NETWORK EXTERNALITIES, MERGERS, AND INDUSTRY CONCENTRATION

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Abstract

I examine how mergers affect performance in network industries. Network providers choose quality for communication within the provider's own network, quality for communication between providers' networks, and output. Cross-border mergers provide firms an incentive to increase output because merged firms internalize positive network externalities. Several results modify conclusions in the current literature. Mergers between larger firms improve consumer surplus more than do mergers between smaller firms. Mergers may improve welfare and lower industry costs even if they increase marginal costs. Horizontal mergers may improve welfare even if marginal costs are unaffected. The Herfindahl-Hirschman index increases even when consumer surplus increases.

1. Introduction

The welfare effects of mergers and market concentration have been analyzed extensively. (See, for example, Salant et al. (1983), Farrell and Shapiro (1990a, 1990b), and Gaudet and Salant (1991).) However, this literature does not examine how network externalities between markets affect incentives to merge and the welfare effects of mergers and market concentration. These issues have taken center stage in recent merger and antitrust cases in network industries. Concerned that the merged company would leverage market dominance in the Internet backbone to capture most of the new growth in Internet access, European Union (EU) regulators required MCI and WorldCom to divest Internet assets as a precondition for their merger (Ungerer, 2000) and later blocked the merger of Sprint and WorldCom. The US and the EU have pursued antitrust actions against Microsoft based in part on the theory that Microsoft attempted to leverage its dominance in the market for PC operating systems to dominate other markets. The US Federal Communications Commission (FCC) placed conditions on the merger of AOL and Time Warner because the FCC believed that AOL was dominant in Instant Messaging and would leverage this dominance to control future markets for advanced Instant Messaging-based services.

In this paper, I extend the Katz and Shaprio (1985) model for a network industry to examine the welfare implications of mergers and market concentration in network industries. I examine an oligopoly model in which firms choose output levels, quality within a network (internal quality), and quality between networks (external quality). There are multiple markets and positive network externalities among the markets. Examples of such situations include: (*i*) telecommunications markets in separate countries or cities (e.g., the merger of Bell Atlantic and

GTE); (*ii*) Internet providers in different markets (e.g., AT&T's purchase of IBM Global Network); (*iii*) computer operating software (e.g., Windows 2000);[1] and (*iv*) markets where customers of existing products and customers of new products wish to share information (e.g., Instant Messaging and advanced Instant Messaging-based services). Network externalities imply that an increase in output in one market stimulates demand in other markets.

When a single firm serves multiple markets with network externalities, the firm internalizes network externalities and may choose higher levels of output than separate firms would choose. When a firm internalizes network externalities, its extra revenue from an increase in output in market *A* reflects not only the effects on price and quantity sold in market *A*, but also the effects that higher market demands have on prices in other markets in which the firm operates. The higher prices in these other markets provide extra revenue from market *A*'s output. I call this additional revenue from market *A*'s output the marginal extra-market revenue.[2] Positive marginal extra-market revenue may cause the merged firm to produce more even if the merger increases marginal production costs or decreases quality.[3] These facts imply that a merger may increase welfare even if it raises marginal production costs, increases market concentration, decreases quality, or extends market dominance. To analyze this result, I consider firms that operate in multiple markets and I consider mergers that combine firms that operate in

[1] Microsoft has different versions of Windows 2000 for different markets. These different versions contain features that allow customers using one to communicate more easily with customers using the other than with customers using non-Microsoft operating systems.

[2] I use the term marginal revenue in the traditional way, i.e., to indicate the effect of a change in output in market *A* on the firm's revenues from its sales in market *A*. In contrast, marginal extra-market revenue refers to the effect of a change in output in market *A* on the firm's revenues from its sales in market *B*.

[3] A change in quality causes customers' willingness to pay to change in the same direction. The resulting increase or decrease in price causes marginal revenue to change in the same direction for every level of output.

different markets (pure cross-border mergers) and mergers that combine firms that operate in some of the same markets, but not all of the same markets (mixed mergers).

I find several results that contradict or revise conclusions in the current literature. I find that a merger between large firms may increase welfare more than does a merger between small firms because large firms profit more from internalizing network externalities than do small firms. A large-firm merger may lower total industry production costs even if industry output increases and the merger creates no cost efficiencies for the merged firm. If pure cross-border mergers cause higher Herfindahl-Hirschman indices, they also improve net consumer surplus. The indices increase with cross-border mergers because the merged firm increases its market share.

My findings combine the existing literatures on mergers and on network effects. Szidarovsky and Yakowitz (1982) and Salant et al. (1983) describe how exogenous mergers may decrease joint profits of the merging firms. Farrell and Shapiro (1990a) identify industry conditions and asset transactions among firms that increase industry concentration and worsen industry performance. Farrell and Shapiro (1990b) develop sufficient conditions for profitable mergers that raise welfare and show that the Herfindahl-Hirschman indices can be misleading indicators of welfare change. Gaudet and Salant (1991) provide rules for determining when a merger increases or decreases welfare.

Katz and Shapiro (1985) show how consumption externalities cause demand-side economies of scale and that firms may choose less product compatibility than is socially optimal. Katz and Shapiro (1994) analyze competition between systems, collections of two or more components that work together. They describe how customer expectations about the future popularity of systems can result in multiple equilibria or the absence of equilibria.[4] Liebowitz and Margolis (1994) describe the limits of network effects in a single market. Analyzing market concentration in a single market, Crémer et al. (2000) show when large networks may discriminate against small networks in terms of the quality of network interconnection.[5]

I develop and explain my results as follows. I first examine pure cross-border mergers and apply the results to multimarket firms. If quality is unchanged or increases because of the merger, the merger improves net consumer surplus as long as marginal production costs do not increase, or, if they do increase, the increase is less than the marginal extra-market revenue. Net consumer surplus improves if quality decreases as long as the marginal extra-market revenue is greater than the decrease in marginal revenue caused by the lower quality plus any increase in marginal production costs.

I then examine the welfare effects of a merger that results in the loss of a competitor in the market where the merging firms were rivals, which I call the common market. I find that welfare increases as long as the effects of internalizing additional network externalities plus the effect of quality changes dominate the effect of losing a competitor in the common market, a situation that may occur if the common market is small relative to the other markets or if customers place high value on network externalities.

The analysis proceeds as follows. Section 2 describes the model. Section 3 presents the results for cross-market integration. Section 4 provides the results for mixed mergers. Section 5 is the conclusion. All proofs are in the Appendix.

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[4] I limit my analysis to situations where equilibria exist by considering only customer expectations of output and quality that are equal to actual output and quality in equilibrium. (Katz and Shapiro, 1985)

[5] I also extend the analysis of multilateral rivalry, the situation in which rivals differ in the mixes of markets that they serve. (Jamison, 1999)

2. The Model

I consider a game in which firms choose qualities simultaneously in the first stage and set output simultaneously in the second stage. These output choices generate market-clearing prices. Lastly, customers choose their preferred network providers. I develop my conclusions using general functions and illustrate the results using simulations based upon linear demand functions similar to those of Katz and Shapiro (1985) and Crémer et al. (2000). I describe the general model first and then the simulation model.

2.1. Demand and Supply

There are as many as three markets for the network service, which I designate as markets *A*, *B*, and *C*. A market is a customer group, such as customers located in a specific geographic region. Customers cannot migrate between markets to buy the service. (For example, a bank in New York cannot purchase telecommunications service from a provider that does not operate in New York.) Each customer buys at most a single unit of output.

As many as three firms may operate in the model. The firms are labeled 1, 2, and 3. A firm may serve more than one market. $q^{i,m} \geq 0$ will denote the number of customers that firm i serves in market m. $\mathbf{q}^i$ will denote the vector of all $q^{i,m}$ for a single firm i, q^m will denote the number of customers served by all firms in market m, and $\mathbf{q}$ will denote the vector of all outputs of all firms in all markets.

I assume that firms can choose to "interconnect" their networks.[6] In the setting of physical communications networks, this interconnection would be the lines and technical arrangements that allow customers to communicate. In the setting of virtual networks, such as computer software, this interconnection could be interpreted as features that allow customers to benefit from other customers. For example, software providers may create features that allow spreadsheet users to exchange data with database users.

