.
\]

On the other hand, the incumbent obtains monopoly profits, \( \pi_I^M \), until the time the new technology is adopted by the entrant, \( \Delta_\tau^* \), and obtains \( \pi_I^* \) thereafter. Hence, the discounted profit of the incumbent is

\[
\Pi_I^* = (1 - \Delta_\tau^*) \pi_I^M + \Delta_\tau^* \pi_I^*.
\]

**Lemma 4** If there is no unbundling, the entrant adopts technology \( H \) if \( a_H/a_L \leq (32q_H/(6 + q_L)^2)^2 \).

**Proof.** The entrant chooses technology \( H \) if and only if \( (\pi_E^H)^2/2a_H \geq (\pi_E^L)^2/2a_L \), which is equivalent to

\[
\frac{a_H}{a_L} \leq \left( \frac{32q_H}{(6 + q_L)^2} \right)^2.
\]

4.2 Unbundled Local Loop

In this section, we show that the type of technology adopted by the entrant is not necessarily the same when the entrant leases loops and when it does not lease loops prior to technology adoption. Hence, we index the technology choice of the entrant with \( \gamma \) when it leases loops prior to technology adoption and with \( \tau \) when it does not (the same choice as if the local loop is not unbundled). We compute the discounted profits for each case.

**Case 1** The entrant adopts a new technology \( \tau, \tau = L, H \) without first competing on the basis of services.

This case is the same as the one where the local loop was not unbundled, hence the equilibrium payoffs are the same as in the previous section,

\[
\Pi_E^* = \max \left\{ \frac{(\pi_E^L)^2}{2a_L}, \frac{(\pi_H^L)^2}{2a_H} \right\},
\]

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and
\[ \Pi_I^e = (1 - \Delta^*_\gamma) \pi_I^{H*} + \Delta^*_\gamma \pi_I^f. \]

**Case 2** The entrant leases local loops, and then adopts a new technology \( \gamma \) (S + H or S + L) at date \( \Delta^*_S \).

If the entrant decides to lease a loop prior to technology adoption, during \((1 - \Delta^*_S)\) it obtains \( \pi_E^S(r) \) from service-based competition. Then, when it adopts the new technology, \( \gamma \), it obtains \( \pi_E^\gamma \). Therefore, given its technology choice, it has the following problem
\[
\max_{\Delta^*_S} \Pi_E^{S+\gamma} = \max \left\{ \Delta^*_\gamma \pi_E^\gamma - \frac{a_\gamma}{2} \left( \Delta^*_S \right)^2 + (1 - \Delta^*_\gamma) \pi_E^S(r) - f \right\}.
\]

The first order condition is
\[
\pi_E^\gamma - a_\gamma \Delta^*_S - \pi_E^S(r) = 0,
\]
which yields the following optimal date of adoption
\[
\Delta^*_{S\gamma}(r) = \frac{\pi_E^\gamma - \pi_E^S(r)}{a_\gamma}.
\]
The entrant chooses technology H if and only if
\[
\frac{(\pi_E^H - \pi_E^S(r))^2}{a_H} \geq \frac{(\pi_E^L - \pi_E^S(r))^2}{a_L},
\]
which is equivalent to
\[
a_H \leq \frac{(\pi_E^H - \pi_E^S(r))^2}{(\pi_E^L - \pi_E^S(r))^2} \equiv K_S(r).
\]
Note that this decision depends on the rental price \( r \).

Equilibrium profits of the firms are
\[
\left( \Pi_E^{S+\gamma}(r) \right)^* = (\Delta^*_\gamma) \pi_E^\gamma - \frac{a_\gamma}{2} \left( \Delta^*_S \right)^2 + (1 - \Delta^*_\gamma) \pi_E^S(r) - f,
\]
and
\[
\left( \Pi_I^{S+\gamma}(r) \right)^* = (1 - \Delta^*_{S\gamma}(r)) \pi_I^S(r) + \Delta^*_{S\gamma}(r) \pi_I^f.
\]

**Lemma 5** The entrant may adopt a different technology when it leases loops prior to technology adoption and when it does not: given that the entrant adopts L when it does not lease loops, for \( a_H \) sufficiently small, there exists \( \tilde{r} \in (v - 5/4, v) \) such that if \( r < \tilde{r} \) the entrant adopts technology H instead of L if it leases loops.
Proof. See Appendix E. ■

This lemma shows that the choice of technology when the local loop is unbundled depends on the rental price.

For the rest of the paper, we focus on the cases in which the entrant adopts the same technology whether it leases loops or does not, prior to technology adoption ($\gamma = \tau$ for all $r$). This is the case either if the entrant always prefers technology L, which is satisfied if

$$a_H > a_L K S(0),$$

or if it always prefers technology H, which is satisfied if

$$a_H < a_L \left(\frac{\pi^H}{\pi^L}\right)^2.$$

Concentrating on those cases simplifies the analysis when we study the incumbent’s unbundling strategy and the social welfare. The results we obtain in the following hold with technology distortion, with different conditions.

We proceed by studying the incentives of the entrant to rent local loops prior to technology adoption.

**Lemma 6** There exists a threshold $\tau$ such that for all $r > \tau$, the entrant does not rent local loops before adopting a new technology.

**Proof.** With unbundling, the optimal adoption date is

$$\Delta_{\tau}^{S*}(r) = \frac{\pi^E - \pi^S(r)}{a_\tau}.$$

The entrant chooses to rent loops before building its own infrastructure, if and only if it gets higher profit when it leases lines than when it does not:

$$\left(\frac{\pi^E - \pi^S(r)}{2a_\tau}\right)^2 + \pi^S(r) - f > \left(\frac{\pi^E}{2a_\tau}\right)^2,$$

or

$$f < \frac{\pi^E(r)}{2a_\tau} \left(\frac{\pi^S(r) - 2\pi^E + 2a_\tau}{\pi^E(r)}\right),$$

$$f < \left(1 - \frac{\Delta_{\tau}^{S*}(r) + \Delta_\tau^*}{2}\right)\pi^E(r). \tag{1}$$

We have $\partial \pi^S_E / \partial r \leq 0$, hence $\partial \Delta_{\tau}^{S*}(r) / \partial r \geq 0$ and $\left(1 - \frac{\Delta_{\tau}^S(r) + \Delta_\tau^*}{2}\pi^E_S(r)\right)$ decreases with $r$. Therefore, there exists a rental price $\tau$ such for all $r > \tau$, (1) is violated, hence the entrant does not lease the loop. If (1) is violated for $r = 0$, we have $\tau < 0$. The entrant leases lines at a given $r = 0$, if

$$f < \frac{1}{2} + \frac{1}{8a_\tau} - \frac{\pi^E}{2a_\tau},$$

which is satisfied by assumption on $f$. 

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As leasing loops at $r = 0$ alone yields positive profits for the entrant, the threshold $\tau$ is characterized by $\tau \in (v - 5/4, v - 3/4)$. We show this with the following. If $(1 - \Delta^S(0)) \pi^S_E(0) < f$, we have $\tau < 0$, and thus, with $r \geq 0$ the entrant never leases the loop. If $(1 - \Delta^S(0)) \pi^S_E(0) > f$, we have $\tau \geq v - 5/4$.

To see that, assume that $\tau \in [0, v - 5/4)$. For any $r$ such that $r \in (\tau, v - 5/4)$, the entrant gets negative profits from service based competition, i.e.,

$$(1 - \Delta^S(r))\pi^S_E(r) < f.$$ 

Since $\Delta^S(x)$ and $\pi^S_E(x)$ are constant for all $x \leq v - 5/4$, the above equation is true for all $\tau \in [0, v - 5/4)$. Therefore, we have $\tau \leq 0$, which establishes a contradiction. Finally, as $f > f$, the entrant gets negative profits from service based competition for all $r > v - 3/4$, hence $\tau < v - 3/4$.

As the adoption date of the new technology differs when the entrant leases lines and when it does not, a relevant question is whether unbundling accelerates or retards adoption.

**Lemma 7** Whenever the entrant leases loops, i.e., for any given $r$ such that $r < \tau$, technology adoption when there is unbundling is later than when there is no unbundling.

**Proof.** Remember that when there is no unbundling, the optimal date for adopting technology $\tau$ is $\Delta^*_\tau = \pi^E/\alpha$. On the other hand, when the local loop is unbundled at a rate $r < \tau$ (so that the entrant leases loops prior to its new technology adoption), the optimal adoption date is $\Delta^S(r) = (\pi^E_E - \pi^S_E(r))/\alpha$.

Then, we have $\Delta^*_\tau > \Delta^S(r)$, i.e., the new technology is adopted earlier when there is no unbundling. The delay introduced by unbundling is

$$\Delta^*_\tau - \Delta^S(r) = \frac{\pi^S_E(r)}{\alpha}.$$ 

Notice that this delay decreases with the rental rate, $r$. This is because the opportunity cost of adopting technology is decreasing with $r$. The delay of technology adoption due to unbundling reflects a "replacement effect"; it takes more time for the entrant to replace its own technology compared to the case where it adopts a technology without having been operating in the relevant market (the higher the profits obtained by service-based competition, the later the new technology is adopted). We also showed the possibility of another replacement effect in Lemma 5, which shows that unbundling may distort the technology choice of the entrant.

Lemma 6 and Lemma 7 represent the trade-off the incumbent faces for its decision on the rental price. Lemma 6 implies that the incumbent might charge a high rental price in order to increase its profits up to the monopoly level, prior to adoption. On the other hand, Lemma 7 implies that it may charge a low rental price in order to delay competition from the new technology.
5 Decision for unbundling and the rental rate of the local loop

We have analyzed the entrant’s strategy for technology adoption given the decision for unbundling and the rental price $r$ for the local loop. Moving backwards, we now study the incentives of the incumbent to unbundle the local loop, and the choice of $r$ if it decides to unbundle it.

**Lemma 8** The incumbent prefers to lease its loops at $r = v - 5/4$ to no unbundling.

**Proof.** The incumbent prefers unbundling at $r = v - 5/4$ to no unbundling if and only if

$$\pi_I^S (1 - \Delta_S^S(r)) + \Delta_S^S(r) \pi_I^T \geq \pi_I^M (1 - \Delta_r) + \Delta_r \pi_I^T,$$

or

$$(\Delta_r - \Delta_S^S(r)) (\pi_I^S - \pi_I^T) \geq (1 - \Delta_r) (\pi_I^M - \pi_I^T).$$  (2)

The above inequality can be interpreted as follows. The left-hand side represents the additional profit that the incumbent makes when it leases lines. Due to retarded technology adoption and high service-based profits, the incumbent makes $\pi_I^S = v - 5$ during $\Delta_r - \Delta_S^S(r)$. The right-hand side represents the additional profit the incumbent makes when it does not lease lines. Due to higher (monopoly) profits, the incumbent gets $\pi_I^M = v - 3$ during $1 - \Delta_r$. Now, remark that $\pi_I^S - \pi_I^T$ increases with $v$, as $\pi_I^S = v - 3/4$, while $\pi_I^M - \pi_I^T$ does not depend on $v$. $\Delta_r$ and $\Delta_S^S(r = v - 5/4)$ do not depend on $v$ either. Therefore, when $v$ increases, the incumbent finds it more profitable to lease lines, as it increases its profit until the time that the entrant adopts the new technology $(\Delta_r - \Delta_S^S(r))$. Replacing the relevant values for $\pi_I^S (r = v - 5/4)$ and $\pi_I^M$, and rearranging inequality (2) yields the following condition on $a_r$

$$a_r \leq v - 3/4 - \pi_I^T + \pi_E^T,$$

which is satisfied by given the assumption on the upper bound for $a_L$ and $a_H$.

From Lemma 6, we know that $r \in (v - 5/4, v - 3/4)$. Therefore, the entrant leases lines at $v - 5/4$. Now, it remains to be checked whether this rental price yields the global maximum of the incumbent’s profit function.

**Proposition 1** If $a_r < v - 5/4 - \pi_I^T + \pi_E^T$, with $\tau = L, H$, in equilibrium the incumbent leases its loops at $r^* = v - 5/4$, and the entrant leases loops prior to its technology adoption.

**Proof.** First, note that the incumbent’s discounted profit function increases with $r$ when $r < v - 5/4$. Indeed, the entrant’s optimal adoption date when it leases lines prior to adoption is constant over that range, while the incumbent’s profit flow under service-based competition increases with $r$. 

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Second, the incumbent’s discounted profit function decreases with \( r \) when \( r \in [v - 5/4, \tau] \) if

\[
a_r < v - 5/4 - \pi^I + \pi^E.
\]

Third, remember that the entrant never leases lines when \( r \geq \tau \), hence the incumbent’s discounted profit is constant when \( r \geq \tau \). From Lemma 8, we know that the incumbent leases lines at \( r^* = v - 5/4 \). Therefore, \( r^* = v - 5/4 \) is the global maximum of the incumbent’s discounted profit function. Furthermore, the entrant leases lines at \( r^* \), as \( r^* < \tau \).

When the incumbent leases its local loops, it faces a trade-off between charging a low rental rate which delays the entrant’s technology adoption, and a high rental rate which increases its revenues. Since the entrant’s adoption date is constant when \( r \in [0, v - 5/4] \), the incumbent has an incentive to increase its rental rate up to \( v - 5/4 \). When \( r \geq v - 5/4 \), increasing the rental rate \( r \) accelerates the entrant’s technology adoption.

Then, when \( a_r \) is sufficiently low, the entrant is likely to adopt a new technology \( \tau \) relatively early, hence, the incumbent is willing to lease its lines in order to delay facility-based competition. Therefore, Proposition 1 implies that if \( a_r \) is sufficiently low (so that the entrant has incentives to adopt a new technology at a relatively earlier date), the incumbent sets an attractive rental price for its loops in order to delay facility-based competition. Intuitively, when \( a_r \) is sufficiently high (so that the incumbent does not expect any facility-based entry to the market in the near future), it chooses not to unbundle -or sets too high a price-, and enjoys monopoly profits.

Finally, remember that we have normalized the discount rate to one. It is clear that whenever the incumbent discounts the future at a very low rate, the threat to its future profits which is driven by facility-based competition becomes less severe. Hence, it may prefer not to unbundle, and to remain as a monopolist until the time the entrant shows up with a new technology.

6 The Social Optimum and Regulatory Tools

In this section, a social welfare maximizing regulator is introduced in order to compare the unregulated and the socially efficient outcomes. The regulator maximizes the social welfare which is defined by the sum of consumers’ surplus and industry profits. In the following, we consider regulation of the rental path as the only regulatory tool. Hence, we assume that the regulator does not control the final prices. We also investigate the case for sunset clauses as a regulatory tool to achieve socially desirable outcomes.

6.1 The Social Optimum

To begin with, we study the optimal rental price for consumers. When the entrant leases lines prior to technology adoption, discounted consumer surplus is

\[
S^{S+\tau} = (1 - \Delta^S_\tau (r)) s (r) + \Delta^S_\tau (r) s_r,
\]
where $s(r)$ denotes consumer surplus under service-based competition and $s_\tau$ denotes consumer surplus under facility-based competition, with

$$s(r) = \begin{cases} v - 13/12 - r & \text{if } r \in [0, v - 5/4) \\ 1/6 & \text{if } r \in [v - 5/4, \tau) \end{cases},$$

$$s_L = v - 13/12 + qL/4 + (qL)^2 / 36,$$

and

$$s_H = v - 1/12 - qH/9.$$

When the entrant does not lease lines, discounted consumer surplus is $S^\tau = (1 - \Delta_\tau) s_M + \Delta_\tau s_\tau$, where $s_M$ denotes consumer surplus under monopoly, with $s_M = 1/6$.

**Lemma 9** There exist $\bar{v}$ such that if $v > \bar{v}$, consumer surplus maximizing rental price is set at the marginal cost, i.e., zero. Otherwise, consumers prefer no unbundling.

**Proof.** If the entrant leases lines prior to technology adoption, discounted consumer surplus is equal to (3). If $r \in [0, v - 5/4)$, $\Delta_\tau^S(r)$ and $s_\tau$ are constant while $s(r)$ decreases with $r$. Therefore, $S^{S+\tau}$ decreases with $r$.