Let $\theta^{i,i} \in [0,\overline{\theta}]$ represent firm *i*'s quality choice for communications between its customers, for all i = 1, 2, 3, and let $\theta^{i,j} \in [0,\overline{\theta}]$ represent firm *i*'s quality choice for external interconnection between its network and *j*'s network, for $j \neq i$. Let θ^i represent the vector of *i*'s quality choices. Quality includes such things as capacity for customers of physical networks to exchange messages, and features, such as instant messaging. A choice of zero represents a refusal to interconnect. Network quality is perfectly observable to firms and customers alike. For simplicity, I assume that each firm chooses a single internal quality and that two firms have a single external quality for interconnecting with each other. Only one quality choice can prevail for each network interconnection because only a single physical capacity can exist at a single point of interconnection, and if a technical feature cannot be used by customers of network *i* to communicate with customers of network *j*, then neither can customers of network *j* use the feature to communicate with customers of network *i*. If two firms have different preferences for external quality between their networks, I assume that the lower external quality preference prevails.

[6] In some industries, regulators require firms to interconnect their networks. Regulated telecommunications is an example of a network industry where regulators require interconnection. The Internet is generally unregulated. Refusal to interconnect is rare in the Internet, but controversy over type of interconnection is common. See Kende (2000) for an excellent overview.

In each market, there is a continuum of customers ordered according to the value they place on the network service. I assume that the customer that values the service the lowest places a sufficiently low value on the service that this customer would receive a negative surplus even if all other customers purchased the network service, all firms chose the highest possible qualities, and prices were zero. When $\hat{\mathbf{q}}$ customers purchase the network service, the q^mth customer in market *m* has a willingness to pay $u^i\left(q^m, \hat{\mathbf{q}}, \theta^i\right)$ for firm *i*'s service and obtains a net surplus from buying from firm *i* at price $p^{i,m}$ equal to $u^i\left(q^m, \hat{\mathbf{q}}, \theta^i\right) - p^{i,m}$. For a customer not purchasing from firm *i*, $u^i\left(q^m, \hat{\mathbf{q}}, \theta^i\right) = 0$. I assume that customers strictly prefer higher quality. Also, if quality is greater than zero customers strictly prefer for more customers to be on the system of networks (reflecting positive network externalities) and that the marginal value of extra customers is decreasing. Quality and network externalities interact in that the marginal value of the size of the system increases with quality, i.e., $\frac{\partial^2 u^i\left(q^m, \hat{\mathbf{q}}, \theta^i\right)}{\partial q^{i,\hat{m}}\, \partial \theta^{i,j}} > 0$ for all *i*, *j* = 1, 2, 3, and *m*, $\hat{m}$ = *A*, *B*, *C*. These assumptions are sufficient to cause firm *i*'s marginal revenue in market *m* to be increasing in quality. If all customer types are identical in their marginal values of network externalities, then the demand curve is linear. I assume that, all other things being equal, a customer is indifferent about which other customers are connected to networks and the networks to which these other customers are connected.

I assume a customer makes purchasing decisions after firms have made their quality and quantity choices. Assuming no price discrimination, each customer chooses the firm for which $u^i\left(q^m, \hat{\mathbf{q}}, \theta^i\right) - p^{i,m}$ is the greatest. I define the marginal customer to be the customer that, in equilibrium, is indifferent between buying and not buying network service.[7] At equilibrium, the marginal customer will receive zero net surplus and so will have a value of $u^{i}\left(q^{m},\hat{\mathbf{q}},\theta^{i}\right)= p^{i,m}$ (Katz and Shapiro, 1985). Prices vary among firms if quality and network size vary among firms.

I express the inverse demand curve for firm *i* in market *m* as $p^{i,m}\left(q^{m},\hat{\mathbf{q}},\theta^{i}\right)$ and hereafter supress the parentheses. I assume that the rate of decrease in value by successively lower customer types is greater in absolute value than the value that customers place on network externalities so that price decreases continuously with quantity sold in the market, i.e., $p^{i,m}_{q^{j,m}} < 0$ for *i*, *j* = 1, 2, 3, where the subscript represents the first partial derivative. Marginal willingness to pay in one market increases with quantity sold in other markets because of positive network externalities, i.e., $p^{i,m}_{q^{j,\hat{m}}} > 0$ for *i*, *j* = 1, 2, 3, and $m \neq \hat{m}$. Price also increases with quality because customers value higher quality network service more than lower quality network service, all other things being equal, i.e., $p^{i,m}_{\theta^{i,j}} > 0$ for *i*, *j* = 1, 2, 3, and a higher price for a given output results in higher marginal revenue at that output. I assume that consumers have decreasing marginal utility for the network service. As a result, the marginal value of quality decreases as quality increases and the marginal value of positive network externalities decreases with the quantity sold in other markets. Quality and network externalities interact in that the marginal value of the size of the system weakly increases with quality, i.e., $p^{i,m}_{q^{j,\hat{m}},\theta^{i,j}} \geq 0$ for all *i*, *j* = 1, 2, 3, and *m*, $\hat{m}$ = *A*, *B*, *C*. To ensure that an internal solution exists for output choices, I assume

[7] The assumptions that $u^{i} = 0$ for the lowest customer type and that marginal costs are strictly positive ensure that there are unserved customers in equilibrium.

that each firm's marginal revenue in a market declines as output increases in the market and that each firm's residual demand curve intersects its marginal cost curve from above (Dixit, 1986).

Costs for production are separable from costs for quality. Firm i incurs fixed costs $K^{i,m} \geq 0$ for each market m and a constant marginal cost $c^{i,m} > 0$ of production. All fixed costs are assumed to be sunk costs.[8] For simplicity, there are no economies of scope across markets. $G^i(\theta^i)$ represents firm i's cost function for quality. In the case of physical networks, firm i establishes a costly physical connection with each other firm and within its own network. In the case of software, there may be no extra costs for a technical feature, such as exporting pure text files, to be available for interfacing with extra software packages. To accommodate this range of possibilities, I assume that $G^i(\theta^i)$ is at least as great as the cost of the highest quality choice, $G^i(\max\{\theta^i\})$, and is no greater than the sum of the stand-alone costs of each quality choice for each interconnection, i.e., $\sum_{j=1}^{3} G^i(\theta^{i,j})$. I assume that the cost of interconnection is weakly increasing in quality and is convex, i.e., $G^i_{\theta^{i,j}} \geq 0$ and $G^i_{\theta^{i,j},\theta^{i,j}} \geq 0$ for i, j = 1, 2, 3. For simplicity, I assume that interconnection costs are independent of the number of customers. The assumption simplifies notation and does not affect results because the effects of customers on quality costs could be included in $c^{i,m}$.

Each firm takes its rivals' quantity choices as given when it chooses its own quantity levels. Firm i's profit maximization problem can be written as:[9]

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[8] Unless otherwise specified, I assume that fixed costs are sufficiently small to allow all firms in the market to receive non-negative profits.

[9] Firm i's production in market m is zero if firm i does not operate in the market.

$$\begin{aligned}&\max_{\mathbf{q}^i,\boldsymbol{\theta}^i}\pi^i = \sum_{m=A}^{C}\left(\left(p^{i,m}-c^{i,m}\right)q^{i,m}-K^{i,m}\right)-G^i\\ &\text{subject to}\qquad \theta^{i,j}\in\left[0,\overline{\theta}\right]\quad \text{for } j=1,2,3\\ &\qquad\qquad\qquad q^{i,m}\geq 0\qquad\quad \text{for } m=A,B,C.\end{aligned}\tag{1}$$

When firm *i* chooses its profit maximizing outputs, its output choice for market *m* reflects the marginal extra-market revenue, $p^{i,\hat{m}}_{q^{i,m}}q^{i,\hat{m}}$, which is strictly positive and represents the portion of network externalities between markets *m* and $\hat{m}$ that firm *i* internalizes when it operates in both markets. If firm *i* did not operate in both markets, its output in market *m* would create network externalities and higher profits for firms in market $\hat{m}$, but these profits would not directly benefit firm *i*. Therefore, firm *i* would not consider these profits when choosing its output for market *m*.

2.2 Welfare

The surplus a customer receives from purchasing depends on the innate value the customer places on the network service, on the internal and external quality choices of the customer's network supplier, and the total number of customers who purchase the network services. In each market, a customer only purchases if he values the service at least as much as the marginal customer does. Recalling that $u^i\left(q^m,\hat{\mathbf{q}},\boldsymbol{\theta}^i\right)$ and $p^{i,m}$ are zero for customers in *m* that do not purchase from firm *i*, integrating over all customers who purchase and summing over all firms and all markets gives the net consumer surplus:

$$U^{net}\equiv\sum_{m=A}^{C}\sum_{i=1}^{3}\left(\int_0^{q^m}\left(u^i\left(\hat{q}^m,\mathbf{q},\boldsymbol{\theta}^i\right)-p^{i,m}\left(q^m,\mathbf{q},\boldsymbol{\theta}^i\right)\right)d\hat{q}^m-T^i\right),$$

and weighted social welfare:

$$Z \equiv \alpha U^{net} + (1-\alpha)\sum_{i=1}^{3}\left(\pi^{i} + T^{i}\right),$$

where α = [1/2, 1] is the weight given to net consumer surplus, assuming that the social planner may give preference to consumers, and T^{i} is a transfer payment from consumers to firm i that may be necessary to ensure non-negative profits, for example. If a social planner chooses **q** and θ to maximize weighted social welfare subject to a non-negative profit constraint for firms, she would: (*i*) equate the sum of the marginal consumer surplus and the positive network externality to the marginal production cost; and (*ii*) equate the marginal consumer surplus from quality and the marginal cost of quality.

2.3 Model for Simulations

Extending Katz and Shapiro (1985) and Crémer et al. (2000), I adopt the following linear demand function for simulations:

$$p^{i,m} = \bar{\tau}^{m} - \sum_{\hat{j}=1}^{3} q^{\hat{j},m} + v\sum_{\hat{m}=A}^{C}\sum_{j=1}^{3}\theta^{i,j}q^{j,\hat{m}}$$

where $\bar{\tau}^{m}$ is the innate value that the market-m customer who values the network service the most places on the service if no other customers purchase, $\hat{j}$ represents firms that operate in market m, v is the constant value that all customers place on communicating with one other customer, and $\bar{\theta} = 1$. I omit fixed costs, assume constant marginal costs, and assume that quality costs follow an exponential function in which the quality choice is multiplied by a constant g. These assumptions give the following profit function:

$$\pi^{i} = \sum_{m=A}^{C}\left(\left(p^{i,m} - c^{i,m}\right)q^{i,m}\right) - \sum_{j=1}^{3} e^{g\theta^{i,j}} .$$

3. Analysis of Cross-Market Integration

In this section I consider pure cross-border mergers, mergers between firms that are not rivals, and apply the results to cross-market integration. First consider the all-monopoly setting in which markets *A* and *B* are initially served by separate monopolies, firms 1 and 2 respectively. (See Table 1.) Now suppose the monopolies merge and let $\tilde{1}$ represent the merged firm. The merger affects output and welfare by affecting quality choices, marginal production costs, and the network externalities that the firms internalize. The merger affects quality by changing the quality of service for communication between customers of markets *A* and *B* from external quality between the pre-merger firms to internal quality for the merged firm. The following lemmas describe quality choices before and after the merger.

Table 1. Arrangements of Firms Before and After Merger Between Firms 1 and 2

Settings		Firms in Markets before and after Mergers		
		Market *A*	**Market *B***	**Market *C***
All-monopoly Setting	Pre-merger	Firm 1	Firm 2	NA
	Post-merger	Firm $\tilde{1}$	Firm $\tilde{1}$	NA
Oligopoly Setting 1	Pre-merger	Firm 1	Firms 2 and 3	NA
	Post-merger	Firm $\tilde{1}$	Firms $\tilde{1}$ and 3	NA
Oligopoly Setting 2	Pre-merger	Firm 1	Firms 1 and 2	Firm 2
	Post-merger	Firm $\tilde{1}$	Firm $\tilde{1}$	Firm $\tilde{1}$

Lemma 1. When firms 1 and 2 increase the external quality between their networks in the all-monopoly setting, both firm's increase their output. That is to say, $\frac{dq^{j,m^*}}{d\theta^{i,j}} > 0$ for *i*, *j* = 1, 2 and $i \neq j$.

Lemma 2: Firm *i* prefers a higher internal quality than external quality with firm *j* if the effect of higher internal quality on the marginal customer's willingness to pay for firm *i*'s service is greater than the combined effects of higher external quality and firm *j*'s higher output on the marginal customer's willingness to pay for firm *i*'s service. That is to say, if

$$p^{i,m^*}_{\theta^{i,i}} > p^{i,m^*}_{\theta^{i,j}} + p^{i,m^*}_{q^{j,\hat{m}}} \frac{dq^{j,\hat{m}}}{d\theta^{i,j}}, \tag{2}$$

then θ^{i,i^*} is greater than firm *i*'s preferred $\theta^{i,j}$. Firm *i* prefers a higher external quality than internal quality if the inequality in (2) is reversed.

[INSERT FIGURE 1 ABOUT HERE]

Figure 1. Pre-merger and Post-merger Quality Choices in the All-Monopoly Setting

Figure 1 illustrates Lemma 2 by showing how optimal choices for quality change as demand changes in market *B*. The vertical axis represents quality values and the horizontal axis shows where market *B*'s inverse demand curve intercepts its price axis as a percent of the corresponding intercept in market *A*, i.e., $\bar{\tau}^B$ as a percent of $\bar{\tau}^A$. The value 100% on the horizontal axis represents the point at which markets *A* and *B* have symmetric demand functions. For this simulation, which I call the all-monopoly simulation, I assume that the inverse demand curve in market *A* intercepts its price axis at 100, *v* is 0.12 in both markets, both firms' constant marginal production costs are 0.10, and *g* is 5 for both firms. I choose these values to obtain internal solutions for quality for at least some levels of demand.

Before the merger the firm in the market with the lower demand determines external quality because this firm has the lower external quality preference. As a result, in Figure 1 firm

2 determines external quality in the range from 25% to 100% on the horizontal axis, the firms have symmetric external quality preferences at 100% (where the markets are symmetric), and firm 1 determines external quality in the area to the right of 100%. Firm 1's pre-merger internal quality line intersects its external quality line at Point A. Point C shows the corresponding situation for firm 2. The pre-merger firms choose identical internal qualities at Point B. Lemma 2 always applies to the firm that chooses external quality, so at Point B, external quality is higher than the pre-merger internal qualities. Post-merger internal quality is higher than all pre-merger qualities, except near the 25% point on the horizontal axis and where qualities reach the technical maximum.[10]

Higher quality increases marginal revenue. Condition 1 describes a possible relationship between changes in marginal revenue, marginal extra-market revenue, and marginal production costs. In the all-monopoly simulation, Condition 1 holds as long as the merger does not increase marginal production costs by a factor of approximately 80.

Condition 1: Any increase in marginal production costs in market *m* is less than the sum of (*i*) the effect that positive network externalities have on the combined firm's marginal extra-market revenue in *m* for firm *j*'s original market and (*ii*) the effect that quality changes have on marginal revenue, i.e.,

$$r_{\theta^{i,i}}^{i,m^*} \Delta\theta^{i,i} + r_{\theta^{i,j}}^{i,m^*}\left(\theta^{\tilde{1},\tilde{1}^*} - \hat{\theta}^{i,j^*}\right) + p_{q^{\tilde{1},m}}^{\tilde{1},\hat{m}^*} \cdot q^{\tilde{1},\hat{m}} > c^{\tilde{1},m} - c^{i,m}, \tag{3}$$

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[10] A merger decreases the number of qualities that must be chosen. This would decrease the total cost of quality, which could improve the welfare effects of a merger. To focus on the effects of internalizing network externalities, in all simulations I compensate for the decrease in the number of qualities chosen by applying a factor that keeps the total cost of quality the same as it would have been if the post-merger quality choices were applied to the pre-merger situation. In the all-monopoly simulation, for example, I multiply the cost of the single post-merger quality choice by two.

where $\Delta\theta^{i,i}$ is the merger's effect on firm i's internal quality and $r^{i,m*}$ is firm i's marginal revenue reflecting optimal output in market m.