If $r \in [v - 5/4, \tau]$, then $s(r)$ and $s_\tau$ are constant, while $\Delta_\tau^S(r)$ increases with $r$. Discounted consumer surplus is given by

$$S^{S+\tau} = \Delta_\tau^S(r) (s_\tau - s(r)) + s(r),$$

and $s(r)$ is constant over the range considered. We find that $s_L > 1/6$ and $s_H > 1/6$ as $q_H < 3$, and hence for all $q_E$, we have $q_E < 9(v - 1/4)$. Therefore, $S^{S+\tau}$ increases with $r$ when $r \in [v - 5/4, \tau]$. Furthermore, remember that the entrant does not lease lines when $r > \tau$, hence consumer surplus is constant over that range. Finally, note that $S^{S+\tau}$ increases with $r$ when $r > v - 5/4$ up to $r = v - 1/4$, where $S^{S+\tau} = S^\tau$.

Therefore, we have two cases. Discounted consumer surplus is maximized either at rental rate $r^{cw} = 0$ or at any rate $r^{cw} > \tau$ (so that the entrant does not lease loops). Consumer surplus under facility based competition can be written as

$$s_\tau = v - x_\tau,$$

with

$$x_\tau = \begin{cases} 13/12 - qL/4 - q_H^2 / 36 & \text{if } \tau = L \\ 1/12 + qH/9 & \text{if } \tau = H \end{cases}.$$

Consumer surplus is higher at $r^{cw} = 0$ than it is at $r > \tau$ if and only if $v > \bar{v}$, where $\bar{v}$ is defined such that the below inequality holds as equality.

$$(v - 13/12) (1 - \Delta_\tau^S(0)) + \Delta_\tau^S(0) (v - x_\tau) \geq (1 - \Delta_\tau) / 6 + \Delta_\tau (v - x_\tau).$$

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If \( v \geq \tilde{v} \), consumer surplus is maximized at rental rate \( r_{cw} = 0 \); otherwise, it is maximized at any rate \( r_{cw} > \tau \), so that there is no service based competition.

Now, we determine the optimal rental price for the industry. Industry profits increase with the rental rate under service-based competition. However, as long as the rental rate distorts the adoption date (the entrant adopts a new technology earlier when the rental rate is high), the adoption cost is higher, which lowers the profits of the entrant. Furthermore the incumbent’s profit also decreases as it earns lower profits when there is facility-based competition.

**Lemma 10** Industry profits are maximized at \( r^* = v - 5/4 \).

**Proof.** When the entrant leases lines at rental rate \( r \) prior to its technology adoption, industry profits are

\[
\Pi_I + \Pi_E = (1 - \Delta_E^S (r)) (\pi_I^S (r) + \pi_E^S (r)) + \Delta_E^S (r) (\pi_I^S + \pi_E^S) - a_r \Delta_E^S (r)^2 / 2 - f.
\]

When \( r \in [0, v - 5/4] \), industry profits increase with \( r \), as in this range \( \Delta_E^S (r) \) is constant, and \( \pi_I^S (r) + \pi_E^S (r) \) increases with \( r \). When \( r \in (v - 5/4, \tau) \), we find that

\[
\partial (\Pi_I + \Pi_E) / \partial r = - (4v - 1 - 8\pi_I^3 + 4r) / 16a_r.
\]

One can verify that industry profits decrease with \( r \) when \( r \in (v - 5/4, v - 3/4) \), as \( v > \pi_I^3 + 3/4 \) holds always (we have \( \pi_I^3 \leq 1/2 \), and \( v > 3 \)).

Finally, \( (\Pi_I + \Pi_E) \) is constant when \( r > \tau \), as the entrant does not lease lines. We find that industry profits are higher at \( r^* = v - 5/4 \) than when there is no unbundling if

\[
f < \frac{2(v - \pi_I^3) - 1}{4a_r}.
\]

Note that right hand side of condition (4) decreases with \( a_r \). Now, remember that the entrant leases lines at zero rental price if \( f < 1/2 + (1 - 4\pi_I^3) / (8a_r) \). Since \( \pi_I^3 > 1/4 \), the right hand side of this latter condition increases with \( a_r \). Finally, remark that \( 1/2 + (1 - 4\pi_I^3) / (8a_r) \) and \( (2(v - \pi_I^3) - 1) / (4a_r) \) cross at \( a_r = v - 3/4 + q_r/3 \), which is the upper bound of \( a_r \). Hence, as we have assumed that the entrant is willing to lease lines at zero rental price, \( r^* = v - 5/4 \) maximizes industry profits.

Finally, we determine the socially optimum rental price. When the entrant leases lines prior to technology adoption, social welfare is

\[
W^{S+\tau} = (1 - \Delta_r^S (r)) w_S (r) + \Delta_r^S (r) w_\tau - \frac{a_r}{2} (\Delta_r^S (r))^2 - f, \tag{5}
\]

and when the entrant does not lease lines, discounted social welfare is

\[
W^\tau = (1 - \Delta_r) w_M + \Delta_r w_\tau - \frac{a_r}{2} (\Delta_r)^2, \tag{6}
\]

where \( w_M, w_S (r) \), and \( w_\tau \) denote social welfare under monopoly, service based competition and facility based competition, respectively. Values of \( w_M, w_S (r) \), and \( w_\tau \) can be found in the Appendix, in the proof of Proposition 2.
Proposition 2 The socially optimum rental price is higher than the price charged by the incumbent.
Proof. See Appendix F.

When unbundling results in no distortion at the technology choice and if unbundling is socially desirable, the social welfare maximizing rental price is

$$r_{sw} = \begin{cases} \min \{v - 5/4 + qL/6 + q_L^2/12, r\} & \text{if } \tau = L \\ r - \epsilon & \text{if } \tau = H \end{cases},$$

where $\epsilon$ is small.

Therefore, the social welfare-maximizing rental price is greater than the rental price the incumbent would charge, as $r_{sw} > r^*$ is always true. The intuition is the following. When $r \geq v - 5/4$, consumer surplus and industry profits under service-based competition do not depend on $r$; the two firms act as two local monopolies and extract all surplus from marginal consumers. Therefore, the social welfare function depends only on the adoption date of the new technology. Recall that $\Delta^S(r) = (\pi^E - \pi^S(r)) / a^S$ increases with $r$, because the greater $r$, the lower the replacement effect is. Proposition 2 shows that the entrant tends to adopt the new technology too late from a welfare point of view, and that it is socially efficient to increase rental price of loops in order to accelerate facility-based competition.

Now, it remains to determine whether unbundling is desirable or not. To that end, we have to compare social welfare with and without unbundling.

Proposition 3 Unbundling the local loop is never desirable when the entrant envisions to adopt technology $H$, while it may be desirable when the entrant envisions to adopt technology $L$.

Proof. First, let $\tau = H$. If the entrant does not lease lines prior to adoption, discounted social welfare is given by equation (6), which yields

$$W^H = (1 - \Delta^H_H) \left( v - \frac{1}{12} \right) + \Delta^H_H \left( v - \frac{1}{12} + \frac{4qH}{9} \right) - \frac{a_H}{2} \left( \Delta^H_H \right)^2,$$

with $\Delta^H_H = \pi^H_H / a^H$. If the entrant leases lines prior to adoption and $r \in (v - 5/4, v - 3/4)$, discounted social welfare is given by equation (5), which yields

$$W^{S+H} (r) = (1 - \Delta^S_H (r)) \left( v - \frac{1}{12} \right) + \Delta^S_H (r) \left( v - \frac{1}{12} + \frac{4qH}{9} \right) - \frac{a_H}{2} \left( \Delta^S_H (r) \right)^2,$$

with $\Delta^S_H = (\pi^H_H - \pi^S (r)) / a^H$. We know from the proof of Proposition 2 that $W^{S+H}$ increases with $r$ when $r \in (v - 5/4, v - 3/4)$. Therefore, we compare $W^H$ to $W^{S+H} (v - 3/4)$. We find that $W^{S+H} (v - 3/4) > W^H$ if and only if...

\[\text{13 There is no welfare loss because demand is (relatively) inelastic. Indeed, remember that consumers buy zero or one access line.}\]
\[ f < (32a_H). \] Therefore, since \( r < v - 3/4 \) and \( f > 0 \), it is never socially desirable to unbundle the local loop when the entrant envisions to adopt technology \( H \).