Proposition 1 provides this section's initial result.

Proposition 1. Suppose Condition 1 holds in the all-monopoly setting. Then a pure cross-border merger results in higher output and net consumer surplus. If the inequality in (3) is reversed, then a pure cross-border merger reduces output and net consumer surplus.

If Condition 1 holds, a pure cross-border merger increases output because the merged firm internalizes positive network externalities. For example, if the merger increases marginal production costs in market *A* by an amount less than (respectively, greater than) the marginal increase in demand in market *B* that the merged firm internalizes, then the merged firm increases (respectively, decreases) output in market *A* above the pre-merger level. Net consumer surplus increases when a pure cross-border merger increases output. To illustrate, consider a pure cross-border merger that increases output of all firms. All new customers enjoy increased net surplus, because these customers received zero net surplus before the merger and now receive a positive net surplus. Also, because the marginal value of network externalities is lower for lower-type customers, the difference between consumer value and price increases for existing customers when quantity increases, causing an increase in net surplus for existing customers. Profits may also be higher even if marginal production costs increase. For example in the simulation, post-merger profits are higher than the sum of the pre-merger profits if marginal production costs increase by less than a factor of approximately 20. When the pure cross-border merger increases both net consumer surplus and profits, the merger increases welfare.

A merger changes how the merging firms choose quality in the all-monopoly setting because (1) the connection between markets *A* and *B* changes from an external connection to an internal connection and (2) the internal quality choice applies to additional markets. However, the merger may not affect quality in some situations. For example, a regulatory authority or industry standards may set quality, the cost of quality may be sufficiently low that firms choose the maximum quality, or it may be costly for the merged firm to change quality from what the pre-merger firms had chosen. The following corollaries provide two rules of thumb for pure cross-border mergers in the all-monopoly situation when quality is not equal to zero and is unaffected by the merger:

Corollary 1A. A pure cross-border merger in an all-monopoly situation increases output and improves net consumer surplus as long as it does not increase marginal production costs if quality is unchanged.

Corollary 1B. In an all-monopoly situation where quality is unchanged by the merger, increasing the number of markets involved in a pure cross-border merger increases output and welfare, all other things being equal.

Corollary 1A is simply a special setting of Proposition 1. Corollary 1B follows directly from Proposition 1 and would apply if, for example, separate monopolies served markets *A* through *C* and all three monopolies merged. Increasing the number of markets involved in the merger increases the positive network externalities that the monopoly internalizes. As a result, the monopoly chooses higher output, which improves net consumer surplus.

Proposition 1 and its corollaries also apply to non-merger situations. Consider the situation in which there is a new market for the network service and both a monopoly and a start-up company are considering entering the market. If Condition 1 holds, then output and net consumer surplus would be higher if the monopoly entered the market. Furthermore, the results of Corollaries 1A and 1B apply.

Now consider oligopoly setting 1 in which, initially, firms 2 and 3 serve market *B* and firm 1 is a monopoly in market *A*. (Recall Table 1.) Now let firms 1 and 2 merge and let $\tilde{1}$ represent the merged firm. The effects on quality are similar to those in the all-monopoly setting because the merger is between two non-rivals in both settings. Using a simulation (which I call the oligopoly setting 1 simulation) to illustrate these effects, the internal quality of the merged firm is approximately 20% higher than firm 1's internal quality, 30% higher than firm 2's internal quality, and 30% high than the external quality between firms 1 and 2 that existed before the merger.[11] Firm 3's internal quality decreases approximately 7% after the merger and its external quality with its rival decreases approximately 3%. Proposition 2 and the following corollaries provide the results for a pure cross-border merger in this oligopoly setting.

Proposition 2. Suppose Condition 1 holds in oligopoly setting 1. Then a pure cross-border merger results in higher output and net consumer surplus. If the inequality in (3) is reversed, then a pure cross-border merger reduces output and net consumer surplus. Furthermore, if the merger increases the Herfindahl-Hirschman index because the merging firm increases output, the merger also results in higher net consumer surplus.

When Condition 1 holds, the merging firms increase their output because they are internalizing network externalities. The merging firms' higher output affects the other firm's output in two ways. First, the higher output in market *A* increases demand in market *B*, which provides the other firm with an incentive to increase its output. Second, the merging firm's

[11] For this simulation, I assume that both market inverse demand curves intersect their respective price axes at 100, marginal production costs for firm 1 are 5, v is 0.12, and g is 5. To compensate for the decrease in the number of quality choices, I multiply the merging firm's internal quality costs by two. I consider various relationships between firm 2's marginal production costs and firm 3's marginal production costs. More specifically, I let firm 3's marginal production costs be αc^B and firm 2's marginal production costs to be $(2-\alpha) c^B$, where $\alpha = [0, 2]$ and c^B represents the unweighted average of the two firms' marginal production costs.

higher output in market *B* is a substitute for the other firm's output in market *B*, which provides this other firm with an incentive to decrease its output, but by an amount that is less than the merging firms' higher output in market *B* (Farrell and Shapiro, 1990b). Others have found that higher Herfindahl-Hirschman indices can be associated with higher welfare in Cournot models (Farrell and Shapiro, 1990b; and Salant and Shaffer, 1999). This finding also holds for pure cross-border mergers in network industries. Specifically, when any increase in marginal production cost for the merging firms is more than outweighed by positive network externality effects of the merger, the market share for the merging firms increases. This causes the Herfindahl-Hirschman index and net consumer surplus to increase in a large-firm merger.

These results from Proposition 2 provide rules of thumb, which are summarized in the following corollaries.

Corollary 2A. A pure cross-border merger in oligopoly setting 1 improves net consumer surplus as long as it does not increase marginal production costs and quality remains unchanged.

Corollary 2B. Increasing the number of markets involved in a pure cross-border merger improves the welfare effects of the merger if marginal production costs and quality remain unchanged.

[INSERT FIGURE 2 ABOUT HERE]

Figure 2. Effects Merger in Oligopoly Setting 1 Simulation on
Net Consumer Surplus, Herfindahl-Hirschman index, and Costs

Figure 2 illustrates Proposition 2 and Corollary 2A using the oligopoly setting 1 simulation. The horizontal axis shows the value for α, which represents the relationship between the marginal production costs of the firms in the competitive markets. When α is 1, the firms' marginal production costs both equal five. When α is 0, firm 3's marginal production cost is zero and firm 2, which merges with firm 1, has a marginal production cost if 10. When α is 2, the marginal production costs of firms 2 and 3 are reversed. The vertical axis shows values for percentage changes. Recalling that net consumer surplus increases when output increases, the line representing change in output shows that the merger always increases net consumer surplus. As I discuss below, a large-firm merger increases net consumer surplus more than does a small-firm merger. The line representing the change in the Herfindahl-Hirschman index shows that the merger increases the index when α is greater than approximately 0.5 and decreases the index when α is less than this.[12] Whenever the index increases, net consumer surplus is also increases.

In oligopoly setting 1, cross-border mergers between larger firms are more beneficial for merging firms and customers than mergers between smaller firms. Before the merger, firm 1 can choose whether to merge with firm 2 or firm 3. Now assume that one of these two rivals has greater output than the other before the merger because it has lower marginal production costs (Farrell and Shapiro, 1990b) and that the merger has the same incremental effect on marginal production costs regardless of which firm merges with firm 1. As Figure 2 illustrates, if firm 1 merges with the larger firm, the merged firm internalizes more network externalities than if firm 1 merges with the smaller firm because the network externalities that it internalizes are

[12] No particular significance can be placed on the value for α at which the change in the Herfindahl-Hirschman index becomes positive because it depends on the marginal production costs in the competitive market.

determined by the output that the merged firm produces in the competitive market. This quantity is greater if firm 1 merges with the larger firm. The following corollary summarizes this result.

Corollary 2C. A large-firm pure cross-border merger in oligopoly setting 1 increases net consumer surplus more than a small-firm merger.