When \( r = L \), we use the same reasoning. We compare \( W_L \) to \( W^{S+H}(r) \) at \( r = v - 5/4 + q_L/6 + q_L^2/12 \). This comparison shows that it is socially desirable to unbundle the local loop if and only if

\[ f < \left( q_L^4 + 4q_L^3 - 20q_L^2 + 48q_L + 144 \right) / (1152a_L). \] (7)

Let

\[ K = \left( q_L^4 + 4q_L^3 - 20q_L^2 + 48q_L + 144 \right) / (1152a_L). \]

We want to determine whether there exists some \( f \) such that \( f > f \) and \( f < K \). Therefore, we have to check whether \( f < K \). First, note that \( 0 < K < 144 / (1152a_L) \) for all \( q_L \) and \( a_L \). Second, note that \( 144 / (1152a_L) \) decreases with \( a_L \). Therefore, we compare \( f \) and \( K \) when \( a_L \) is at its minimum. We find that \( 144 / (1152a_L) > (v - 1) / (4v - 3) \) for all \( v \) when \( a_L = (6 + q_L)^2 / 72 \).

Therefore, for \( q_L \) very low, there exists some values of \( a_L \) and \( f \) such that unbundling is desirable.

This result shows that the incumbent has stronger incentives to unbundle its local loop than a social welfare maximizing regulator. Indeed, while the incumbent is willing to lease lines at a relatively low price, a welfare maximizing regulator might choose not to unbundle the local loop. The intuition is that the quality improving innovation threatens the incumbent’s position, whereas it increases social welfare. Therefore, the incumbent is willing to delay adoption by charging a lower rental price, while the regulator is willing to hasten adoption by setting a high rental price. The same reasoning explains why the regulator never unbundles when the entrant envisions to adopt technology \( H \), and might choose to unbundle when the entrant envisions to adopt technology \( L \). Indeed, remark that technology \( H \) provides higher social welfare than technology \( L \).

However, note that our welfare analysis ignores some aspects of unbundling. For instance, it may be socially desirable to unbundle the local loop to prevent incumbent operators from preempting the market for high bandwidth services.

### 6.2 A Regulatory Tool: Sunset Clauses

Subsection 6.1 dealt with the determination of the socially optimal rental price. Another important supply condition is the timing of introduction of local loops for leasing. Sunset clauses specify ex ante a period of time after which the incumbent’s facilities are no longer regulated. Sunset clauses have been specified in the unbundling regulations in Canada and in the Netherlands. For example, Opta, the Dutch regulatory authority, has specified a five-year period after which the incumbent operator, KPN Telecom, would be “in principle, free to set a tariff on a commercial basis”. Similarly, the Canadian Radio-Television and Telecommunications Commission issued a decision (CRTC-97-8), which stated

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14See Guidelines on access to the unbundled loop, March 1999.
that following a five-year mandatory unbundling, the incumbent’s services and components that are deemed to be essential facilities (including local loops in certain areas) would not be subject to mandatory unbundling and the rental rate would not be regulated any longer. In March 2001, CRTC has extended this sunset period without specifying a termination date. The motivation behind these sunset clauses is to provide the entrants with incentives to build their own facilities. In this respect, deregulating the rental rate is assumed to render leasing lines an unattractive option to the entrant.

However, we show that in our setting, sunset clauses do not enhance the incentives of the entrant to build up its own infrastructure. To validate the assertion, we start by computing the socially optimum adoption date. Since unbundling is never desirable when the entrant adopts technology $H$ after leasing lines, we only consider technology $L$. Furthermore, we assume that $f$, $a_L$, and $q_L$ are such that condition (7) is satisfied, i.e., unbundling is socially desirable when the entrant adopts technology $L$ after leasing lines. When the entrant leases lines, and then adopts technology $L$ at date $\Delta$, total discounted welfare is

$$ W = (1 - \Delta) w_S + \Delta w_L - \frac{a_L}{2} \Delta^2 - f, $$

hence the socially optimal adoption date is

$$ \Delta^{sw}_L = \frac{w_L - w_S}{a_L}. $$

**Proposition 4** Assume that the entrant adopts technology $L$ after leasing lines and that condition (7) is satisfied. If the regulator sets the rental price at the socially optimum level, then the technology adoption date of the entrant is either socially optimal or too late.

**Proof.** See Appendix G. □

This proposition implies that when it is socially desirable to unbundle the local loop, regulating the rental price is sufficient to maximize welfare if the entrant leases lines at the social welfare maximizing rental price, $r^{sw} = v - 5/4 + q_L/6 + q_L^2/12$. In that case, introducing a sunset clause does not improve social welfare. When the social welfare maximizing rental price is $r^{sw} = r - \epsilon$, adoption occurs too late from a social point of view. However, sunset clause does not improve welfare neither in this case. To see why, let $\Delta > \Delta^{sw}_L (r^*)$ be the date from which the local loop will not be regulated any longer, and let $r^{uw}$ be the regulated rental price. Discounted welfare is given by

$$ (1 - \Delta) w_S + (\Delta - \Delta^*) w_S + \Delta_L w_L - \frac{a_L}{2} \Delta^2 L - f. $$

During $(1 - \Delta)$, the entrant leases lines at the regulated price $r^{uw}$. Once the sunset clause applies, the entrant leases loops at the unregulated rental price, $r^{uw}$.

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15 See Order CRTC 2001-184.
Finally, once it adopts technology $L$, it obtains $\pi_L^E$. Therefore, its discounted profit is

$$(1 - \Delta) \pi_L^S(r^{sw}) + (\Delta - \Delta_L) \pi_L^S(r^u) + \Delta_L \pi_L^E - \frac{a_L}{2} \Delta_L^2 - f.$$  

(8)

The incumbent’s discounted profit is

$$(1 - \Delta) \pi_I^S(r^{sw}) + (\Delta - \Delta_L) \pi_I^S(r^u) + \Delta_L \pi_I^E.$$  

A similar analysis as the one we used in Proposition 1 shows that $r^u = r^* = v - 5/4$. The entrant chooses the date of adoption, for given $r^{sw}$, $r^u$, and $\Delta$. The maximization of (8) yields the following adoption date

$$\left(\Delta_E^S\right)^* = \frac{\pi_I^E - \pi_E^S(r^u)}{a_L}.$$  

Note that the adoption date of the entrant neither depends on the regulated price, $r^{sw}$, nor on the sunset clause, $\Delta$. It depends solely on the unregulated rental rate, $r^u$. As $r^u = r^*$, the entrant adopts at the same time as in the unregulated case. Therefore, the regulator sets $r^{sw}$ and $\Delta$, but it cannot influence $\left(\Delta_E^S\right)^*$ whenever $\Delta > \left(\Delta_E^S\right)^*$. As we know that the entrant adopts the new technology too late from a social point of view when the rental price is not regulated, the sunset clause does not improve welfare.

7 Conclusion

This paper provides a formal analysis of the effects of unbundling on the investment decisions for alternative technologies to the traditional loops. Our analysis suggests that the incumbent, as the owner of the loops, has an incentive to provide attractive terms for lease, and hence sets a low rental price. By doing so, it delays the date of entrant’s adoption of a new and better technology. Although we did not consider the possibility of entry deterrence, a slight modification in the adoption cost function can extend the analysis in that direction. Adding a fixed component to the adoption cost would give rise to a deterred entry of a new technology if the rental price is set sufficiently low.

The incumbent faces the following trade off regarding its pricing decision: If it sets the price too high, which is tantamount to not unbundling, it can maintain its monopoly profits, at least in the short run. However, in this case the entrant will face no replacement effects and will adopt a new technology at an earlier date. The incumbent will face fiercer competition than it would had it leased its loops to the entrant. On the other hand, if the consumer valuation for the high bandwidth services is sufficiently high, the incumbent has no incentives to give away its loops at a very low price as its rental revenues and the benefits from retarding the fierce competition should balance off the loss from foregone high monopoly profits. It is important to note that, along with the standard replacement effect on the entrant decision for the date of adoption, there may also be a distortion on the type (quality) of technology to be adopted.
From a social welfare point of view the main trade off is clear; service-based competition through unbundling is good since it promotes immediate competition, and it increases the variety of services. On the other hand, facility-based entry brings forth a better quality of service, and a greater flexibility for the entrant for customer targeting. The regulator who is concerned with promoting facility-based competition should regulate the rental price of the loops. It may also choose to determine the duration of the lease contract if the rental price is unregulated. However in our setting, a sunset clause, another regulatory tool that has been claimed to improve socially desirable outcomes, does not improve social welfare. Sunset clauses are expected to give entrants an incentive to invest in alternative technologies as the regulators commit not to regulate the rental price after the clause. As we have shown that the incumbent sets a lower price than what is socially efficient, and not a higher price, a sunset clause does not put a pressure on the entrant for technology adoption.