A large-firm merger reduces average industry production costs relative to a small-firm merger. Because the larger firm initially has lower marginal production costs than the smaller firm, increasing the larger firm's market share through the merger provides lower average industry costs. Conversely, a small-firm merger increases average industry production costs. These results hold even though the sum and variance of the marginal costs are both constant and even if marginal production costs rise slightly. This modifies the results of Bergstrom and Varian (1985) and Salant and Shaffer (1999), who respectively found that the sum of the marginal costs determine market output and that there is a negative correlation between aggregate production costs and the variance of marginal costs. Corollary 2D provides conditions for a decrease in total industry production costs when output increases.

Condition 2. The merger's effects on the non-merging firm's output from (1) positive network externalities from greater output in the monopoly market and (2) the changes in quality, are greater in absolute value than the effect of greater output in the competitive market, i.e.,

$$r_{q^{1,A}}^{3,B}\Delta q^{\tilde{1},A} + r_{\theta^{3,3}}^{3,B}\Delta\theta^{3,3} + \sum_{i=1,2} r_{\theta^{i,3}}^{3,B}\left(\theta^{\tilde{1},3^*} - \hat{\theta}^{i,3^*}\right) > \left| r_{q^{\tilde{1},B}}^{3,B}\Delta q^{\tilde{1},B} \right|, \tag{4}$$

where $\Delta q^{\tilde{1},A}$ and $\Delta q^{\tilde{1},B}$ are the changes in the merged firm's output in markets A and B respectively.

Condition 2 is sufficient for the non-merging firm to increase its output after the merger.

Corollary 2D. Suppose Condition 1 holds in oligopoly setting 1 and the inequality in (4) is reversed. Then a large-firm merger decreases total industry production costs in the competitive market, even though output in the competitive market increases, if the smaller firm's decrease in output is greater in absolute value than the increase in the larger firm's total cost in competitive market divided by the smaller firm's marginal production cost, i.e., $-\Delta q^{3,B} > \frac{1}{c^{3,B}}\left(c^{\tilde{1},B} q^{\tilde{1},B^*} - c^{1,B} q^{1,B^*}\right)$. If the merger does not affect marginal production costs, then the large-firm merger decreases total industry production costs in the competitive market if $-\frac{\Delta q^{3,B}}{\Delta q^{1,B}} > \frac{c^{1,B}}{c^{3,B}}$.

Figure 2 illustrates this result. There are two lines that show percentage changes in total industry production costs resulting from the merger. If marginal production costs are unchanged by the merger, then total industry production costs decrease if α exceeds approximately 1.65 even though total industry output increases. If the merger causes a diseconomy of production, such that marginal production costs increase 10% for the merging firms, then total industry production costs always increase after the merger.

The results for Proposition 2 and its corollaries hold for monopoly entry into an existing market. Output and net consumer surplus are higher if the monopoly gains a large market share than if it obtains a small market share. Industry production costs may decrease in the competitive market even if output increases.

4. Mixed Mergers

In this section I consider an oligopoly setting where a merger decreases the number of firms in one market and the number of markets served by the merged firm is greater than the number of markets served by individual merging firms before the merger. Mixed mergers, such as the one in this model, raise issues of market concentration and extending monopoly or dominant market position for regulators.

Consider oligopoly setting 2 where firm 1 is a monopolist in market *A* and a duopolist in market *B*. Also suppose firm 2 is a monopolist in market *C* and serves as the second duopolist in market *B*. (Recall Table 1.) Now assume that firms 1 and 2 merge and let $\tilde{1}$ represent the merged firm. After the merger, the merged firm selects its quality and output to maximize its profits. I call market *B* the common market because both merging firms operated in it before the merger, and I call the markets *A* and *C* the individual markets because only one merging firm operates in each of these markets before the merger.

The mixed merger has two effects on output. First, the merger removes a competitor in the common market. All other things being equal, this causes output to decrease in this market. Second, the merger increases the network externalities that merging firms internalize in individual markets. To illustrate, consider how the merging firms choose output in market *A*. Before the merger, when firm 1 chooses its output, it internalizes network externalities from its output in the common market. After the merger, when the merged firm chooses its output in market *A*, it internalizes network externalities from its output in both markets *B* and *C*. Not only does the merged firm internalize network externalities from an extra market, but it also internalizes network externalities from a larger output in the common market because the merged firm serves the entire market. The corresponding analysis for market *C* would provide analogous conclusions.

Proposition 3 describes the mixed merger's effect on quality. Conditions 3 and 4 are useful for the analysis. Condition 3 addresses the merger's effect on internal quality by relating internal quality's effect on the merging firms' revenue to the merger's effect on the cost function for internal quality. Condition 4 addresses the effect of changing external quality to internal quality because of the merger.

Condition 3: For firm *i*, the effects of internal quality on post-merger revenue in all markets divided by the effects of internal quality on pre-merger revenue in firm *i*'s pre-merger markets, is greater than one, i.e., using firm 1 to illustrate,

$$\frac{\sum_{m\in A,B,C} p_{\theta^{\tilde{1},\tilde{1}}}^{\tilde{1},m^*} q^{\tilde{i},m^*}}{\sum_{m\in A,B} p_{\theta^{1,1}}^{1,m^*} q^{1,m^*}} > 1. \tag{5}$$

Condition 4: The effects of internal quality on post-merger revenue in all markets are greater than the effects of external quality on firm 1’s pre-merger revenue in both of its markets, i.e.,

$$\sum_{m\in A,B,C} p_{\theta^{\tilde{1},\tilde{1}}}^{\tilde{1},m^*} q^{\tilde{1},m^*} > \sum_{m\in A,B} p_{\theta^{1,3}}^{1,m^*} q^{1,m^*} + \sum_{\hat{m}=B,C} \frac{\partial q^{2,\hat{m}}}{\partial \theta^{1,2}} \sum_{m\in A,B} p_{q^{2\hat{m}}}^{1,m^*} q^{1,m^*}, \tag{6}$$

where firm 1 has the lower external quality preference and so determines external quality.

Proposition 3. A mixed merger in oligopoly case 2 increases (respectively, decreases) internal quality if Condition 3 holds (respectively, the inequality in (5) is reversed) and results in an internal quality that is higher (respectively, lower) than the pre-merger external quality if Condition 4 holds (respectively, the inequality in (6) is reversed).

Figure 3 shows a simulation of quality choices in oligopoly setting 2.[13] For this simulation, I contrast quality choices across various levels of demand in the common market. The vertical axis represents the quality values and the horizontal axis represents values at which the inverse demand curve in market *B* intersects its price axis. The lines in the graph show pre-merger internal and external quality and post-merger internal quality. I show only one pre-merger internal quality because I assume for the simulation that the firms are symmetric. Figure 3 shows that the merger increases quality in the simulation.

[INSERT FIGURE 3 ABOUT HERE]

Figure 3. Pre-merger and Post-merger Quality Choices in Oligopoly Setting 2 Simulation

Proposition 4 describes how the mixed merger affects output and net consumer surplus.

Proposition 4. A mixed merger in oligopoly case 2 increases net consumer surplus if marginal production costs are unchanged and if the effect on output of the greater marginal extra-market revenue and the changes in quality are greater in absolute value than the effect of the decrease in the number of competitors in the common market.

I use the oligopoly setting 2 simulation to illustrate the mixed merger's effect on output and welfare. For the simulation, I allow both *v*, the value that customers place on network externalities, and the demand in market *B* to vary. Figure 4 shows the effect the merger has on

[13] For this simulation, which I call the oligopoly setting 2 simulation, I assume that the inverse demand curves in markets *A* and *C* intersect their vertical axes at $100, marginal production costs for firm 1 are $0.10, *v* is 0.12, and *g* is 7. I multiply post-merger internal quality costs by two. I consider various levels of demand for market *B*, letting the intercept of the inverse demand curve vary from 25 to 200.

net consumer surplus for various sizes of the common market relative to the individual markets and for various values of network externalities. The curved surface shows the region where the merger increases net consumer surplus. The region where no curved surface is shown represents a decrease in net consumer surplus. The merger increases industry profits for all regions, so the curved surface also shows the region for which the merger unambiguously increases welfare. The following corollaries result from Proposition 4.

[INSERT FIGURE 4 ABOUT HERE]

Figure 4. Effect of Mixed Merger on Net Consumer Surplus

Corollary 4A. The merger in the simulation always decreases net consumer surplus when quality is zero because consumers and firms receive no benefits from network externalities.

Corollary 4B. The increase in net consumer surplus from the merger increases with the value consumers place on network externalities.

Corollary 4B holds because higher customer values for communication increase the firm's marginal extra-market revenue, which causes the firm to increase output. Furthermore, when the marginal value of network externalities approaches the ratio of the slope of the inverse demand curve to the number of markets (a value of one-third in the simulation), the marginal extra-market revenue always exceed marginal production cost, which gives the firm an incentive to serve the entire market.

Corollary 4C. The change in net consumer surplus decreases as the innate value that customers in the common market place on the network service increases relative to the innate value that customers in the individual markets place on the network service.

Corollary 4C holds because, as the value customers place on the service in the common market increases, the effect of losing a competitor in the common market rises relative to the benefit of extra internalized network externalities in the individual markets. The merger always increases welfare in the simulation when the innate value in the common market is equal to the sum of the innate values in the individual markets and the ratio of the value of network externalities to the slope of the inverse demand curve is greater than 0.13.

5. Conclusion

In this paper, I extend Katz and Shapiro (1985) to examine how mergers affect industry performance in network industries. I find that internalizing network externalities may cause mergers to improve efficiency. This has several results that contradict or modify conclusions in the current literature. These mergers may improve efficiency even when they increase marginal production costs. Also, mergers between larger firms have more beneficial affects on consumer surplus than do mergers between smaller firms. This study implies that conventional views of mergers and market entry in network industries may be inappropriate and that careful study is needed before mergers are rejected or antitrust actions taken on grounds of extending monopoly or dominant market positions or of increasing industry concentration.

Additional research is needed. I do not develop a general model for welfare-improving cross-border mergers. I also do not examine the case where products provided by merging firms are imperfect substitutes rather than completely separate products. Lastly, empirical research is needed to measure network externalities, how they vary across markets and networks, and whether network firms that serve multiple markets respond to the incentives that I model.

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APPENDIX

Proofs of Lemmas 1 and 2. From (1), firm *j*'s first order conditions for optimally choosing output in the all-monopoly setting include:

$$p^{j,m} + p^{j,m}_{q^{j,m}} q^{j,m} - c^{j,m} = 0 .$$

Totally differentiating gives:

$$\frac{dq^{j,m}}{d\theta^{i,j}} = -\frac{p^{j,m}_{\theta^{i,j}} + p^{j,m}_{q^{j,m},\theta^{i,j}} q^{j,m}}{2p^{j,m}_{q^{j,m}} + p^{j,m}_{q^{j,m},q^{j,m}} q^{j,m}} .$$

The sign of firm *j*'s reaction in its output to a change in external quality depends on the sign of the numerator because $2p^{j,m}_{q^{j,m}} + p^{j,m}_{q^{j,m},q^{j,m}} q^{j,m}$ is the slope of firm *j*'s marginal revenue curve with respect to its own output, which is negative by assumption. By assumption, $\frac{\partial^2 u^i\left(q^m,\hat{\mathbf{q}},\boldsymbol{\theta}^i\right)}{\partial q^{i,m}\,\partial\theta^{i,j}} > 0$ and $\frac{\partial u^i\left(q^m,\hat{\mathbf{q}},\boldsymbol{\theta}^i\right)}{\partial\theta^{i,j}} > 0$, so the numerator is positive. This confirms Lemma 1.

Now consider firm *i*'s first order conditions for optimally choosing $\theta^{i,i}$ and $\theta^{i,j}$:

$$p^{i,m^*}_{\theta^{i,i}} q^{i,m^*} = G^i_{\theta^{i,i}} \text{ , and} \tag{A1}$$

$$q^{i,m^*}\left(p^{i,m^*}_{\theta^{i,j}} + p^{i,m^*}_{q^{j,\hat{m}}} \frac{\partial q^{j,\hat{m}}}{\partial\theta^{i,j}} \right) = G^i_{\theta^{i,j}} \text{ .} \tag{A2}$$

From (A1) and (A2), if firm *i* prefers a higher internal quality than external quality, then $G^1_{\theta^{1,1}}\left(\theta^{1,1^*}\right) > G^1_{\theta^{1,2}}\left(\theta^{1,2^*}\right)$, or

$$p^{i,m^*}_{\theta^{i,i}} q^{i,m^*} > q^{i,m^*}\left(p^{i,m^*}_{\theta^{i,j}} + p^{i,m^*}_{q^{j,\hat{m}}} \frac{\partial q^{j,\hat{m}}}{\partial\theta^{i,j}} \right). \tag{A3}$$

Dividing both sides of (A3) by q^{i,m^*} confirms Lemma 2.

In the all-monopoly simulation, the sign for (2) is reversed. Because the markets are symmetric, $q^{i,m^*} = q^{j,\hat{m}^*}$. In the simulation, (A3) becomes $vq^{i,m^*} < vq^{j,\hat{m}^*} + v\theta^{i,j}\dfrac{\partial q^{j,\hat{m}}}{\partial \theta^{i,j}}$.

Canceling equal terms shows that the sign in (2) is reversed because $v\theta^{i,j}\dfrac{\partial q^{j,\hat{m}}}{\partial \theta^{i,j}} > 0$. □

Proof of Proposition 1 and Corollaries 1A-B. Consider firm *i*'s first order condition for choosing output before the merger:

$$p^{i,m} + p^{i,m}_{q^{i,m}} q^{i,m} - c^{i,m} = 0 \tag{A4}$$

and firm $\tilde{1}'s$ first order condition for choosing output if it serves two markets:

$$p^{\tilde{1},m} + p^{\tilde{1},m}_{q^{\tilde{1},m}} q^{\tilde{1},m} - c^{\tilde{1},m} + p^{\tilde{1},\hat{m}}_{q^{\tilde{1},m}} q^{\tilde{1},\hat{m}} = 0. \tag{A5}$$

Combining (A4) and (A5) gives:

$$p^{\tilde{1},m} + p^{\tilde{1},m}_{q^{\tilde{1},m}} q^{\tilde{1},m} - p^{i,m} - p^{i,m}_{q^{i,m}} q^{i,m} + p^{\tilde{1},\hat{m}}_{q^{\tilde{1},m}} q^{\tilde{1},\hat{m}} = c^{\tilde{1},m} - c^{i,m}. \tag{A6}$$

A merger can decrease marginal revenue by decreasing quality, increasing output, or both. The effect a change in quality on marginal revenue is

$$\left(p^{i,m^*}_{\theta^{i,i}} + p^{i,m}_{q^{i,m},\theta^{i,i}}{}^{*} \cdot q^{i,m^*}\right)\Delta\theta^{i,i} + \left(p^{i,m^*}_{\theta^{i,j}} + p^{i,m}_{q^{i,m},\theta^{i,j}}{}^{*} \cdot q^{i,m^*}\right)\left(\theta^{\tilde{1},\tilde{1}^*} - \hat{\theta}^{i,j^*}\right)\Big), \tag{A7}$$

which is positive if quality increases and negative if quality decreases for customers in market *m*. If the marginal revenue after the merger is less than the original marginal revenue plus (A7), then the merger increases output. The text explains why an increase in output results in an increase in net consumer surplus. This confirms Proposition 1.