References


A Appendix

Determination of the lower bound of the fixed cost. Let $\psi_H = (1 - \Delta_H) \pi_H^E$, with

$$\Delta_H = \frac{\pi_H^E - \pi_S^E}{a_H}.$$ 

We show that when

$$f > \frac{18v - 13}{72v - 6},$$

we have $\psi_H < f$, i.e., the entrant has a negative profit flow in case of service based competition, for $r > v - 3/4$.

First, note that $\frac{\partial \psi_H}{\partial \pi_S^E} > 0$, as

$$\frac{\partial \psi_H}{\partial \pi_S^E} = (1 - \Delta_H) + \frac{\pi_S^E}{a_H}$$

and $\Delta_H < 1$. Second, note that as $\pi_S^E$ decreases with $r$, $\psi_H$ also decreases with $r$. Therefore, if $\psi_H < f$ is true for $r = v - 3/4$, then it is also true for all $r > v - 3/4$. Replacing for $r$ in $\psi_H$, we find

$$\left(1 - \frac{4q_H}{9 - (2\sqrt{3}/9(v - (v - 3/4)^{3/2})} \right) \left(2\sqrt{3}/9(v - (v - 3/4)^{3/2}) \right) < f,$$

which simplifies to

$$\frac{1}{4} + \frac{9 - 16q_H}{144\alpha_H} < f.$$

As $9 - 16q_H < 0$ for all $q_H \in (2, 3)$, the left hand side of the above inequality is increasing in $a_H$. Remember that we have assumed $a_H < v - 3/4 + q_H/3$. Hence if the above inequality holds for $a_H = v - 3/4 + q_H/3$, it holds for all $a_H$. Replacing for $a_H$, we find

$$\frac{18v - 9 - 2q_H}{6(12v - 9 + 4q_H)} < f.$$

Observe that the left hand side of the above inequality decreases with $q_H$, which implies that if the inequality is satisfied for $q_H = 2$, then it is satisfied for all $q_H$. Replacing for $q_H$, we derive the following condition

$$f > \frac{18v - 13}{72v - 6}.$$

Same reasoning applies for the technology $L$, and we derive the following condition

$$f > \frac{v - 1}{4v - 3}.$$

We have assumed that $v > 3$, hence, it is easy to verify that

$$\frac{18v - 13}{72v - 6} < \frac{v - 1}{4v - 3}.$$
Hence, the binding condition is the one we have derived for technology $L$, and hence, for any given $r > v - 3/4$, the entrant obtains a negative profit flow when it competes on the basis of services if

$$f > \frac{v - 1}{4v - 3} \equiv \frac{1}{4}.$$

Note that $f$ increases with $v$ and that $f < 1/4$ for all $v$. Furthermore, we have $\bar{f} > f$. Indeed, let us assume that

$$\bar{f} = \frac{1}{2} + \frac{1}{8a_L} - \frac{(6 + q_L)^2}{144a_L}.$$

We have $\bar{f} > f$ if and only if

$$\frac{1}{2} + \frac{1}{8a_L} - \frac{(6 + q_L)^2}{144a_L} > \frac{v - 1}{4v - 3}.$$

Since $f$ increases with $v$ and goes to $1/4$ when $v$ goes to the infinite, it is sufficient to show that

$$\frac{1}{2} + \frac{1}{8a_L} - \frac{(6 + q_L)^2}{144a_L} > \frac{1}{4},$$

which is equivalent to

$$a_L > \frac{(6 + q_L)^2}{36} - \frac{1}{2},$$

which is the lower bound for $a_L$. The same analysis applies when $\bar{f} = 1/2 + 1/(8a_H) - 2q_H/(9a_H)$.

**B Appendix**

**Computation for Service Based Competition.** We proceed in three steps. We first derive the profit functions. Then, we determine the reaction functions. And finally, we solve for the Nash equilibrium of the game.

**Step One:** We start by deriving the profit function of firm $i \in \{I, E\}$ for any price charged by firm $j \neq i$, $\pi_i(p_i \mid p_j)$. Notice that the demands for the two firms overlap only when $p_i \in (p_j - 1, p_j + 1)$. First, assume that $p_j \geq v$, then $\pi_i(p_i \mid p_j)$ is independent of $p_j$, as firm $j$ serves no consumer. Second, assume that $p_j < v$, then, the marginal consumer is defined by $x = (p_E - p_I + 1)/2$. The marginal consumer obtains a positive surplus if and only if

$$p_i \leq \overline{p}_i(p_j) \equiv p_j - 1 + 2\sqrt{v - p_j}.$$

If $p_i > \overline{p}_i(p_j)$, firm $i$ and $j$ get the following local monopoly profits

$$\pi^{M}_i(p_I, p_E) = p_I\sqrt{v - p_I} + r\sqrt{v - p_E},$$

where $r \geq 0$.
and
\[ \pi_E^M (p_I, p_E) = (p_E - r) \sqrt{v - p_E}. \]
If \( p_i < \overline{p}_i (p_j) \), firm \( i \) and \( j \) get the following duopoly profits
\[ \pi_I^D (p_I, p_E) = p_I D_I + r D_E, \]
and
\[ \pi_E^D (p_I, p_E) = (p_E - r) D_E, \]
where
\[ D_I = \begin{cases} 
0 & \text{if } \overline{p} \leq 0, \\
(p_E - p_I + 1) / 2 & \text{if } \overline{p} \in (0, 1), \\
1 & \text{if } \overline{p} \geq 1
\end{cases} \]
and \( D_E = 1 - D_I \).

**Step Two:** Now, we can determine the reaction functions of the firms. The reaction function of firm \( i \) is defined as the optimal choice of \( p_i \) given \( p_j \). Let \( p_i^M \) and \( p_i^D \) denote the prices that maximize \( \pi_i^M \) and \( \pi_i^D \), respectively. We find \( p_i^D (p_j) = (p_j + 1 + r) / 2 \), \( p_i^M = v - 1 \), and
\[ p_E^M = \begin{cases} 
(2v + r) / 3 & \text{if } r \geq v - 3, \\
v - 1 & \text{if } r < v - 3.
\end{cases} \]
We start by deriving the reaction function of firm \( I \). We have four possible cases. The optimal price for firm \( I \) is
1. \( p_I^M \) if \( \overline{p}_I (p_E) \leq p_I^M \),
2. \( p_I^D (p_E) \) if \( \overline{p}_I (p_E) > p_I^M \), \( p_I^D (p_E) < \overline{p}_I (p_E) \) and \( p_E - 1 < p_I^D (p_E) \leq p_E + 1 \),
3. \( p_E - 1 \) if \( \overline{p}_I (p_E) > p_I^M \), \( p_I^D (p_E) < \overline{p}_I (p_E) \) and \( p_I^D (p_E) \leq p_E - 1 \),
4. \( \overline{p}_I (p_E) \) if \( p_I^M < \overline{p}_I (p_E) < p_I^D (p_E) \).

To begin with, consider case (1). We find that \( \overline{p}_I (p_E) \leq p_I^M \) if \( p_E > v \). Now, consider cases (2) to (4). First, we look for the conditions for case (2). We find that \( p_I^D (p_E) < \overline{p}_I (p_E) \) if and only if \( p_E < r - 5 + 4\sqrt{v - r + 1} \) and that \( p_I^D (p_E) > p_E - 1 \) if and only if \( p_E < r + 3 \). We have to compare these two conditions. The comparison yields that \( r - 5 + 4\sqrt{v - r + 1} \geq r + 3 \) if and only if \( r \leq v - 3 \). Firm \( I \) gets positive demand when it charges \( p_I^D (p_E) \) if and only if \( p_I^D (p_E) \leq p_E + 1 \), which is satisfied if \( p_E \geq r - 1 \). When \( p_E < r - 1 \), firm \( I \) prefers firm \( E \) to serve all customers and to pay \( r \) for leasing lines than to charge a retail price lower than \( r \).