If quality and marginal production costs are unaffected by the merger, then (A7) is zero and the right-hand side of (A6) is zero. Because $p^{\tilde{1},\hat{m}}_{q^{\tilde{1},m}} q^{\tilde{1},\hat{m}} > 0$, the merger decreases marginal revenue, which means that output increases because quality is constant. This confirms Corollary 1A. Furthermore, increasing the number of markets increases the marginal extra-market revenue. This means marginal revenue decreases more if there are extra markets, which, because quality is constant, means that output is higher when the pure cross-border merger involves more markets. This confirms Corollary 1B. □

Proofs of Proposition 2 and Corollaries 2A-2B: The proof for Proposition 1 and its corollaries with respect to output and net consumer surplus applies to Proposition 2 and Corollaries 2A and 2B. Furthermore, the Herfindahl-Hirschman index increases in oligopoly setting 1 any time the merged firm chooses a higher output in market B and its post-merger market share is greater than the non-merging firm's pre-merger market share. □

Proof of Corollary 2C: Consider (A5). Firm $\tilde{1}$'s optimal output in market m is an increasing function of $p^{\tilde{1},\hat{m}}_{q^{\tilde{1},m}} q^{\tilde{1},\hat{m}}$, which is higher with a large-firm merger than with a small-firm merger, all other things being equal. This confirms Corollary 2C. □

Proof of Corollary 2D: If Condition 1 holds and the inequality in (4) is reversed, then firm 3 chooses a lower output after the merger than before the merger, i.e., $\Delta q^{3,B} < 0$. The decrease in firm 3's production costs are greater in absolute value than the change in the merged firm's production costs in market B if $c^{\tilde{1},B} q^{\tilde{1},B^*} - c^{1,B} q^{1,B^*} < -c^{3,B} \Delta q^{3,B}$, which confirms Corollary 2D. □

Proof of Proposition 3: Using firm 1 to illustrate, consider its pre-merger and post-merger first order conditions for choosing internal quality:

$$p_{\theta^{\tilde{1},\tilde{1}}}^{\tilde{1},A^*} q^{\tilde{1},A^*} + p_{\theta^{\tilde{1},\tilde{1}}}^{\tilde{1},B^*} q^{\tilde{1},B^*} + p_{\theta^{\tilde{1},\tilde{1}}}^{\tilde{1},C^*} q^{\tilde{1},C^*} = G_{\theta^{\tilde{1},\tilde{1}}}^{\tilde{1}} \text{, and} \tag{A8}$$

$$p_{\theta^{1,1}}^{1,A^*} q^{1,A^*} + p_{\theta^{1,1}}^{1,B^*} q^{1,B^*} = G_{\theta^{1,1}}^{1} . \tag{A9}$$

If internal quality is higher after the merger, then its post-merger marginal cost is greater at $\theta^{\tilde{1},\tilde{1}^*}$ than at $\theta^{1,1^*}$. Using this and combining (A8) and (A9) gives (5).

Now compare first order conditions for pre-merger internal quality with post-merger internal quality. Before the merger, the firm with the lower external quality preference determines external quality. Assuming this is firm 1, its first order conditions for choosing external quality include:

$$\begin{aligned} &p_{\theta^{1,3}}^{1,A^*} q^{1,A^*} + \frac{\partial q^{2B}}{\partial \theta^{1,2}}\left(p_{q^{2B}}^{1,A^*} q^{1,A^*} + p_{q^{1,B}}^{1,B^*} q^{1,B^*}\right) + p_{\theta^{3,3}}^{1,B^*} q^{1,B^*} \\ &+ \frac{\partial q^{2C}}{\partial \theta^{1,2}}\left(p_{q^{2C}}^{1,A^*} q^{1,A^*} + p_{q^{2C}}^{1,B^*} q^{1,B^*}\right) = G_{\theta^{1,2}}^{1} \end{aligned} . \tag{A10}$$

After the merger, the merged firm's internal quality choice is defined by (A8). If this quality is higher than the pre-merger external quality, then its marginal cost is greater valued at $\theta^{\tilde{1},\tilde{1}^*}$ than at $\theta^{1,2^*}$. Using this and combining (A8) and (A10) gives (6). This confirms Proposition 3. □

Proof of Proposition 4 and Corollary 4C: Before the merger, firm *i*'s first order conditions for choosing output in its individual market *m* include:

$$p^{i,m} + p_{q^{i,m}}^{i,m} q^{i,m} - c^{i,m} + p_{q^{i,m}}^{i,B} q^{i,B} = 0 ,$$

and for choosing output in the common market include:

$$p^{i,B} + p^{i,B}_{q^{i,B}} q^{i,B} - c^{i,B} + p^{i,m}_{q^{i,B}} q^{i,m} = 0\,.$$

Firm $\tilde{1}'s$ first order conditions for choosing output in any market m include:

$$p^{\tilde{1},m} + p^{\tilde{1},m}_{q^{\tilde{1},m}} q^{\tilde{1},m} - c^{\tilde{1},m} + \sum_{\substack{\hat{m}=A,B,C \\ \hat{m}\neq m}} p^{\tilde{1},\hat{m}}_{q^{\tilde{1},m}} q^{\tilde{1},\hat{m}} = 0\,.$$

Let Δq^m represent the change in output market m resulting from the merger. For market A, $\Delta q^A > 0$ if:

$$r^{\tilde{1},A}\Big|_{q^{1,A^*}} + \sum_{\theta=\theta^{1,1},\theta^{1,2}} \left(\theta^{\tilde{1},\tilde{1}} - \theta\right) r_{\theta}^{\tilde{1},A}\Big|_{q^{1,A^*}} - c^{\tilde{1},A} + \rho^{\tilde{1},B}_{q^{\tilde{1},B}} \Delta q^B + \sum_{m=B,C} \rho^{\tilde{1},m} > 0\,, \qquad \text{(A11)}$$

where $\rho^{i,m}$ is the marginal extra-market revenue for firm i from market m. (A11) says that the firm would increase output in market A if the sum of the effects of the change in quality on marginal revenue and on marginal extra-market revenue from market B is either positive or is less in absolute value than the additional marginal extra-market revenue from markets B and C. A corresponding analysis for market C would provide analogous results. For market B, $\Delta q^B > 0$ if:

$$2\left(r^{\tilde{1},B}\Big|_{Q^{B^*}} - c^{\tilde{1},B}\right) + \sum_{\theta=\theta^{1,1},\theta^{1,2},\theta^{2,2}} \left(\theta^{\tilde{1},\tilde{1}} - \theta\right) r_{\theta}^{\tilde{1},B}\Big|_{Q^{B^*}} + \sum_{m=A,C} \left(\rho^{\tilde{1},m} + \rho^{\tilde{1},m}_{q^{\tilde{1},m}} \Delta q^m\right) > 0\,,$$

where Q^{B*} is the pre-merger output in market B, which says that the firm would increase output in market B if the difference between its marginal revenue and marginal cost, valued at the pre-merger industry output for market B, times two (to compensate for the loss of the competitor), plus the pre-merger marginal extra-market revenues, was greater than the effect of the quality changes on marginal revenue and the changes in output in the individual markets on marginal extra-market revenue. The merger increases industry output if the sum of the changes in output across markets is greater than zero. This confirms Proposition 4.

Corollary 4C holds because, as the value customers place on the service in the common market increases, the effect of losing a competitor in the common market rises relative to the benefit of extra internalized network externalities in the individual markets. Losing the competitor allows the merged firm to lower output and raise price in the common market. When the common market is small relative to the non-common markets, the increased output in the non-common markets (from additional internalized network externalities) causes an increase in demand in the common market. When the common market is relatively small, this demand increase dominates the effect of losing the competitor. □