This analysis shows that when \( r \leq v - 3 \), the optimal price for firm \( I \) is \( p_I^D (p_E) \) if \( p_E \in [r - 1, r + 3] \) and \( p_E - 1 \) if \( p_E > r + 3 \). When \( r > v - 3 \), the optimal price for firm \( I \) is \( p_I^D (p_E) \) if \( p_E \in [r - 1, r - 5 + 4\sqrt{v - r + 1}] \) and \( \overline{p}_I (p_E) \) if \( p_E > r - 5 + 4\sqrt{v - r + 1} \). To summarize, we have two cases. If \( r \in (0, v - 3) \), then
\[ R_I (p_E) = \begin{cases} 
 r & \text{if } p_E \in [0, r - 1) \\
 (p_E + 1 + r) / 2 & \text{if } p_E \in [r - 1, r + 3) \\
 p_E - 1 & \text{if } p_E \in [r + 3, v) \\
 v - 1 & \text{if } p_E \in [v, \infty) 
\end{cases} \]

If \( r \geq v - 3 \), then

\[ R_I (p_E) = \begin{cases} 
 r & \text{if } p_E \in [0, r - 1) \\
 (p_E + 1 + r) / 2 & \text{if } p_E \in [r - 1, r - 5 + 4\sqrt{v - r + 1}] \\
 p_E - 1 & \text{if } p_E \in [r - 5 + 4\sqrt{v - r + 1}, v) \\
 v - 1 & \text{if } p_E \in [v, \infty) 
\end{cases} \]

We proceed the same way to derive the reaction function of firm \( E \). The only difference is that when \( r > v - 3 \), firm \( E \) does not serve all customers when it charges its monopoly price, \( p_E^M = \frac{2v + r}{3} \). When \( r > v - 3 \), firm \( E \) can charge its monopoly price if \( p_I^M > \overline{p}_E (p_I) \), which is satisfied if and only if \( p_I > \frac{(2v + r)}{3} - 1 + 2\sqrt{v - r}/\sqrt{3} \). To summarize, we have two cases. If \( r \in (0, v - 3) \), then

\[ R_E (p_I) = \begin{cases} 
 r & \text{if } p_I \in [0, r - 1) \\
 (p_I + 1 + r) / 2 & \text{if } p_I \in [r - 1, r + 3) \\
 p_I - 1 & \text{if } p_I \in [r + 3, v) \\
 v - 1 & \text{if } p_I \in [v, \infty) 
\end{cases} \]

If \( r \in [v - 3, \infty) \), then

\[ R_E (p_I) = \begin{cases} 
 r & \text{if } p_I \in [0, r - 1) \\
 (p_I + 1 + r) / 2 & \text{if } p_I \in [r - 1, r - 5 + 4\sqrt{v - r + 1}] \\
 \overline{p}_E (p_I) & \text{if } p_I \in [r - 5 + 4\sqrt{v - r + 1}, \frac{(2v + r)}{3} - 1 + 2\sqrt{v - r}/\sqrt{3}) \\
 (2v + r) / 3 & \text{if } p_I \in \left(\frac{(2v + r)}{3} - 1 + 2\sqrt{v - r}/\sqrt{3}, \infty\right) 
\end{cases} \]

**Step Three:** Now, we can determine the equilibrium of the game. First, for all \( r \in (0, v - 3) \), \( p_I^S = p_E^S = 1 + r \) is an equilibrium and it is the unique equilibrium. Second, let us assume that \( r \geq v - 3 \). The competitive equilibrium \((1 + r, 1 + r)\) exists if and only if \(1 + r \in [r - 1, r - 5 + 4\sqrt{v - r + 1}] \), which is satisfied if \( r < v - 5/4 \). There is an equilibrium such that firm \( I \) charges its monopoly price, \( v - 1 \), only if \( v - 1 < r - 1 \), i.e., \( r > v \).

When \( r \in (v - 5/4, v - 3/4) \), there is a corner equilibrium such that the marginal consumer gets zero surplus, i.e., \( p_I^S = p_E^S = v - (1/2)^2 = v - 1/4 \). Indeed, we find that \( v - 1/4 > r - 5 + 4\sqrt{v - r + 1} \) if and only if \( r > v - 5/4 \). Besides, we find that \( v - 1/4 < (2v + r) / 3 - 1 + 2\sqrt{v - r}/\sqrt{3} \) if \( v - 27/4 < r < v - 3/4 \).

Finally, when \( r \in (v - 3/4, v) \), there is an equilibrium such that firm \( E \) charges its monopoly price, \( p_E^M = \frac{2v + r}{3} \) and firm \( I \) charges \( p_I^M \). Indeed, when \( r > v - 3/4 \) and firm \( I \) charges \( p_I^S = v - 1/4 \), the optimal price for firm \( E \) is \( p_E^M = \frac{2v + r}{3} \). The best response of firm \( I \) is then to charge \( \overline{p}_I (p_E^M) = \frac{(2v + r)}{3} - 1 + 2\sqrt{v - r}/\sqrt{3} \). We check that \( R_I (p_E^M) = \overline{p}_I (p_E^M) \), as \( (2v + r) / 3 > r - 5 + 4\sqrt{v - r + 1} \) when \( r > v - 21/2 + 3\sqrt{10} \approx v - 1.01 < \)
We also check that $R_E \left( \pi_I^M (p_E^M) \right) = p_E^M$, as $\pi_I^M (p_E^M) = (2v + r) / 3 - 1 + 2\sqrt{v-r}/\sqrt{3}$.

To summarize, for $r \leq v - 5/4$ we have a competitive equilibrium; for $r \in (v - 5/4, v - 3/4)$, we have a corner equilibrium; for $r \in (v - 3/4, v)$, we have a quasi-monopolistic equilibrium. Equilibrium prices and profits are

\[
p_I^S = \begin{cases} 
1 + r & \text{if } r \in [0, v-5/4) \\
v - 1/4 & \text{if } r \in [v - 5/4, v-3/4) \\
(2v + r)/3 - 1 + 2\sqrt{v-r}/\sqrt{3} & \text{if } r \in [v-3/4, v)
\end{cases}
\]

\[
p_E^S = \begin{cases} 
1 + r & \text{if } r \in [0, v-5/4) \\
v - 1/4 & \text{if } r \in [v - 5/4, v-3/4) \\
(2v + r)/3 & \text{if } r \in [v-3/4, v)
\end{cases}
\]

\[
\pi_I^S = \begin{cases} 
1/2 & \text{if } r \in [0, v-5/4) \\
(2v - r)/2 & \text{if } r \in [v - 5/4, v-3/4) \\
2\sqrt{3}/9 & \text{if } r \in [v-3/4, v)
\end{cases}
\]

and

\[
\pi_I^S = \begin{cases} 
1/2 + r & \text{if } r \in [0, v-5/4) \\
(v - 1/4 + r)/2 & \text{if } r \in [v - 5/4, v-3/4) \\
r - 1 + \sqrt{3}/2 - 2\sqrt{3}(v-r)/9 & \text{if } r \in [v-3/4, v)
\end{cases}
\]

C Appendix

Proofs of Lemma 1 and Lemma 2. We first start by deriving the demand function. Marginal consumers are defined by

\[
\overline{\theta}(x) = (\pi_E - p_I) + (y_E^2 - y_I^2) - 2(y_E - y_I) x / q_H.
\]

Let $p_I^{(0,0)}$ s.t. $\overline{\theta}(x = 0) = 0$ for a given $\pi_E$ ($\overline{\theta}(x)$ passes through south-west corner of the unit square, so superscript $(0,0)$ stands for $(x = 0, \theta = 0)$). As $\partial \overline{\theta}/\partial x < 0$, this is the only point where $\overline{\theta}(x)$ touches the unit square. $p_I^{(0,0)}$ is found by

\[
0 = \pi_E - p_I + (y_E^2 - y_I^2),
\]

which implies

\[
p_I^{(0,0)} = \pi_E + (y_E^2 - y_I^2).
\]

Then, whenever

\[
p_I \geq p_I^{(0,0)}
\]

demand for the incumbent is zero.