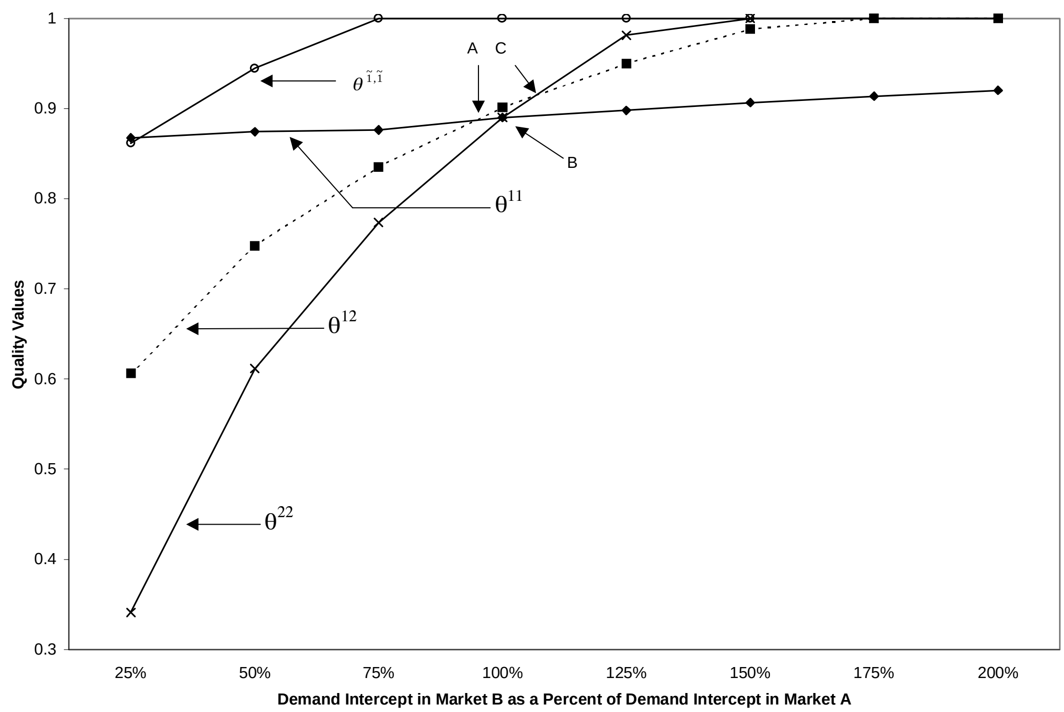

Figure 1. Pre-merger and Post-merger Quality Choices in the

All-Monopoly Setting

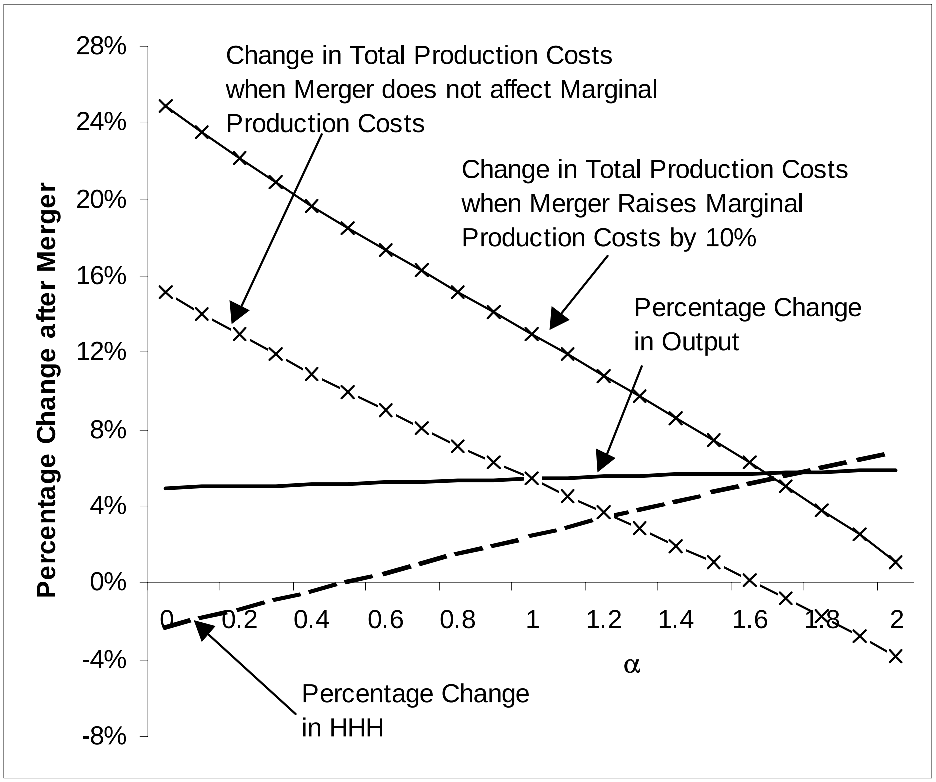

Figure 2. Effects Merger in Oligopoly Setting 1 Simulation on Net Consumer Surplus, Herfindahl-Hirschman index, and Costs

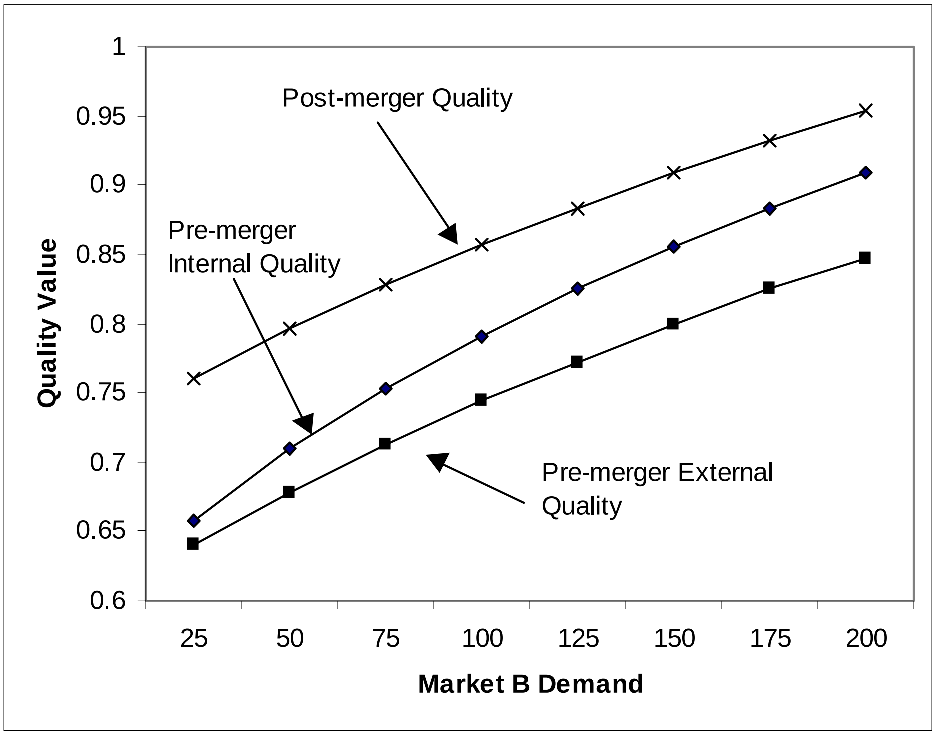

Figure 3. Pre-merger and Post-merger Quality Choices in Oligopoly Setting 2 Simulation

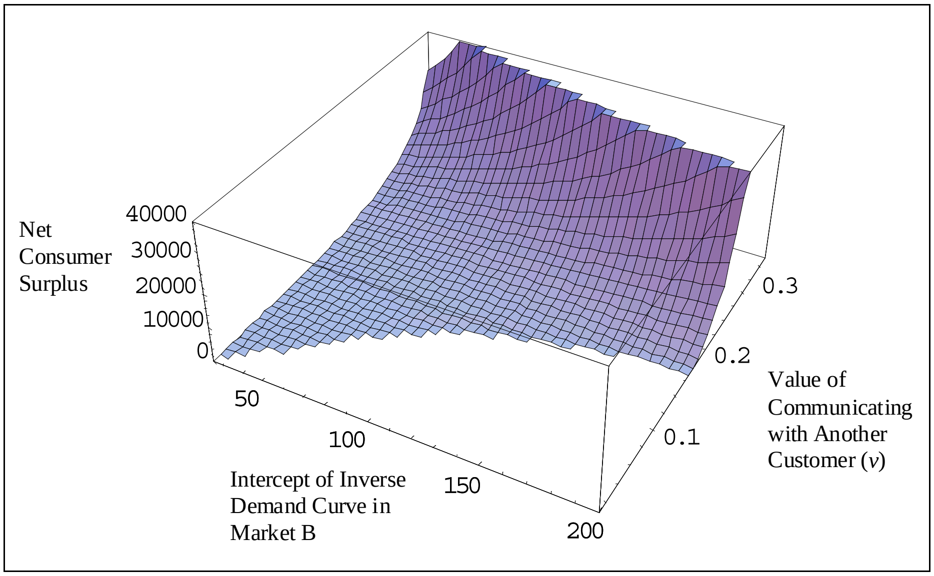

Figure 4. Effect of Mixed Merger on Net Consumer Surplus