Similarly, let $p_I^{(1,1)}$ s.t. $\overline{\theta}(x = 1) = 1$ for a given $\pi_E$, so that $\overline{\theta}(x)$ passes through north-east corner of the unit square. Hence,

\[
1 = \left(\pi_E - p_I  + (y_E^2 - y_I^2) - 2(y_E - y_I) \right)/q_H.
\]
which implies
\[ p_I^{(1,1)} = E_p + (y_E^2 - y_I^2) - 2(y_E - y_I) - qH \]
Whenever
\[ p_I \leq p_I^{(1,1)} \]
the incumbent has a unit demand (all the market).
Similarly we find \( p_I^{(0,1)} \) and \( p_I^{(1,0)} \).
\[ p_I^{(0,1)} = E_p + (y_E^2 - y_I^2) - qH \]
\[ p_I^{(1,0)} = E_p + (y_E^2 - y_I^2) - 2(y_E - y_I). \]
The demand for the incumbent is formed by three segments. Let \( \bar{x} \) the point where \( \overline{g}(x) \) touches the lower side of the unit square, i.e., \( \overline{g}(x) = 0 \). We find \( \bar{x} \) by solving
\[ 0 = ((p_E - p_I) + (y_E^2 - y_I^2) - 2(y_E - y_I) \bar{x}) / qH \]
for \( \bar{x} \), which yields
\[ \bar{x} = (p_E - p_I) + (y_E^2 - y_I^2) \]
\[ 2(y_E - y_I) \]
For
\[ p_I \in (p_I^{(1,0)}, p_I^{(0,0)}) \]
the demand is
\[ D_I^1 = \int_0^{\bar{x}} g(x) dx \]
\[ D_I^1 = ((p_E - p_I) + (y_E^2 - y_I^2))^2 / 4(y_E - y_I)qH. \]
On the other hand, for
\[ p_I \in (p_I^{(0,1)}, p_I^{(1,0)}) \]
we have the linear segment of the demand, which is defined by
\[ D_I^2 = \int_0^1 g(x) dx, \]
\[ D_I^2 = ((p_E - p_I) + (y_E^2 - y_I^2) - (y_E - y_I)) / qH. \]
Finally, let \( \bar{\tau} \) s.t. \( \overline{g}(\bar{\tau}) = 1 \) so that
\[ 1 = ((p_E - p_I) + (y_E^2 - y_I^2) - 2(y_E - y_I)\bar{\tau}) / qH. \]
This implies that
\[ \bar{\tau} = (p_E - p_I) + (y_E^2 - y_I^2) - qH \]
\[ (y_E - y_I) \]
For all
\[ p_I \in (p_I^{(1,1)}, p_I^{(0,1)}) \]
demand is
\[ D_I^3 = \bar{\tau} + \int_{\bar{\tau}}^1 g(x) dx, \]
\[ D_1^H = (2q_H (p + (y_E^2 - y_I^2)) - q_H^2 - (p + (y_E^2 - y_I^2) - 2(y_E - y_I))^2) / 4(y_E - y_I)q_H, \]

where \( p \) stands for \( (p_E - p_I) \).

The entrant’s demand can be found by \( D_1^E = (1 - D_1^H) \), \( D_2^E = (1 - D_2^H) \), and \( D_3^E = (1 - D_3^H) \). We look at the linear segment, \( D_2^H \), and solve for the equilibrium. Then, we show that the equilibrium exists, given our assumptions on \( q_H \) and \( q_L \).

Letting \( y = (y_E^2 - y_I^2) - (y_E - y_I) \),

and

\[ D_2^H \equiv D_3^H, \]

we have

\[ D_2^H = (p_E - p_I + y) / q_H \]

and

\[ D_3^H = 1 - ((p_E - p_I) + y) / q_H. \]

The incumbent and the entrant maximize their profit flows which are

\[ \pi_I^H = ((p_E - p_I + y) / q_H) p_I, \]

and

\[ \pi_E^H = (1 - (p_E - p_I + y) / q_H) p_E, \]

respectively. If it exists, Nash equilibrium of this price game yields

\[ p_I^H = (q_H + y) / 3 \]

and

\[ p_E^H = (2q_H - y) / 3. \]

We have \( D_1^H = (q_H + y) / 3q_H \), and \( D_2^H = (2q_H - y) / 3q_H \), and corresponding profits are

\[ \pi_I^H = (q_H + y)^2 / 9q_H, \]

and

\[ \pi_E^H = (2q_H - y)^2 / 9q_H. \]

Now, it remains to check whether price equilibrium exists, i.e., if \( p_I^H \) and \( p_E^H \) are valid in the linear part of the demand, i.e., if \( p_I^H \in [p_I^{(0,1)}, p_I^{(1,0)}] \). We know that \( p_I^{(0,1)} = p_E^H - 1 \), and \( p_I^{(1,0)} = p_E^H + 1 \). Furthermore, \( p_E^H = (2q_H - y) / 3 \).

Thus we have

\[ p_I^{(0,1)} = (2q_H - y) / 3 - 1 \]

and

\[ p_I^{(1,0)} = (2q_H - y) / 3 + 1. \]

We have \( p_I^H \in [p_I^{(0,1)}, p_I^{(1,0)}] \) if \( 2y + 3 > q_H \) and \( 2y/3 < q_H \) holds. The former inequality holds if \( q_H < 3 \) and the latter holds if \( q_H > 2/3 \). Remember that vertical dominance is defined by \( q_H > 2 \), hence second inequality always holds.
We find same conditions for \( p^H_E \in [p_L^{(0,1)}, p_L^{(1,0)}) \). Hence our assumption on \( q_H \), i.e., \( q_H \in (2,3) \) ensures the existence of price equilibrium, and it is easy to verify that the equilibrium horizontal locations are \( y_E = y_l = 1/2 \). Thus we have \( p^H_I = q_H/3 \), \( p^H_E = 2q_H/3 \), \( p^I_I = q_H/9 \), and \( p^I_E = 4q_H/9 \).

Similarly, we find the relevant payoffs for competition with technology \( L \) (horizontal dominance). Computations are available upon request.

## D Appendix

**Proof of Lemma 3.** We know that \( \pi^S_I(r) = 1/2 + r \) and \( \pi^L_I = (6-q_L)/72 \) for \( r \in [0, v-5/4) \). As \( (6-q_L)/72 < 1/2 \), \( \pi^I_I < \pi^S_I \) holds for all \( r \geq 0 \). Now assume that it adopts technology \( H \), so that \( \pi^H_I = q_H/9 \). Then, \( \pi^H_I < \pi^S_I(r) \) holds for all \( r \geq 0 \) as \( q_H/9 < 1/2 \).

Note that \( \pi^S_I(r) \) is increasing with \( r \) when \( r \in [v-5/4, v-3/4) \). Then if \( \pi^S_I(r) > \pi^L_I \) in this range, i.e., if \( (v-1/4+r)/2 > (6-q_L)/72 \) holds for \( r = v-5/4 \), it holds for all \( r \) in this range. Furthermore, observe that the right hand side of the inequality is decreasing with \( q_L \). As \( q_L \in (0,3/2) \), if this inequality is satisfied for \( q_L = 0 \), it is satisfied for all \( q_L \). It is indeed the case, as when we replace \( r = v-5/4 \) and \( q_L = 0 \), the inequality becomes \( 72v - 60 > 0 \), which is always satisfied as we assumed \( v > 3 \). Finally we can show that for \( r \in [v-5/4, v-3/4) \), we have \( \pi^S_I(r) > \pi^L_I \). This is true if \( (v-1/4+r)/2 > q_H/9 \) holds for \( r = v-5/4 \), and for \( q_H = 3 \), as \( q_H \in (2,3) \). And this is always true as \( 9v - 9.75 > 0 \).

It remains to check for \( r \in [v-3/4,v) \). Remember that

\[
\pi^S_I(r) = r - 1 + \sqrt{3}\sqrt{v-r} - 2\sqrt{3}(v-r)^{3/2}/9
\]

in this range of rental price. Let \( \rho = \sqrt{v-r} \). Then,

\[
\pi^S_I(r) = \frac{3\rho}{9} (9-2\rho^2).
\]

As \( v-3/4 < r < v \), we have \( \rho \in (0, \sqrt{3}/4) \). Hence, \( (9-2\rho^2) > 0 \) as \( \rho < 3/\sqrt{2} \), which implies that for \( r \in [v-3/4,v) \), we have \( \pi^S_I(r) > r - 1 \). Finally, as \( r \geq v-3/4 \), and \( v > 3 \), it is true that \( \pi^S_I > 3-7/4 \). Since \( \pi^I_I \leq 1/2 \), \( \pi^S_I(r) > \pi^I_I \) holds for all \( r \in [v-3/4,v) \).

## E Appendix

**Proof of Lemma 5.** We begin by showing that the entrant adopts technology \( H \) when there is no unbundling, it also does so when there is unbundling.
First, assume that the entrant prefers technology $H$ when there is no unbundling

$$\frac{(\pi_E^H)^2}{a_H} > \frac{(\pi_L^E)^2}{a_L} \Rightarrow \frac{\pi_E^H}{\sqrt{a_H}} > \frac{\pi_L^E}{\sqrt{a_L}}.$$ 

And assume that, when there is unbundling it prefers technology $L$ instead of $H$, which is true if

$$(\pi_E^H - \pi_E^S(r))^2 / a_H < (\pi_L^E - \pi_L^S(r))^2 / a_L$$

or

$$\frac{\pi_E^H}{\sqrt{a_H}} - \frac{\pi_E^S(r)}{\sqrt{a_L}} < (1/\sqrt{a_H} - 1/\sqrt{a_L}) \pi_E^S(r)$$

holds. Since $a_H > a_L$, we have $1/\sqrt{a_H} < 1/\sqrt{a_L}$. Therefore, the right-hand side of the above expression is negative, while the left-hand side is positive. Therefore, if the entrant prefers technology $H$ when there is no unbundling, it also prefers technology $H$ when there is unbundling.

Now we show that when the entrant prefers technology $L$ when there is no unbundling, it may prefer to adopt technology $H$ when there is unbundling. Observe that the entrant chooses technology $L$ when there is unbundling if

$$\frac{\pi_L^E}{\sqrt{a_L}} > \frac{\pi_E^H}{\sqrt{a_H}}.$$ 

On the other hand, it entrant prefers technology $H$ when there is unbundling if and only if

$$(\pi_E^H - \pi_E^S(r)) / \sqrt{a_H} > (\pi_L^E - \pi_L^S(r)) / \sqrt{a_L} \Leftrightarrow$$

$$\frac{\pi_L^E}{\sqrt{a_L}} - \frac{\pi_E^H}{\sqrt{a_H}} < (1/\sqrt{a_H} - 1/\sqrt{a_L}) \pi_E^S(r)$$

Note that both sides of the inequality are positive. Therefore, the entrant prefers technology $H$ if and only if

$$\pi_E^S(r) > \frac{\frac{\pi_L^E}{\sqrt{a_L}} - \frac{\pi_E^H}{\sqrt{a_H}}}{1/\sqrt{a_H} - 1/\sqrt{a_L}} \equiv \tilde{\pi}_E^S > 0.$$ 

If $\pi_E^S(r) < \tilde{\pi}_E^S$, the entrant prefers technology $L$. We know that $\pi_E^S(r)$ decreases with $r$ when $r \in [v - 5/4, v]$, and is constant otherwise. Therefore, if $\pi_E^S(r) > \tilde{\pi}_E^S$ is true for some $r$, there exists $\tilde{r} \in (v - 5/4, v]$ such that $\pi_E^S(r) > \tilde{\pi}_E^S$ for $r < \tilde{r}$ and $\pi_E^S(r) < \tilde{\pi}_E^S$ for $r > \tilde{r}$. We find that $\tilde{r} = v - 1/4 - 2\tilde{\pi}_E^S$ when $\tilde{\pi}_E^S \in (1/4, 1/2)$ and that $\tilde{r} = v - 3/16 \sqrt{3} \, \pi_E^{S/3}$ when $\tilde{\pi}_E^S \in (0, 1/4)$.

It remains to show that, $\pi_E^S(r) > \tilde{\pi}_E^S$ is true for some $r$. Indeed, $\tilde{\pi}_E^S < 1/2$ is true if and only if

$$a_H < a_LK_v(0),$$

in other words, if $a_H$ is sufficiently small.
Appendix

Proof of Proposition 2. Social welfare is defined as the sum of consumer surplus and industry profits. We study discounted welfare $W$ as a function of $r$. First, we compute welfare flows for all $r \leq \tau$ (all rental prices for which the entrant leases loops). When the incumbent is a monopolist, social welfare flow is

$$w_M = v - 1/12,$$

as

$$w_M = s_M + \pi^M = 1/6 + v - 1/4.$$  

When the firms compete on the basis of services, and if $r \in [0, v - 5/4)$ social welfare flow is

$$w_S (r) = v - 1/12,$$

as

$$w_S (r) = s_S (r) + (\pi^S_H (r) + \pi^S_E (r)) = v - 1/12 - (1 + r) + (1 + r).$$

If $r \in [v - 5/4, v - 3/4)$ social welfare flow is

$$w_S (r) = v - 1/12,$$

as

$$w_S (r) = s_S (r) + (\pi^S_H (r) + \pi^S_E (r)) = v - (v - 1/4) - 1/12 + (v - 1/4).$$

Hence, we can conclude that $w_S (r) = v - 1/12$ for all $r \leq \tau$. When the firms compete on the basis of facilities, social welfare flow is

$$w_H = v - 1/12 + 4q_H/9,$$

if the entrant adopts technology $H$, as

$$w_H = s_H + (\pi^H_H + \pi^H_E) = v - 1/12 - q_H/9 + 5q_H/9.$$  

And if the entrant adopts technology $L$, social welfare flow is

$$w_L = v - 1/12 + q_L/4 + q_L^2/18.$$  

First, remark that since $\Delta^S_E (r)$ is constant when $r \leq v - 5/4$, which implies that $W^{S+\tau}$ is also constant. Second, if $r > \tau$, $W^{S+\tau} = W^\tau$, as the entrant does not lease lines. Third, assume that $r \in [v - 5/4, v - 3/4)$, and that $\tau$ is near $v - 3/4$. Let’s also assume that the entrant adopts technology $L$. We find that $W' (r)$ decreases with $r$, as $W'' (r) = -1/(4\alpha_L) < 0$. We also have $W' (r = v - 5/4) > 0$ and $W' (r = v - 3/4) < 0$, as $q_L < 3/2$. We find that

$$W' (r) = 0 \iff r = r^*_L = v - 5/4 + q_L/6 + q_L^2/12.$$  

Notice that $r^*_L \in (v - 5/4, v - 3/4)$, as $q_L \in (0, 3/2)$. Therefore, $r^*_L$ maximizes welfare if $r^*_L < \tau$, otherwise $r$ maximizes welfare. Now assume that the the entrant adopts technology $H$. Then, we find that $W' (r) = -(1 - 4v + 4r) / 16\alpha_H$, and it is decreasing with $r$. Furthermore, $W' (r = v - 3/4) > 0$. Therefore, $W$ increases with $r$ up to $r = \tau - \epsilon$ (with $\epsilon$ very small), and the social welfare maximized with $r = \tau - \epsilon$.  

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G Appendix

Proof of Proposition 4. We have

\[ \Delta_K^{sw} = \frac{(w_L - w_S)}{a_L}, \]

and

\[ \Delta_L^{sw} = \frac{(v - 1/12 + q_L/4 + q_L^2/18) - (v - 1/12)}{a_L}. \]

Hence,

\[ \Delta_L^{sw} = \frac{q_L}{4} + \frac{q_L^2}{18} \]

\[ /a_L < \Delta_L^{*}. \]

It is then easy to check that \( \Delta_L^{sw} = \Delta_L^{*} (r) \iff r = r_L^{*}. \) Therefore, when the rental price is set at the socially optimal level, \( r_L^{*} \), the entrant adopts technology \( L \) at the socially optimal date. When \( r < r_L^{*} \), the welfare maximizing rental price, \( r \), leads to too late an adoption.